Questions and Answers

Chapter 10

Questions

(Q1) *Summarize a series of procedures for multiscale enhancement filtering described in this chapter from input original images to output final filter-enhanced images.*

(Q2) *Explain the parameters involved in the procedures and discuss how to select these parameters.*

(Q3) *Derive the mathematical formula of the width response curves shown in Fig. 1.3.*

(Q4) *Discuss the effect of the anisotropic resolution (voxel shape) of input volume data on multiscale enhancement filtering.*

Answers

(A1) A series of procedures is described below.

Step 1 Sinc interpolation: Sinc interpolation is performed for an original input 3D image to obtain isotropic voxel sampling in the three directions. Let Δ be voxel intervals in the x- and the y-directions, and Δ_z be that in the z-direction. Assume that $\Delta < \Delta_z$, sinc interpolation in the z-direction described in Section 1.2.3 is performed so that the interval in the z-direction becomes Δ. Hereafter, we assume $\Delta = 1$ without loss of generality.

Step 2 Computation of filter response: Filter response $S_\xi\{f; \sigma_i\}$ at each scale σ_i is calculated in the following, where $\xi \in \{sheet, line, blob\}$, and $\sigma_i = s^{i-1}\sigma_1 (i = 1, 2, \ldots n)$.

Step 2.1 Normalized Gaussian derivative computation: Let $f[\vec{x}]$ be a 3D image, in which signal sampling is isotropic in all three directions. Here, $\vec{x} = (x, y, z)^\top$, and x, y, and z are integers. The normalized partial second derivative combined with Gaussian blurring for the 3D image $f[\vec{x}]$ is given by

$$fxx[\vec{x}; \sigma_i] = \sigma_i^2 f[\vec{x}] \otimes g_{xx}[\vec{x}; \sigma_i], \tag{1.77}$$

where $\otimes$ represents discrete convolution, and $g_{xx}[\vec{x}; \sigma_i] = \frac{\partial^2}{\partial_{i^2}}\mathrm{Gauss}(\vec{x}; \sigma_i)$. The above convolution can be decomposed into three separate one-dimensional convolution and the radius of the convolution kernel should be $5 \cdot \sigma_f$ as described in Section 1.2.3.

Step 2.2 Eigenvalue computation: The Hessian matrix at each voxel position $\vec{x}$ can be obtained from six 3D images of Gaussian second derivatives computed in the previous step, that is $f_{xx}[\vec{x}; \sigma_i]$, $f_{xy}[\vec{x}; \sigma_i]$, $f_{xz}[\vec{x}; \sigma_i]$, $f_{yy}[\vec{x}; \sigma_i]$, $f_{yz}[\vec{x}; \sigma_i]$, and $f_{zz}[\vec{x}; \sigma_i]$. The eigenvalues λ_1, λ_2, and λ_3 of the Hessian matrix are computed at each voxel position $\vec{x}$. Jacobi's method for eigenvalue computation of a symmetric matrix as found in "Numerical Recipes in C" can be used for this purpose.

Step 2.3 Filter response (similarity measure) computation: At each voxel position, $S_\xi\{f; \sigma_i\}$ (where $\xi \in \{sheet, line, blob\}$) is calculated using λ_1, λ_2, and λ_3 obtained in the previous step according to the formula of the similarity measures shown in Section 1.2.1.

Step 3 Multiscale integration: At each voxel position, n filter responses, $S_\xi\{f; \sigma_i\}(i = 1, 2, \ldots n)$, are obtained. The maximum response is selected as the final response out of the n filter responses.

(A2) The parameters involved in the multiscale computation (Steps 2 & 3 in A1), are σ_1, s, and n. The parameter values σ_1, s, and n are determined so that the width response curve (as shown in Fig. 1.3) covers the width range of anatomical structures of interest. The scale factor s is recommended to be $\sqrt{2}$ or 1.5 or smaller as analyzed in Section 1.3. The minimum scale σ_1 should not be smaller than 0.8 (voxels) so as not to be considerably affected by the discretization error, which are experimentally shown in [7].

The parameters involved in the filter response computation at each scale (Step 2.3 in A1), are γ_{st} and α. γ_{st} should be 0.5 or 1. When $\gamma_{st} = 1$, the response becomes more selective to a specific local structure but lower in branches and curves. $\alpha = 0.25$ is experimentally found to be appropriate in [7].

(A3) The normalized response of the sheet filter to the sheet model, $S_{sheet}\{h_{sheet}(X, \sigma_r); \sigma_f\}$, is written as

$$S_{sheet}\{h_{sheet}(X, \sigma_r); \sigma_f\} = \beta \cdot \left\{-\frac{d^2}{dx^2} G(x; \sigma_f)\right\} * h_{sheet}(X, \sigma_r), \quad (1.78)$$

where β is the normalization factor. The width response curve for the sheet structure is defined by $S_{sheet}\{h_{sheet}(\mathbf{0}, \sigma_r); \sigma_f\}$, where $\mathbf{0} = (0,0,0)$.

