Questions and Answers

Chapter 3

Questions

(Q1) *Describe the concept of image registration.*

(Q2) *Describe the steps for voxel-based image registration.*

(Q3) *Given two images A and B, describe how to compute their mutual information value.*

(Q4) *Describe three visual inspection methods for the evaluation of prostate registration.*

(Q5) *Describe the quantitative evaluation methods for the prostate and pelvic registration*

(Q6) *Given two sets of corresponding control points, describe how to compute the thin plate transformation.*

Answers

(A1) Image registration is the process of aligning images and relating correspondence features of spatial information, which is fundamental to medical image interpretation and analysis.

(A2) There are four steps for voxel-based image registration. First, transform one image with respect to the other. For two-dimensional image, the transformation parameters are 2 translations and 1 rotation. For three-dimensional image, the parameters are 3 translations and 3 rotations. Second, after transform an image, perform image interpolation in order to get the new transformed image. Interpolation methods include linear interpolation or sinc interpolation. Third, compute the similarity between the reference image and the transformed image. The similarity measure can be correlation coefficient or mutual information. Finally, check if the two images are matched. Actually, this is an optimization procedure. If not, go back to the first step and repeat the procedure until meet the ending criteria.

(A3) In the equation to compute the mutual information, MI(A,B) is the mutual information between images A and B, the small a is the intensity value of image A, the small b is the intensity value of image B. PAB is the joint probability distribution of image A and B. PA is the marginal probability distribution of image A. PB is the marginal probability distribution of image A. Given two images, first compute their histograms and use the normalized histogram to approximate the marginal probability distribution. Then, compute the joint histogram and use the normalized joint histogram to approximate the joint probability distribution. Plug into this equation and get the mutual information value.

(A4) First, manually segment prostate boundaries in image slices and copy them to corresponding slices from the other volume. This enables visual determination of the overlap of prostate boundaries over the entire volume. Second, color overlay displays can be used to evaluate overlap of structures. One image is rendered in gray and the other in the "hot—iron" color scheme available in IDL. To visualize potential differences, it is quite useful to interactively change the contribution of each image using the transparency scale. Third, use a sector display, which divided the reference and registered images into rectangular sectors and created an output image by alternating sectors from the two input images. Even subtle shifts of edges could be clearly seen.

(A5) First, evaluate the registration of the pelvis by measuring the displacement of bony landmarks following registration. There are six easily found bony landmarks consisting of 2 great sciatic notches, 2 lesser sciatic notches, the pubic symphysis, and the coccyx. Following registration, calculate the root-mean-squared (RMS) distance over the six landmarks. Second, to test the dependency of registration on algorithmic features such as image cropping, one can compare transformation parameters. However, one can choose a more meaningful approach that consist of finding the average displacement of voxels in a region-of-interest (ROI). The 3D distances between transformed voxels are calculated in millimeters and averaged over a cubic ROI just covering the prostate. Third, measure potential displacements of the 3D centroid as obtained from manually segmented prostates.

(A6) First, let $P_1 = (x_1, y_1, z_1)$, $P_2 = (x_2, y_2, z_2), \ldots, P_n = (x_n, y_n, z_n)$ be n control points in the image coordinate of the *reference* volume. Write $r_{ij} = |P_i - P_j|$ for the distance between point i and j. We define matrices:

$$P = \begin{bmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ \ldots & \ldots & \ldots & \ldots \\ 1 & x_n & y_n & z_n \end{bmatrix}, \text{n} \times 4;$$

$$K = \begin{bmatrix} 0 & r_{12} & r_{13} & \ldots & r_{1n} \\ r_{21} & 0 & r_{23} & \ldots & r_{2n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ r_{n1} & r_{n2} & r_{n3} & \ldots & 0 \end{bmatrix}, \text{n} \times \text{n};$$

and

$$L = \begin{bmatrix} K & P \\ P^T & O \end{bmatrix}, (\text{n} + 4) \times (\text{n} + 4);$$

where T is the matrix transpose operator and O is a 4×4 matrix of zero.

Second, let $Q_1 = (u_1, v_1, w_1)$, $Q_2 = (u_2, v_2, w_2), \ldots, Q_n = (u_n, v_n, w_n)$ be n corresponding control points in the image coordinate of the *floating* volume. We get matrices:

$$V = \begin{bmatrix} u_1 & u_2 & \ldots & u_n \\ v_1 & v_2 & \ldots & v_n \\ w_1 & w_2 & \ldots & w_n \end{bmatrix}, 3 \times \text{n},$$

$$Y = (V|0 \quad 0 \quad 0 \quad 0)^T, 3 \times (\text{n} + 4),$$

and define the vector $W = (w_1, w_2, \ldots, w_n)$ and the coefficients $\alpha_1, \alpha_x, \alpha_y$ and α_z by the equation

$$L^{-1}Y = (W|\alpha_1 \quad \alpha_u \quad \alpha_v \quad \alpha_w)^T.$$

Third, use the elements of $L^{-1}Y$ to define a function $f(u', v', w')$ everywhere in the entire volume:

$$f(u', v', w') = \alpha_1 + \alpha_u u + \alpha_v v + \alpha_w w + \sum_{i=0}^{n} w_i |P_i - (u, v, w)|.$$