4.4 Phase conjugation

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4.4.1 Introduction

Phase conjugation is a nonlinear optical process which generates a light beam having the same wavefronts as an incoming light beam but opposite propagation direction, see Fig. 4.4.1. Therefore phase conjugation is also called wavefront reversal. A nonlinear optical device generating a phase-conjugated wave is called a *phase conjugator* or *Phase-Conjugate Mirror* (*PCM*).

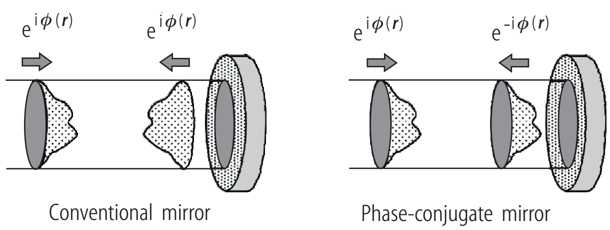

Fig. 4.4.1. Wavefront reflection at a conventional mirror and at a phase-conjugate mirror (PCM).

In Fig. 4.4.2 we consider the conjugation property of a PCM on a probe wave emanating from a point source. A diverging beam, after "reflection" from an ideal PCM, gives rise to a converging conjugate wave that precisely retraces the path of the incident probe wave, and therefore propagates in a time-reversed sense back to the same initial point source.

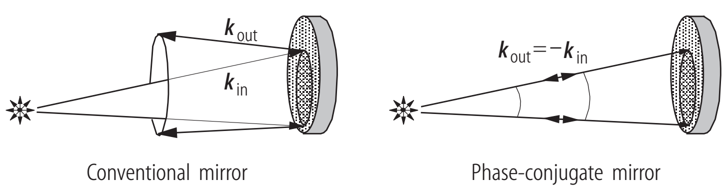

Fig. 4.4.2. Beam propagation after reflection at a conventional mirror and a PCM, both illuminated by a point source.

A phase conjugator reflects light, mostly laser beams, only if the incident power is high enough (*self-pumped phase conjugator*) or if the nonlinear material in the phase conjugator is pumped by additional laser beams, e.g. two additional beams in a *degenerate four-wave mixing* arrangement. In principle phase conjugation could be achieved also by a *deformable mirror* which is controlled by a wavefront sensor adapting the local mirror curvature to the incoming wavefront. Instead of a deformable mirror also a *2-dimensional phase modulator* could be used. However, deformable mirrors and other phase modulators up to now are more complicated set-ups with longer reaction periods than nonlinear optical phase conjugators to solve practical problems requiring phase conjugation.

4.4.2 Basic mathematical description

The incoming wave E_{in} is given by (4.4.1) with frequency f, where the amplitude E_0 and phase Φ are combined to the complex amplitude A. The complex conjugate is denoted by c.c.

$$E_{\mathrm{in}}(x,y,z,t) = \frac{1}{2}\, E_0(x,y,z)\, \mathrm{e}^{2\,\pi\,\mathrm{i}\,(ft+\Phi(x,y,z))} + \text{c.c.} = A(x,y,z)\, \mathrm{e}^{\mathrm{i}\,\omega t} + \text{c.c.}\ , \tag{4.4.1}$$

$$A(x,y,z) = \frac{1}{2}\, E_0(x,y,z)\, \mathrm{e}^{2\,\pi\,\mathrm{i}\,\Phi(x,y,z)}\ . \tag{4.4.2}$$

The phase-conjugated wave exhibits the same wavefronts, however the sign of the phase Φ is inverted due to the inverted propagation direction. Thus, the phase-conjugated wave E_{pc} can be written as (4.4.3):

$$E_{\mathrm{pc}}(x,y,z,t) = \frac{1}{2}\, E_0(x,y,z)\, \mathrm{e}^{2\,\pi\,\mathrm{i}\,(ft-\Phi(x,y,z))} + \text{c.c.} = A_{\mathrm{pc}}(x,y,z)\, \mathrm{e}^{\mathrm{i}\,\omega t} + \text{c.c.}\ , \tag{4.4.3}$$

$$A_{\mathrm{pc}}(x,y,z) = \frac{1}{2}\, E_0(x,y,z)\, \mathrm{e}^{-2\,\pi\,\mathrm{i}\,\Phi(x,y,z)} = A^*(x,y,z)\ . \tag{4.4.4}$$

As can be seen A_{pc} equals the complex-conjugated A^*, which explains the term *phase conjugation*. From (4.4.1) and (4.4.3) we derive that the incident and phase-conjugated wave are also related to each other by

$$E_{\mathrm{in}}(x,y,z,-t) = E_{\mathrm{pc}}(x,y,z,t)\ . \tag{4.4.5}$$

Thus, the phase-conjugate wave E_{pc} propagates as if one would reverse the temporal evolution of the incident wave E_{in} . Therefore the term "*time-reversed replica*" is sometimes used to describe the phase-conjugate wave.