The sheet structure with variable width σ_r is modeled by

$$\begin{aligned} h_{sheet}(X, \sigma_r) &= \exp\left(-\frac{x^2}{2\sigma_r^2}\right) \\ &= \sqrt{2\pi}\sigma_r G(x; \sigma_r), \end{aligned} \quad (1.79)$$

where $G(x; \sigma_r)$ is the 1D Gaussian function. By combining Eqs. (1.78) and (1.79), we have

$$\begin{aligned} S_{sheet}\{h_{sheet}(X, \sigma_r); \sigma_f\} &= \beta \cdot \sqrt{2\pi}\sigma_r \left\{-\frac{d^2}{dx^2} G(x; \sigma_f) * G(x; \sigma_r)\right\} \quad (1.80) \\ &= \beta \cdot \sqrt{2\pi}\sigma_r \left\{-\frac{\partial^2}{\partial x^2} G\left(x; \sqrt{\sigma_f^2 + \sigma_r^2}\right)\right\}, \quad (1.81) \end{aligned}$$

in which we used the relation $G(x; \sigma_1) * G(x; \sigma_2) = G(x; \sqrt{\sigma_1^2 + \sigma_2^2})$. Since

$$\frac{d^2}{dx^2} G(x; \sigma) = \left\{\left(\frac{x^2}{\sqrt{2\pi}\sigma^5} - \frac{1}{\sqrt{2\pi}\sigma^3}\right) \exp\left(-\frac{x^2}{2\sigma^2}\right)\right\}, \quad (1.82)$$

the width response curve for the sheet is given by

$$S_{sheet}\{h_{sheet}(\mathbf{0},\sigma_r);\sigma_f\} = \frac{\beta\sigma_r}{\left(\sqrt{\sigma_f^2+\sigma_r^2}\right)^3} = \frac{\beta}{\sigma_f^2}\cdot\frac{\left(\frac{\sigma_f}{\sigma_r}\right)^2}{\left(\sqrt{(\frac{\sigma_f}{\sigma_r})^2+1}\right)^3}. \quad (1.83)$$

Thus, $S_{sheet}\{h_{sheet}(\mathbf{0},\sigma_r);\sigma_f\}$ is essentially a function of $\frac{\sigma_f}{\sigma_r}$ when $\beta=\sigma_f^2$, and its maximum is $\frac{2}{(\sqrt{3})^3}(\approx 0.385)$ when $\frac{\sigma_f}{\sigma_r}=\sqrt{2}$.

Using similar derivations, when the normalization factor $\beta=\sigma_f^2$ is multiplied, the width response curve for the line is given by

$$S_{line}\{h_{line}(\mathbf{0},\sigma_r);\sigma_f\} = \frac{\sigma_f^2\sigma_r^2}{\left(\sqrt{\sigma_f^2+\sigma_r^2}\right)^4} = \frac{\left(\frac{\sigma_f}{\sigma_r}\right)^2}{\left(\sqrt{(\frac{\sigma_f}{\sigma_r})^2+1}\right)^4}, \quad (1.84)$$

whose maximum is $\frac{1}{4}(=0.25)$ when $\frac{\sigma_f}{\sigma_r}=1$.

The width response curve for the blob is given by

$$S_{blob}\{h_{blob}(\mathbf{0},\sigma_r);\sigma_f\} = \frac{\sigma_f^2\sigma_r^3}{\left(\sqrt{\sigma_f^2+\sigma_r^2}\right)^5} = \frac{\left(\frac{\sigma_f}{\sigma_r}\right)^2}{\left(\sqrt{(\frac{\sigma_f}{\sigma_r})^2+1}\right)^5}, \quad (1.85)$$

whose maximum is $\frac{2}{3}(\sqrt{\frac{3}{5}})^5(\approx 0.186)$ when $\frac{\sigma_f}{\sigma_r}=\sqrt{\frac{2}{3}}$

(A4) Precisely speaking, the final filter responses can be modeled as a convolution of multiscale filter responses and the point spread function (PSF) of the imaging process, which is determined by the imaging scanner and imaging conditions. When the PSF is anisotropic, that is, the input volume data has anisotropic resolution (voxel shape), the final responses depend on the orientation of the local structure. Once the PSF of the imaging process is obtained, the effect of voxel anisotropy in the final responses can be analyzed systematically by taking the PSF of the imaging process into account using approaches similar to those described in Section 1.5.