Questions and Answers

Chapter 10

Questions

(Q1) *What is image registration?*

(Q2) *Why is the algorithm-based registration still required given the success of hybrid imaging devices?*

(Q3) *What is cross-entropy? What is reversed cross-entropy? What is symmetric divergence?*

(Q4) *Why the cross-entropy is a generalized similarity measure for image registration?*

(Q5) *When is cross-entropy (reversed cross-entropy, symmetric divergence) maximized? When is it minimized?*

(Q6) *How can one get a favorable priori pdf?*

(Q7) *What are common unfavorable priori pdf's?*

(Q8) *What are the general steps for a similarity-based registration?*

(Q9) *Why does cross-entropy, reversed cross-entropy, and symmetric divergence based registration suffer from numerical instability? How to deal with it?*

(Q10) *What are the requirements for a good similarity measure?*

Answers

(A1) Image registration is a process that quantitatively relates the information in one image to that in another image by determining a one-to-one transformation between coordinates in the two image spaces.

(A2) In many cases, hardware registration is impractical or impossible and one must rely on software-based registration techniques. For example, when monitoring treatment effectiveness over time, software image registration is necessary since the single or multi-modality images are acquired at different times. In addition, applications involving inter-subject or atlas comparisons require software registration since the images originate from different subjects. Other applications for software registration include the correction of motion that occurs between sequential transmission and emission scans in PET and SPECT as well as the positioning of patients with respect to previously determined treatment plans. The need to offer multiple different combinations of imaging modalities (i.e., PET/MR, SPECT/MR, PET/CT, etc.) would be impractical. As most researchers agree, the hybrid devices will likely play a major role primarily in radiation oncology.

(A3) Cross-entropy, also known as Kullback-Leibler divergence, is an information-theoretic measure that quantifies the difference between two probability density functions. It's mathematic definition is given in Eq. 1. For reversed cross-entropy, the roles of two pdf's are reversed and its mathematic definition is given in Eq. 2. Symmetric divergence is the sum of cross-entropy and reversed cross-entropy.

(A4) Cross-entropy maximization degenerates to mutual information maximization when the priori pdf is the product of two marginal pdf's, conditional entropy minimization when the priori pdf is proportional to one of the marginal pdf's, or joint entropy minimization when the priori pdf is a uniform distribution.

(A5) When a favorable (desirable or likely) priori pdf is given, an estimate of the true pdf can be found by minimizing the cross-entropy since we want our estimated pdf at registration to be as close to the priori as possible. If an unfavorable priori pdf is given, an estimate of the true pdf can be obtained by maximizing the cross-entropy since we want our estimated pdf at registration to be different from the priori as much as possible. The same would be true when reversed cross-entropy and symmetric divergence are used as similarity measures for image registration. They can be maximized and minimized, which is an unusual property for similarity measures. If more than one likely and unlikely priori pdf is available, one can define cross-entropy, reversed cross-entropy, and symmetric divergence for them and maximize or minimize them collectively.

(A6) A favorable pdf can be computed based on previous registration results (see Leventon *et al.*, Volume III, Chapter 10). Theoretical analysis can also provide information regarding a favorable priori. For example, the voxel values in images of the same modality and of the same patient are linearly related and it has been shown that MR image can be used to simulate a PET image (see Andersson *et al.*, Volume III, Chapter 10).

(A7) Unfavorable pdfs can be uniform, proportional to one of the marginal pdf's, or the product of two marginal pdf's.

(A8) The general steps are illustrated in Fig. 1. The floating image will undergo rotation and translation to match the reference image. Before the automatic registration starts, an initial set of registration parameters must be set and the floating image is appropriately transformed (i.e. rotated and translated). The similarity measures between the reference image and the transformed floating image are then computed. If the similarity number is not optimal, the registration parameters are updated, otherwise the registration process stops and the optimal registration parameters are output. The scheme to update the registration parameters is determined by the optimization algorithm employed.

(A9) The instability is caused when the priori or joint probability is zero. Ideally one could sample a large dataset to get a better estimate of the priori or use sophisticated sampling schemes to better estimate the joint probability. Alternatively one can use the simple, heuristic approach developed in this chapter. There is no instability for the cross-entropy maximization. The zero pdf's can be ignored in the cross-entropy minimization. For the reversed cross-entropy maximization, the zero joint pdf's can be substituted with a small positive number. For the reversed cross-entropy minimization, the zero pdf's can be ignored. The symmetric divergence has the cross-entropy and reversed cross-entropy components and can be treated accordingly.

(A10) A good similarity measure shall have a peak or valley around the true registration. The similarity profile in a hyperspace around the true registration shall be smooth. The second requirement makes sure the optimization process is easy, and the first requirement ensure that the optimized solution is acceptable. The cross-entropy, reversed cross-entropy, and symmetric divergence discussed in this chapter all satisfy these two requirements.