Questions and Answers

Chapter 12

Questions

(Q1) *How are the principles of continuum mechanics used to regularize the deformable image registration problem involving the deformation of a template image into alignment with a target image? What are the primary advantages of this approach to regularization of the deformable image problem in comparison to ad-hoc methods?*

(Q2) *What is the purpose of the regularization term W in the deformable image registration problem?*

(Q3) *What is meant by treating the image data as a "hard constraint" in the deformable image registration problem?*

(Q4) *In hyperelastic Warping, in the limit as the penalty parameter A, $\lambda \to \infty$, the image-based energy converges to a finite value. Explain.*

(Q5) *Treating the image data as a hard constraint may cause the stiffness matrix to become ill-conditioned. How does the augmented Lagrangian method solve this problem?*

(Q6) *What is the role of the stiffness quantities in the solution procedure?*

(Q7) *How is sequential low pass filtering used in hyperelastic Warping to keep from converging to local minima in the solution?*

(Q8) *When using a regular mesh for hyperelastic Warping, why is rezoning needed?*

(Q9) *How is mechanical stress calculated with hyperelastic Warping?*

Answers

(A1) A continuum mechanics-based approach models the template image as a deformable continuum that is analogous to a physical material. This method generates a one-to-one correspondence between template and target images and, with the use of appropriate constitutive models, is objective for arbitrarily large deformations and rotations.

(A2) Deformable image registration presents an ill-posed problem, which is solved by minimizing a potential energy cost function. Without regularization, this problem admits multiple solutions. W is the regularization term of the potential energy that constrains the solution space. This does not necessarily eliminate the possibility of multiple solutions, but it constrains the solution to provide solutions with some desirable quality (i.e., one-to-one mapping).

(A3) For hyperelastic Warping, the energy function $E = \int_\beta W(X, C)\frac{dv}{J} - \int_\beta U(T(X), S(\varphi))\frac{dv}{J}$ combines the effect of image data and the effect of mechanics. In order to reach a solution, a hard constraint of the image data is applied with the use of a penalty parameter in the image energy density functional. The mechanics is used to regularize the problem. By contrast, a solution could be reached for the image deformation problem by using the image data as a soft constraint and using mechanics to drive the solution.

(A4) The image energy density functional is given by $U(X, \varphi) = \frac{\lambda}{2}(T(X) - S(\varphi))^2$. As the penalty parameter $\lambda \to \infty$, $(T(X) - S(\varphi))^2 \to 0$, and the image energy converges to a finite value.

(A5) As the penalty parameter λ is increased, some of the diagonal terms in the stiffness matrix $\mathbf{K}_1$ become very large with respect to others, leading to numerical ill-conditioning of the matrix. Augmented Lagrangian methods use a small penalty parameter λ to generate an initial solution at each computational timestep, then incrementally increase the image-based body force in an iterative loop.

(A6) The image stiffness is given $\mathbf{k} := \partial^2 U/\partial_\varphi \partial_\varphi$ and the regularization stiffness is $\mathbf{D} := d^2 W/\partial_\varphi \partial_\varphi$. These 2nd derivative terms (Hessians) describe how small perturbations of the current configuration affect the contributions of W and U to the overall energy of the system.

(A7) By using sequential low pass filtering, fine textural details are reduced and the image is first registered to larger image features. The cut-off frequency of the spatial frequency is then changed over computational time to remove the spatial filter and attain registration of fine textural detail after global registration is achieved.

(A8) Regular meshes are subject to element inversion during the solution process due to the fact that they often cross large intensity gradients in the image data and thus are subjected to large distortional forces during the registration process. A rezoning algorithm resets the finite element mesh and interpolates the current results on the reset mesh, avoiding element inversion and thus allowing the image registration process to continue.

(A9) The template image is modeled as a deformable continuum with material properties defined via a hyperelastic strain energy function. Material properties combined with strain information from the finite element deformation yield stress distributions for the deformed image.