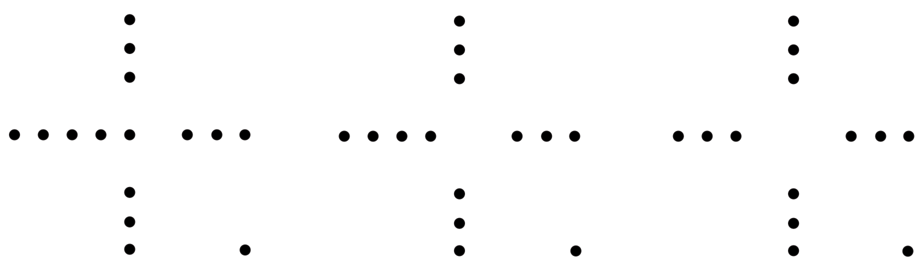
Questions and Answers

Chapter 12

Questions

(Q1) *Characterize Algorithm 1 according to category and interactivity level.*

The following figure and definitions are pertinent to Questions 2 to 6.



(a) (b) (c)

Figure 1: Three examples of a set of spels V; in each case the spels are dots in the plane and $V = T \cup L \cup R \cup B \cup \{o\}$, where T contains the top three dots, L contains the five (a), four (b), or three (c) horizontally-centered dots on the left, R contains the three horizontally-centered dots on the right, B contains the three vertically-centered dots on the bottom, and o is the dot on the bottom-right.

Assuming that the unit of length is such that the distance between the nearest distinct points in the Vs of Figure 1 is 1, we can define a fuzzy *spel affinity on V as any of the following:*

$$\psi(c,d) = \begin{cases} 0, & \text{if } c = d, \\ 1/\|c-d\|, & \text{otherwise,} \end{cases} \tag{1}$$

where $\|c-d\|$ is the Euclidean distance between the dots c and d,

$$\bar{\psi}(c,d) = \begin{cases} \psi(c,d), & \text{if } \|c-d\| \le 3, \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

and

$$\bar{\bar{\psi}}(c,d) = \begin{cases} 1/3, & \text{if } \|c-d\| \le 4, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

(Q2) *Are the sets V of Figure 1 ψ-connected? If not, why?*

(Q3) *Are the sets V of Figure 1 $\bar{\psi}$-connected or $\bar{\bar{\psi}}$-connected? If not, why?*

(Q4) *Consider the seeded 2-fuzzy graph $(V, \Psi, \mathcal{V})$ where V is the set (a) of Figure 1, $\Psi = (\psi, \psi)$, V_1 contains the leftmost spel of V and V_2 contains the rightmost spel of V. Compute the 2-segmentation σ using Theorem (as shown in Chapter 12, Volume II).*

(Q5) *Does the 2-segmentation σ change if we use $\Psi = (\psi, \bar{\bar{\psi}})$?*

(Q6) *Is $(V, (\psi, \bar{\psi}), \mathcal{V})$, where V_1 contains the leftmost spel of V and V_2 contains the rightmost spel of V, a connectable 2-fuzzy graph for any of the sets of Figure 1?*

(Q7) *What does the concept of blocking of chains mean?*

(Q8) *Why should one use the fcc grid instead of the traditional sc (cubic) grid?*

(Q9) *Suppose that the fuzzy spel affinities defined for a specific application can only assume values from a small set (around 1000 elements). Discuss an alternative data structure for implementing the algorithm more efficiently.*

The following definitions are pertinent to Questions 10 and 11.
Using the notation of this chapter, the Relative Fuzzy Connectedness (RFC)[1] *of (as shown in Chapter 12, Volume-II) defines a 2-segmentation*

[1] The definitions of RFC and IRFC of [27] are restricted to 2-fuzzy graphs where $\psi_1 = \psi_2$ with a single seed spel per object.

as follows. For $1 \le m \le 2$ and for any $c \in V$, let μ_m^c denote the ψ-strength of the strongest chain from (the unique element of) V_m to c. Then, let

$$\sigma_1^c = \begin{cases} \mu_1^c, & \text{if } \mu_1^c > \mu_2^c, \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

$$\sigma_2^c = \begin{cases} \mu_2^c, & \text{if } \mu_1^c \le \mu_2^c, \\ 0, & \text{otherwise,} \end{cases} \tag{5}$$

and $\sigma_0^c = max\left\{\sigma_1^c, \sigma_2^c\right\}$ for all $c \in V$.

