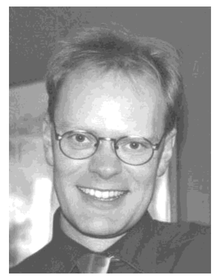
Modelling and monitoring epidemics by means of spatio-temporal lattices



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There are many infectious diseases — viral (e.g. flu, AIDS, SARS), bacterial (e.g. anthrax, cholera, salmonellosis) and para-sitic (e.g. sleeping sicknes, malaria, scabies) — in need of accurate control in order to take justified preventive action in various situations.

Model

For properly adressing the main characteristics of the problem of detecting a sudden break-out of some infectious disease, a model with binary states (”ill” coded as 0 and ”healthy” coded as 1 according to some standard of dichotomization) according to a simple Markov chain Markov field (Guyon, 1995) is considered. In this model, the local states interact directly causing global features like phase transition in contrast to multivariate models where states interact via a covariance matrix. For spatial interaction between neighbouring states, x and y, a pair-potential energy function is $1 - xy$ used. For time intertia a temporal pair-potential, and for skewness an external field are added to the total energy component. In full explicy the random field process is

$X = \{X_{i,t} : i \in A, t \in \mathbb{Z}^+\}$

where A is a generalised lattice (i.e. a set of sites) and

$p(x_t|x_{t-1}) \propto \exp(\phi Q_t + \psi R_t + \lambda S_t)$

where the spatial energy is

$Q_t = \sum_{(i,j)\in\partial_t} x_{i,t}x_{j,t}$

the temporal energy is

$R_t = \sum_i I(x_{i,t} = x_{i,t-1})$

and the external field is

$S_t = \sum_i x_{i,t}$

(sufficient statistics for the parameters ϕ, ψ and λ respectively.)

Dynamic neighbourhoods

As time varies, the individuals under observation may change their location. This is implemented into the model as a *neighbourhood process*

$\partial_t = \{(i,j) : i,j \in A, i \overset{t}{\sim} j\}$

meaning that sites i and j are neighbours at time t if the pair (i, j) is an element of the set ∂_t. The process $\{\partial_t\}$ is modelled as a Markov chain.

Semi-perfect simulation

At time t it is possible to simulate a pattern, X_t, perfectly (Propp and Wilson, 1996) conditional on the pattern at time *t-1* and the neighbourhood configuration at time t. In this sense it is possible to simulate sequences of the process “semi-perfectly” by sampling after a run-in period.

Sudden change

A sudden break-out of an epidemic is modelled by an abrupt change of the value of the parameter ϕ for spatial interaction at a random time-point τ. Thus the model is

$$p(x_t|x_{t-1},\partial_t) \propto \begin{cases} \exp(\phi_0 Q_t + \psi R_t + \lambda S_t) & t < \tau \\ \exp(\phi_1 Q_t + \psi R_t + \lambda S_t) & t \geq \tau \end{cases}$$

The problem is to descide when a change has occurred.

Monitoring

To solve the problem of detecting a change, quickly and accurately, a stopping rule

$T = \min\{t : a_t > C\}$

is defined where a_t is called *alarm function* and *C* is called *threshold*. It is desired that this stopping rule is constructed so that the probability of “false alarm” is low, or correspondingly that

$\mathrm{ARL}^0 = E(T|\tau > T)$

is large. At the same time it is also desired that the probablity of “motivated alarm” is high, or correspondingly that the expected delay

$E(T-\tau|\tau \leq T)$

is low. Usually methods of monitoring are evaluated by calibrating the threshold so that some particular ARL^0 is achieved for all methods and then e.g. the expected delay may be used to compare the methods. In this project two of the most commonly used methods are considered, the *Cusum method* and the *Shiryaev-Roberts method.* In the case with the spatio-temporal model considered, the implementation of these stopping rules is very difficult. Not only do they have to be derived in the case of this highly structured model but in order to be of any practical use, the methods need to be expressed recursively for turning into program code.