An ideal PCM also maintains the polarization state of an incident wave after phase conjugation. As an example, a probe wave that is Right-Handed Circularly Polarized (RHCP) will result in a RHCP-reflected wave after conjugation. This is in contrast to a conventional mirror, which reflects an incident RHCP field to yield a Left-Handed Circularly Polarized (LHCP) wave [82Pep].

One should realize that an ideal phase-conjugated wave exhibits the same frequency f as the incident wave and reveals the same polarization state. Often, real phase conjugators do not have these properties. However, if an PCM maintains the polarization state it is called a "*vector phase conjugator*".

The nonlinear optical process which comes closest to yielding an ideal phase-conjugate wave is the backward-going, degenerate four-wave mixing interaction. Other classes of interaction (e.g. stimulated effects) result in nonideal conjugate waves due to frequency shifts, nonconjugated field polarization, etc. Although the application of stimulated effects, especially Stimulated Brillouin Scattering (SBS), yields to nonideal phase-conjugate mirrors they are used the most to solve practical problems requiring phase conjugation (e.g. compensation of phase distortions in high average power laser systems [99Eic]).

4.4.3 Phase conjugation by degenerate four-wave mixing

Four-wave mixing can be understood as a real-time holographic process, which facilitates phase conjugation. If the frequencies of the incoming wave, the two additionally required pump waves, and the phase-conjugated or reflected wave are equal the process is called *Degenerate Four-Wave Mixing* (*DFWM*).

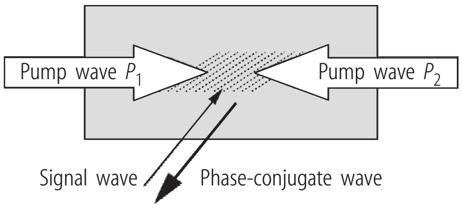

Fig. 4.4.3. Setup for phase conjugation by four-wave mixing.

In Fig. 4.4.3 the setup for phase conjugation by four-wave mixing is shown.

Interference of the incoming wave $E_{\text{in}}(x, y, z, t)$ with the pump wave $P_1(x, y, z, t)$ results in a spatially periodic intensity pattern which modulates the absorption coefficient or refractive index of the optical material resulting in a dynamic or transient amplitude or phase grating. The other pump $P_2(x, y, z, t)$ is diffracted at this grating producing the phase-conjugated wave. This corresponds to the conventional holographic process where the read-out wave is replaced by the second pump wave counterpropagating to the first pump or reference wave.

Recording of a hologram is the first step in phase conjugation and leads to a transmission function t in the hologram plane (variables will not be noted furthermore to simplify the readability):

$$t \propto |P_1 + E_{\text{in}}|^2 = \cdots = |P_1|^2 + P_1 E_{\text{in}}^* + P_1^* E_{\text{in}} + |E_{\text{in}}|^2 \ . \tag{4.4.6}$$

During the read-out the phase-conjugate wave can be generated. Therefore, the hologram is illuminated with a second pump wave P_2, propagating in the opposite direction to P_1. This is in contrast to standard holography. Since P_2 precisely retraces the path of P_1 in the opposite propagation direction, P_2 equals P_1^*. This means, that the two pump beams should be phase-conjugated to each other, so that their spatial phases cancel and do not influence the phases of the reflected beam.

In the hologram plane we obtain a field strength distribution as follows:

$$P_2\, t = P_1^*\, t \propto P_1^* \,|P_1|^2 + |P_1|^2 \, E_{\text{in}}^* + (P_1^*)^2 \, E_{\text{in}} + P_1^* \,|E_{\text{in}}|^2 \ . \tag{4.4.7}$$

The second term $|P_1|^2 \, E_{\text{in}}^*$ corresponds to the phase-conjugate wave of E_{in}. The other expressions lead to three additional waves which are not of interest here. They can be suppressed in thick nonlinear media in case of Bragg diffraction.

Common dynamic grating materials for phase conjugation are:

- photorefractive crystals ($LiNbO_3$, $BaTiO_3$, ...),
- liquid crystals (molecular reorientation effects),
- laser crystals (spatial hole-burning, excited-state absorption),
- saturable absorbers,
- absorbing gases and liquids (thermal gratings),
- semiconductors (Si, GaAs, ...).