The Iterative Relative Fuzzy Connectedness (IRFC) of [27] produces a sequence ${}^0\psi_2, {}^1\psi_2, \ldots$ of spel-adjacencies and a sequence of ${}^0\sigma, {}^1\sigma, \ldots$ of 2-segmentations defined as follows. ${}^0\psi_2 = \psi$ and ${}^0\sigma$ is the 2-segmentation defined by RFC. Now assume that, for some $i > 0$, we have already obtained ${}^{i-1}\psi_2$ and ${}^{i-1}\sigma$. For all $c, d \in V$, we define

$${}^i\psi_2(c, d) = \begin{cases} 1, & \text{if } c = d \\ 0, & \text{if } {}^{i-1}\sigma_1^c > 0 \text{ or } {}^{i-1}\sigma_1^d > 0, \\ \psi(c, d), & \text{otherwise.} \end{cases} \tag{6}$$

Then ${}^i\sigma$ is defined just as σ is defined in RFC using (4) and (5), but with μ_m^c replaced by ${}^i\mu_m^c$ everywhere. Whenever ${}^i\sigma = {}^{i-1}\sigma$, then that 2-segmentation is considered to be the final output of IRFC.

(Q10) *Consider the seeded 2-fuzzy graph $(V, \Psi, \mathcal{V})$ where V is the set (c) of Figure 1, $\Psi = (\psi, \psi)$, V_1 contains the leftmost spel of V and V_2 contains the rightmost spel of V. Compute the 2-segmentations σ using Theorem (as shown in Chapter 12, Volume-II) and RFC and compare them.*

(Q11) *Consider the seeded 2-fuzzy graph $(V, \Psi, \mathcal{V})$ where V is the set (a) of Figure 1, $\Psi = (\psi, \psi)$, V_1 contains the leftmost spel of V and V_2 contains the bottommost spel of B. Compute the 2-segmentations σ using Theorem (as shown in Chapter 12, Volume-II) and IRFC and compare them.*

Answers

(A1) Algorithm (as shown in Chapter 12, Volume-II) is a region growing algorithm since it starts with preselected seed points forming the initial regions that grow according to the fuzzy spel affinities, and it is a semi-automatic algorithm because the user provides input in the form of seed spels.

(A2) Yes, since there are chains with ψ-strength greater than 0 connecting all elements in the sets (a), (b) and (c).

(A3) None of the sets (a), (b) and (c) are $\bar{\psi}$-connected. In all three sets the spel o is not $\bar{\psi}$-connected to the rest of the set since its closest neighbors are at a distance of 4 units, and thus, $\bar{\psi}(o, d) = 0$, for all $d \in V$. The sets (a), (b) and (c) are $\bar{\bar{\psi}}$-connected since there are chains with $\bar{\bar{\psi}}$-strength greater than 0 connecting all elements in the sets (a), (b) and (c).

(A4) The 2-segmentation σ computed using Theorem (as shown in Chapter 12, Volume-II) is shown in Figure 2.

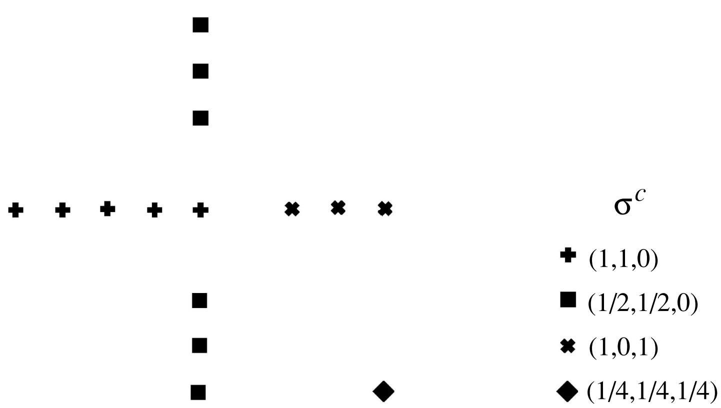


Figure 2: Answer to Question 4.