A *Cusum method* is defined by letting

$a_t = \log c_t + \left(\log \frac{d_t}{c_{t-1}} + a_{t-1}\right)^+$

A *Shiryaev-Roberts method* is defined by

$a_t = c_t\left(1 + \frac{d_t a_{t-1}}{c_{t-1}}\right)$

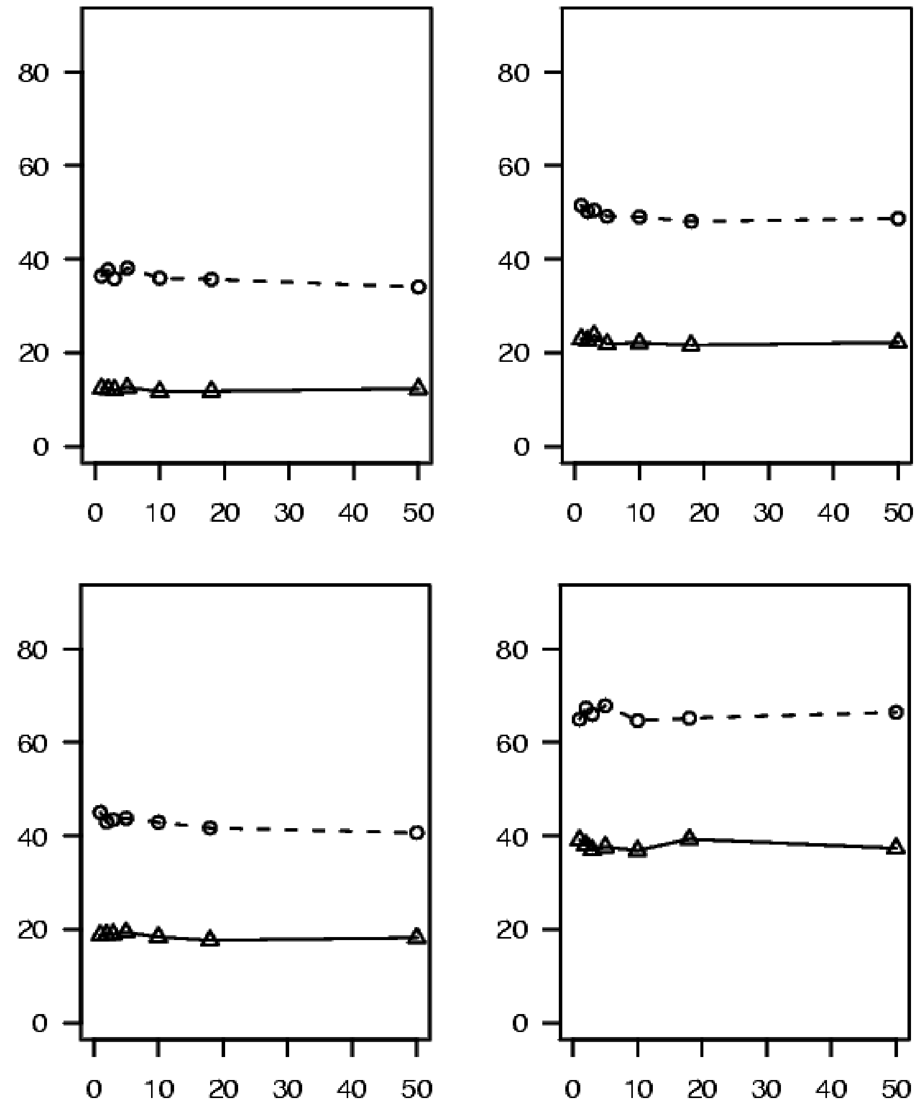
where

$$c_t = \frac{p(q_t|x_{t-1},\partial_t;\phi_1)}{p(q_t|x_{t-1},\partial_t;\phi_0)}$$

$$d_t = \prod_{i\in A} \frac{p(x_{i,t-1}|\{x_{j,t-1}: j \overset{t-1}{\sim} i\},\partial_{t-1},x_{t-2};\phi_0)}{p(x_{i,t-1}|\{x_{j,t-1}: j \overset{t-1}{\sim} i\},\partial_{t-1},x_{t-2};\phi_1)}$$

Evaluation

For $\mathrm{ARL}^0 = 100$ we have the following values of expected delay of the Cusum method for some parameter values.



Remaining

- Conditions for stationarity.
- Empirical evaluation.
- Software development.

References

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