Spatio-temporal geostatistical analyses of runoff and precipitation

Jon Olav Skøien and Günter Blöschl
Institute of Hydraulic and Water Resources Engineering,
Vienna University of Technology

Abstract

Traditionally, interest has resided in the stochastic analyses and filtering of runoff time series, keeping catchment size fixed. We propose that catchment size can be viewed as the spatial support and a mean runoff time as the temporal support of runoff measurements. We are then better able to explain the variance reduction of runoff from increasing catchment size. We suppose that the proposed concepts will provide a better understanding of hydrological space-time variability as needed in regionalisation and ungauged catchment problems.

Method

Spatio-temporal variograms:

Three alternative models, all of them accounting for non-stationarity:

1. Exponential model (non-separable) $_{1st}(h_s,h_t) \quad a_1 h_s^b h_t^c (1 \quad \exp(\quad ((d_1 h_t \quad h_s)/e_1)^{f_1}))$
2. Cressie-Huang (1999) model (Non-separable) $_{st}(h_s,h_t) \quad a_2 h_s^b h_t^c \; 1 \quad \frac{1}{(d_2 h_t \quad 1)} \exp \quad \frac{e_2^2 h_s^2}{d_2 h_t \quad 1}$
3. Product-sum model (separable) $_{st}(h_s,h_t) \quad h_s^b h_t^c \cdot {}_{st}(h_s,0) \quad {}_{st}(0,h_t) \quad k \quad {}_{st}(h_s,0) \; {}_{st}(0,h_t)$
   (De Cesare et al., 2001)

One dimensional variogram:

$$(h) \quad a(1 \quad \exp(\quad (h/e)^f))$$

Linear spatio-temporal aggregation - extended from Skøien et al. (2003):

$$_{st}(h_s|S,h_t|T) \quad \int_0^{R\max}\int_0^{T\max} {}_{st}(r, \;) \; f_{2s}(r|(h_s,S)) f_{2t}(\; |(h_t,T)) drd \quad \int_0^{R\max}\int_0^{T\max} {}_{st}(r, \;) \; f_{1s}(r|S) f_{1t}(\; |T) drd$$

Temporal support:

$T \qquad S$

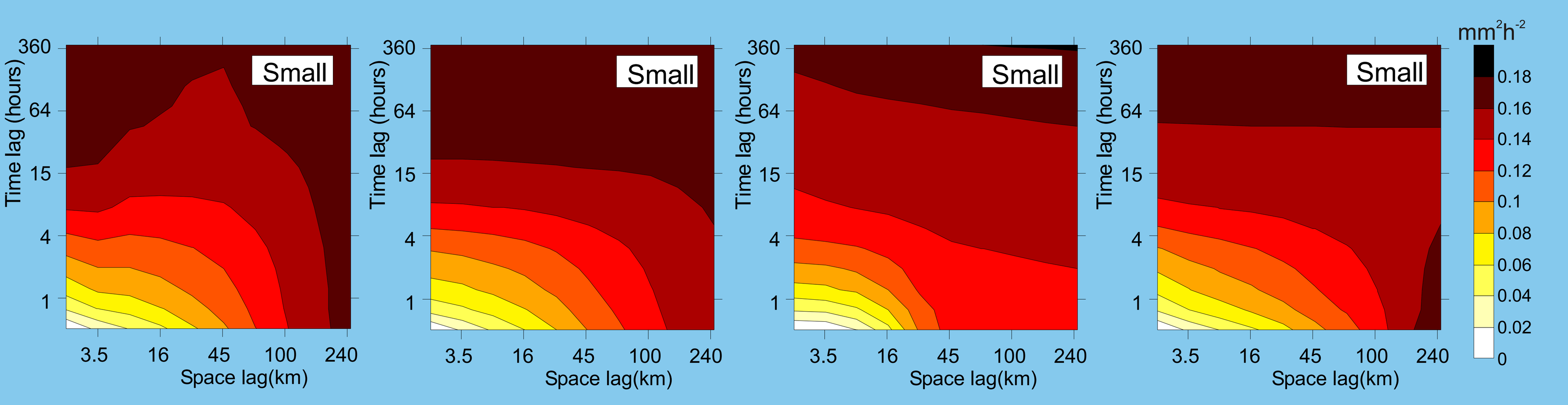
Notation:

$_{st}$ are spatiotemporal variograms
h_s, h_t are spatial and temporal lags, respectively
S, T are spatial support (square root of area) and temporal support, respectively
f_{1s}, f_{1t}, f_{2s}, and f_{2t} are pdfs of spatial (*s*) and temporal (*t*) distances. 1 is within a catchment/time interval, while 2 is between catchments/time intervals
Rmax and Tmax are maximum spatial or temporal separation
a-f are parameters to be fitted

References
Cressie, N., and Huang, H.-C. (1999) Classes of nonseparable, spatio-temporal stationary covariance functions, *J. Am. Stat. Ass.* 94, 1330-1340
De Cesare, L., Myers, D. E. and Posa, D. (2001) Estimating and modelling space-time correlation structures, *Stat. & Prob Lett* 51, 9-14
Skøien, J.O., Blöschl, G., and Western, A. W. (2003) Characteristic space-time scales in hydrology. Water Resources Research 39(10), 1304

Fitting of spatio-temporal variogram models

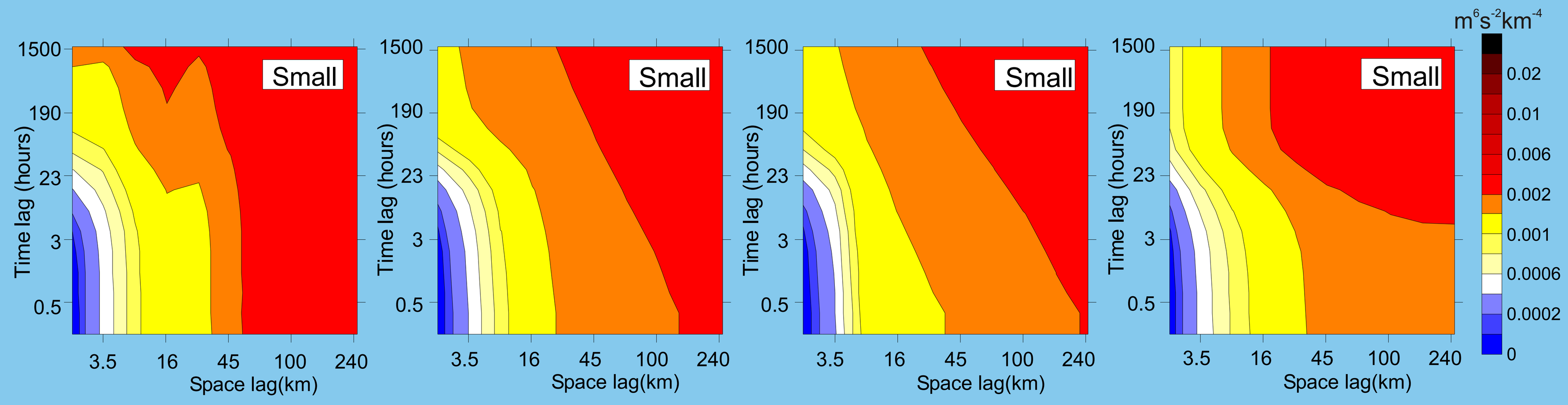
Precipitation



Objective function from variogram models fitted independently to spatio-temporal sample variograms of precipitation

Variogram model	Small catchments	Medium catchments	Large catchments	Total
Exponential model	0.66	0.48	0.79	1.93
Cressie-Huang model	13.8	15.0	13.0	41.8
Product-sum model	0.40	0.40	0.63	1.43