(A5) 2-segmentation σ computed using Theorem (as shown in Chapter 12, Volume-II) is shown in Figure 3.

(A6) No, $(V, (\psi, \bar{\psi}), \mathcal{V})$ is not connectable for any of the sets in Figure 1 because they are not $\bar{\psi}$-connected (see Question 3).

(A7) It means that the strongest chains connecting a seed spel of an object to any other spel of that object must be fully contained within the object.

(A8) The fcc grid is superior to the sc grid because it samples the 3D space more efficiently, and thus the fcc grid can represent a 3D image with

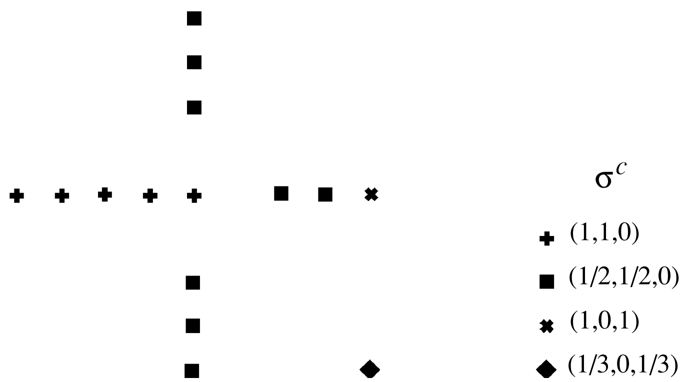


Figure 3: Answer to Question 5.

the same accuracy as that of the sc grid but using fewer grid points. Another advantage of using the fcc grid over the sc grid is that if we have an object that is a union of Voronoi neighborhoods of the fcc grid, then for any two faces on the boundary between this object and the background that share an edge, the normals of these faces make an angle of 60° with each other. This results in a less blocky image than if we used a surface based on the cubic grid with voxels of the same size.

(A9) Due to the restricted number of values $({}^1r,\ {}^2r, \dots, {}^{|R|}r)$ that the fuzzy affinity functions return, instead of using a heap to maintain a partial order of the spels, one can use an array with $|R|$ positions in which each element contains M linked lists of spels. Such lists contain the spels c for which the strongest chains (and thus σ_0^c) from a seed spel in V_m, for $1 \le m \le M$ has (currently) the value ${}^ir$, for $1 \le i \le |R|$. Then, Step 11 can be replaced by the instruction "**for** $l \leftarrow 1$ **to** $|R|$" and Steps 25–26 are then executed during Step 11 (moving to the next element of the array). Moreover, now insertions and updates of values of σ^c only take a constant number of operations since no heap maintenance is required.

(A10) The 2-segmentation σ computed using Theorem (as shown in Chapter 12, Volume-II) (a) and the one computed using RFC (b) (see (4) and (5)) are shown in Figure 4.

(A11) The 2-segmentation σ computed using Theorem (as shown in Chapter 12, Volume-II) (a) and the ones (${}^{0}\sigma$ and ${}^{1}\sigma$) computed using IRFC (b) (see (4), (5) and (6)) are shown in Figure 4.