The disadvantage of phase conjugation by four-wave mixing is the requirement of two additional pump waves for the nonlinear medium. However, this facilitates amplification of the phase-conjugate wave in the nonlinear medium at the same time. Vector phase conjugation is not achieved by this simple DFWM scheme, but requires polarization-dependant interactions.

4.4.4 Self-pumped phase conjugation

Self-pumped phase conjugation of continuous-wave laser beams in the lower power range (mW ... W) can be realized in Four-Wave Mixing (FWM) loop arrangements using photorefractive media, see Sect. 4.4.6 for detailed discussion.

Table 4.4.1. Brillouin gain coefficient g and phonon lifetime τ for different SBS media.

SBS medium	Brillouin gain coefficient g [cm/GW]	Phonon lifetime τ [ns]
SF_6 (20 bar)	25	15
Xe (50 bar)	90	33
C_2F_6 (30 bar)	60	10
CS_2	130	5.2
CCl_4	6	0.6
Acetone	20	2.1
Quartz	2.4	5

For pulsed lasers, self-pumped phase conjugation is achieved by stimulated scattering. For practical application, stimulated Brillouin scattering [72Kai] in

- gases (SF_6, Xe, C_2F_6, CH_4, N_2, ...) under high pressure,
- liquids (CS_2, CCl_4, acetone, freon, $GeCl_4$, methanol, ...), and
- solids (bulk quartz glass, glass fibers)

is used.

Table 4.4.1 shows the Brillouin gain coefficient g and the phonon lifetime τ for different gaseous, liquid, and solid-state SBS media.

A phase-conjugate mirror consists simply of a gas or liquid cell or a fiber piece. The incoming wave is focused into the material where an oppositely traveling wave is generated initially by spontaneous scattering. This wave interferes with the incoming wave and induces a sound wave or another type of phase grating reflecting the incoming beam similarly as a dielectric multilayer mirror. The induced density variations have the frequency of the initial sound wave, which is amplified therefore and reinforces the backscattering. A detailed discussion of stimulated Brillouin scattering is given in Sect. 4.3.3.2.

The amplification depends strongly on the extension of the interference area. Therefore the phase-conjugated backscattered part dominates, leading to an exponential rise of the reflected phase-conjugated signal. The wavefronts of the sound-wave grating match the wavefronts of the incoming beam. Any disturbance of the incident wavefront will result in a self-adapted mirror curvature with response times in the ns range.

For applications the “threshold”, reflectivity, and conjugation fidelity are the most important parameters that characterize the performance of a Brillouin-scattering phase-conjugate mirror. A sharply defined threshold does not exist for the nonlinear SBS process. However, after exceeding a certain input energy a steep increase of reflectivity can be observed. Often this is called the energy threshold of the phase-conjugate medium. For long pulses as compared to the phonon lifetime (typically several ns) the SBS is expected to become stationary. In this case the energy threshold can be substituted by a power threshold. Well above this threshold, the reflectivity is not stationary but exhibits statistical fluctuations because SBS starts from noise.

It is important to emphasize that the power and not the intensity determines the “threshold” in case of strongly monochromatic input waves. Slight focusing leads to lower intensity, but also to a longer Rayleigh length and a larger interaction area. Stronger focusing reduces the interaction length, but results in stronger refractive-index modulation. Both effects compensate each other if the interaction length is not limited by the coherence length.

Practically, for most laser sources the coherence length is rather short. Here the interaction length should be short compared to the coherence length. This requires adequate focusing of the beam into the SBS medium. Focal length and scattering material have to be chosen suitable to achieve a high SBS reflectivity and a good reproduction of the wavefront. Side effects in the material like absorption, optical breakdown, or other scattering processes have to be avoided. Figure 4.4.4

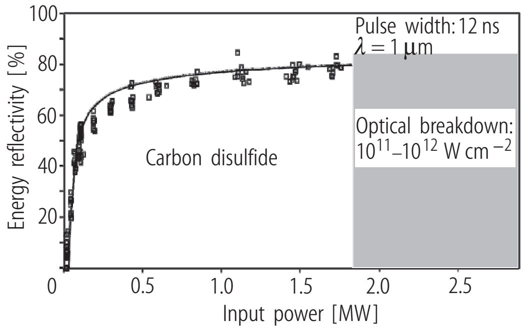

Fig. 4.4.4. Commonly used carbon disulfide (CS_2) shows an SBS threshold of about 18 kW (pulse peak power) under stationary conditions.