Runoff

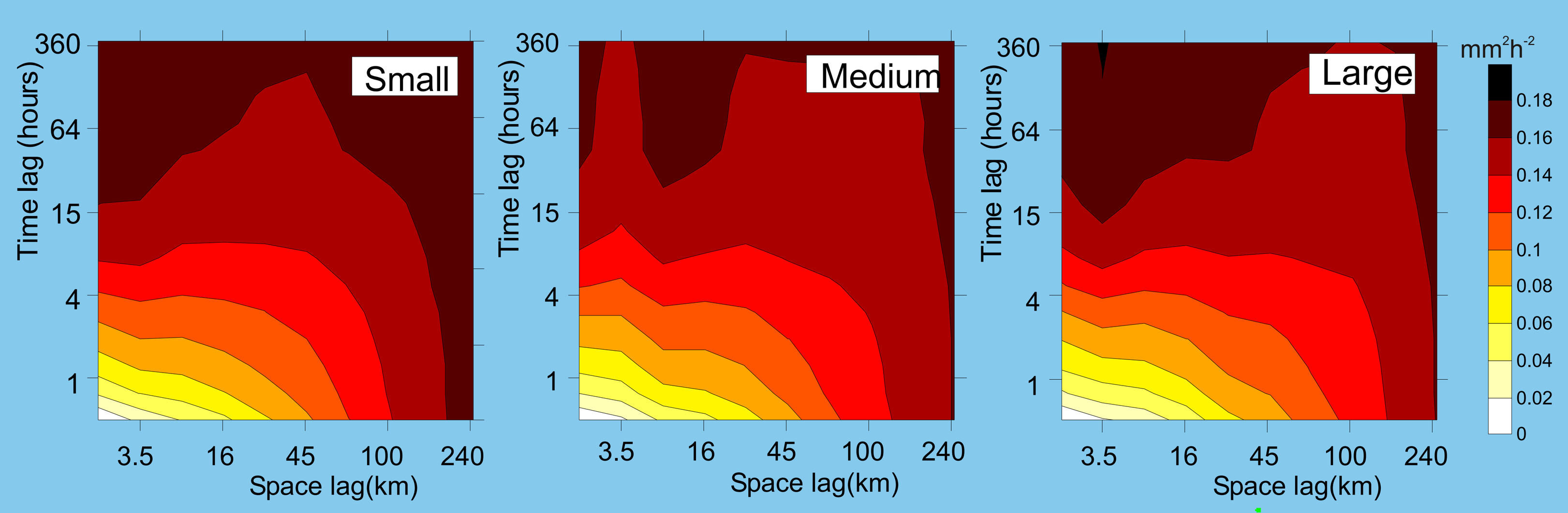


Objective function from variogram models fitted independently to spatio-temporal sample variograms of runoff

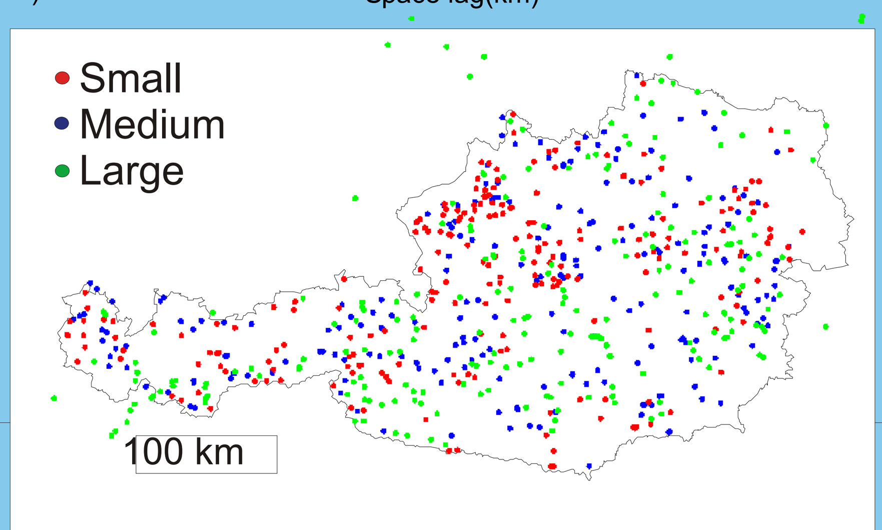
Variogram model	Small catchments	Medium catchments	Large catchments	Total
Exponential model	4.8	6.7	4.6	16.1
Cressie-Huang model	7.5	17.8	24.1	49.4
Product-sum model	9.1	7.2	6.9	23.2