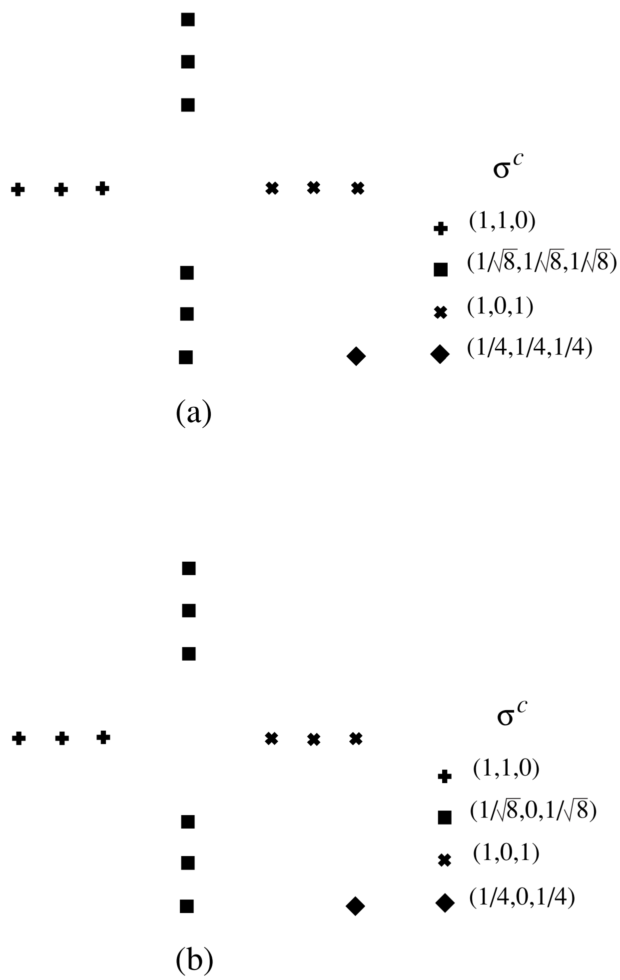


Figure 4: Answer to Question 10.

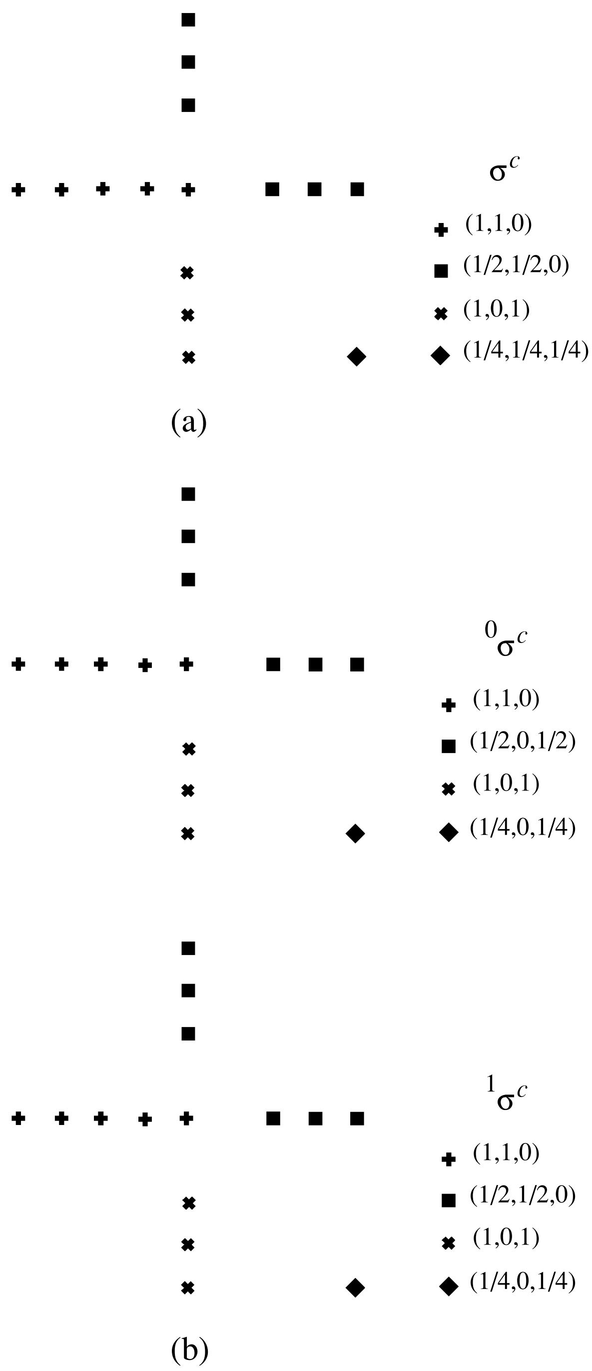


Figure 5: Answer to Question 11.