Spatio-temporal sample variograms

Precipitation



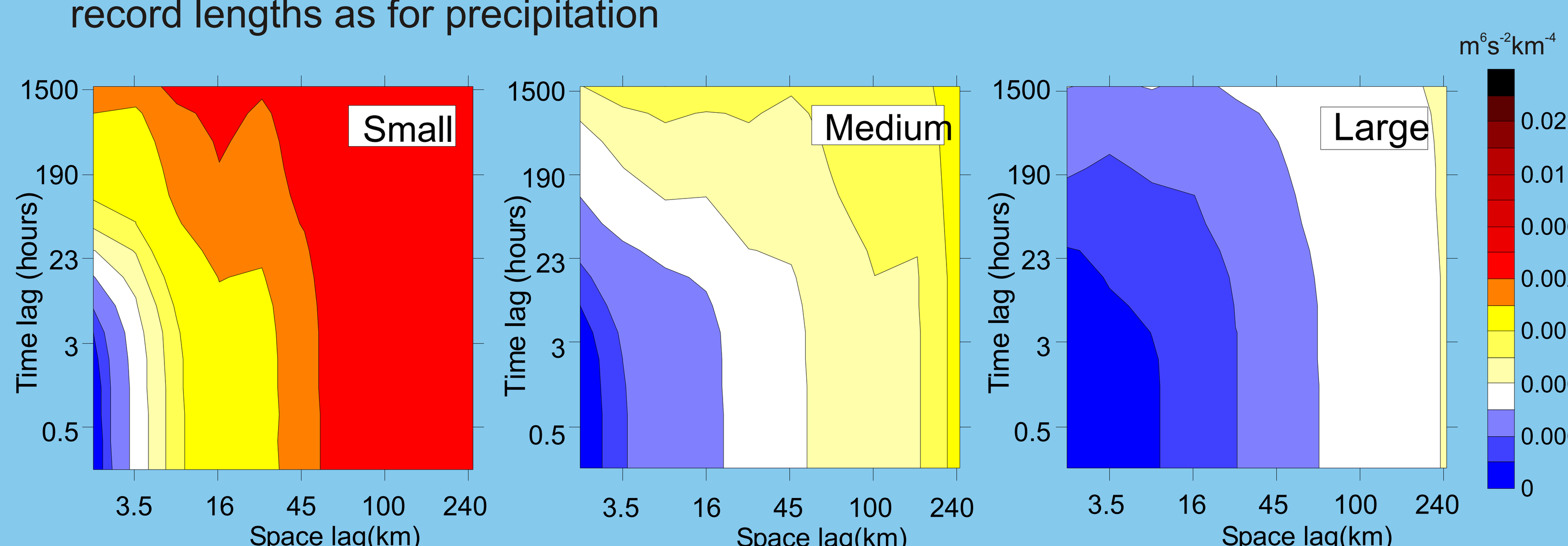
Spatio-temporal sample variograms of hourly catchment precipitation for catchments of three size classes. Almost 600 data series from Austria. 20 years of recordings. Median size in each class: 38 km², 119 km², and 668 km².



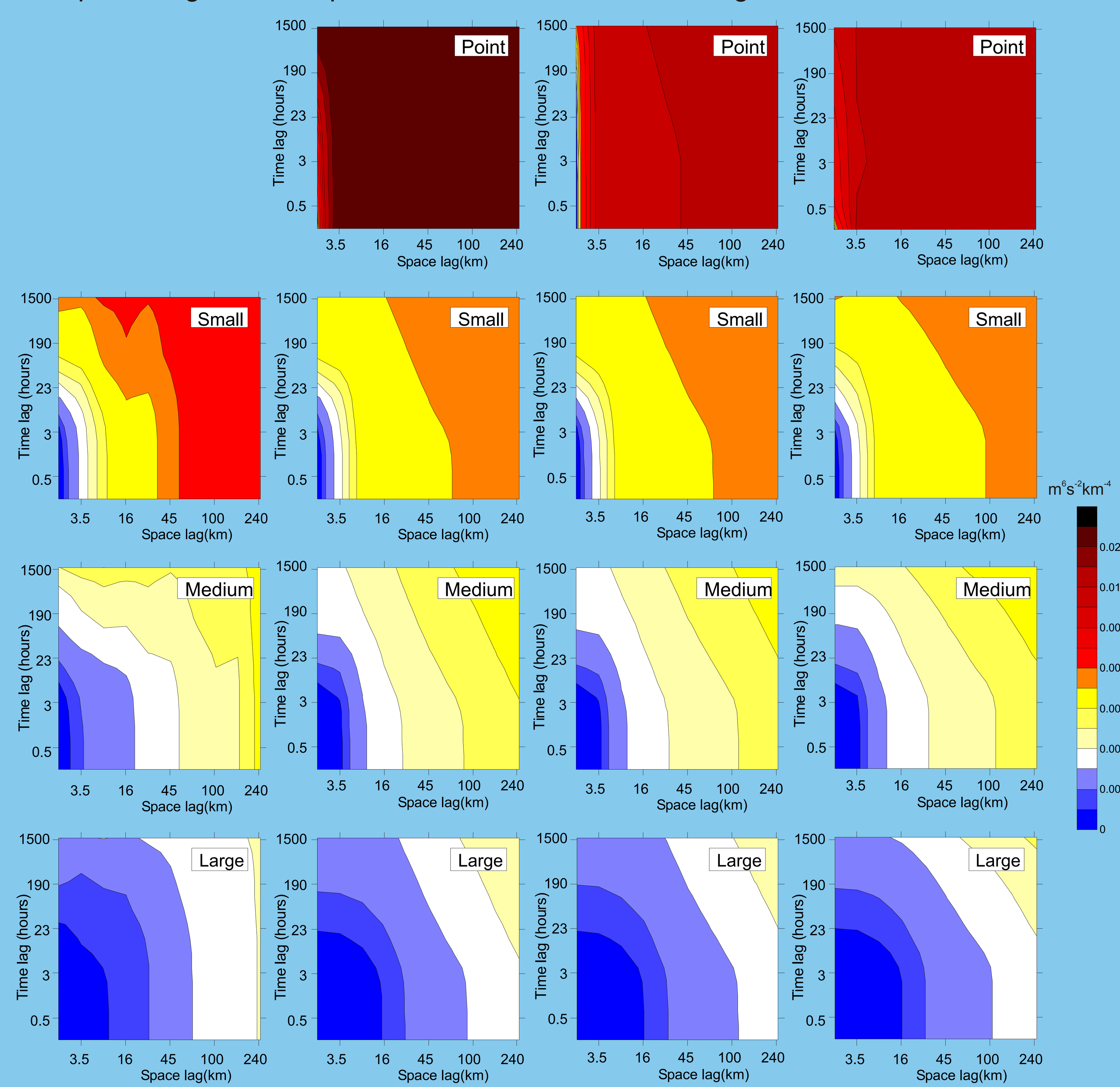
Centroids of gauged catchments. Catchments were stratified into three size classes.

Runoff

Spatio-temporal sample variograms of quarter hourly runoff measurements. More or less same catchments and record lengths as for precipitation



Spatio-temporal aggregation of runoff



Objective function of jointly fitted variograms from all three catchment classes, by regularising variogram models of point runoff.

Variogram model	Total
Exponential model	44.9
Cressie-Huang model	46.2
Product-sum model	46.0

Conclusion

- Exponential and product-sum model are best suited for modelling spatio-temporal variograms of precipitation and runoff
- Spatio-temporal aggregation models are able to explain runoff variograms of different catchment size classes
- All the variogram models are equally well suited to model variograms of point runoff
- Variograms of point runoff can be used for interpolation of ungaged basins

geoENV 2004
Fifth European Conference on Geostatistics for Environmental Applications.
Neuchâtel, Switzerland, October 13-15, 2004.

Contact:
Jon Olav Skøien
skoien@hydro.tuwien.ac.at