Table 4.4.2. SBS threshold, max. reflectivity, far-field fidelity, M^2-limit, and power limit for different fiber phase conjugators, coherence length 1.5 m. The reflectivity is corrected with respect to Fresnel and coupling losses.

Core diameter [µm]	SBS threshold [kW]	Maximum reflectivity [%]	Far-field fidelity [%]	M^2-limit	Power limit [kW]
200	17	80	93	63	160
100	6.4	80	91	31	40
50	2.0	88	70	16	10
25	0.3	86	–	8	2.5

shows the energy reflectivity of carbon disulfide as a function of the input power at 1 µm wavelength. Carbon disulphide shows one of the smallest power thresholds for liquids of about 18 kW. Applying gases as SBS media, the power thresholds are about one order of magnitude higher. A saturation of energy reflectivity close to 80 % is a typical value for liquid SBS media, although reflectivities up to 96 % had been demonstrated [91Cro].

For high-power input pulses bulk solid-state media like quartz are investigated as SBS media, too [97Yos]. To reduce the power threshold of SBS a waveguide geometry can be applied [95Jac]. The beam intensity inside the waveguide is high within a long interaction length resulting in low power thresholds. To avoid toxic liquids and gases under high pressure multimode quartz fibers can be used [97Eic]. The lower Brillouin gain of quartz glass compared to suitable SBS gases and liquids can be overcome using fibers with lengths of several meters resulting in SBS thresholds down to 200 W peak power [98Eic].

The power threshold P_{th} can be estimated from (4.4.8), where A_{eff} is the effective mode field area inside the fiber core, L_{eff} the effective interaction length, which depends on the coherence length, and g is the Brillouin gain coefficient; for quartz g is about 2.4 cm/GW [89Agr].

$$P_{\text{th}} = \frac{21\ A_{\text{eff}}}{L_{\text{eff}}\ g}\ . \tag{4.4.8}$$

Table 4.4.2 shows the power threshold, the maximum energy reflectivity, the far-field fidelity, the M^2-limit (see below), and an approximated power limit of fiber phase conjugators with different core diameters. The used quartz-quartz fibers had a step-index geometry and a numerical aperture of 0.22. They were investigated with an Nd:YAG oscillator amplifier system generating pulses of 30 ns (FWHM) at 1.06 µm wavelength. Regarding applications it is important to couple also spatially aberrated beams into the fiber. The upper limit for the beam parameter product is due to the finite numerical aperture and the core diameter of the fiber. This can by expressed by a "times diffraction limit value" M^2, see Chap. 2.2 for further information about beam characterization. The upper power limit is approximated assuming a damage threshold above 500 MW/cm^2 for ns pulses.

An important feature of a fiber phase conjugator is the threshold behavior for different M^2-values of the incoming beam. In case of a fiber the SBS threshold is nearly independent of the incoming beam quality. This is caused by mode conversion inside the fiber resulting in homogeneous illumination and therefore in constant SBS reflectivity. In case of a Brillouin cell the reflectivity depends on the far-field distribution of the incoming beam. Here phase distortions result in amplitude fluctuations in the focal region. A comparison between a diffraction-limited beam ($M^2 = 1.0$) and a highly distorted beam ($M^2 = 10$) showed an increase of the SBS threshold of 300 % in case of the Brillouin cell. For the fiber phase conjugator no remarkable changes of the power threshold were observed [97Eic].

Practically, the reproduction of the initial wavefront is not perfect after phase conjugation. To characterize the deviation with respect to the reference wave the term *fidelity* F is introduced [77Zel]:

$$F = \frac{\left|\int E_{\text{in}}\, E_{\text{p}}^{*}\, \mathrm{d}^2\, r\right|^2}{\int \left|E_{\text{in}}\right|^2 \mathrm{d}^2\, r \cdot \int \left|E_{\text{p}}\right|^2 \mathrm{d}^2\, r}\,. \qquad (4.4.9)$$

The fidelity equals unity in case of perfect wavefront reproduction and is smaller than unity for practical cases. To calculate the fidelity, the electric field distribution of the incident signal E_{in} and the not perfectly phase-conjugated wave E_{p} – the perfectly phase-conjugated wave is denoted E_{pc} in Sect. 4.4.2 – has to be known. The determination requires sophisticated measurement equipment. In contrary, the *far-field fidelity* can be measured with less effort and is therefore often used. The transmission through an aperture of the phase-conjugate signal is compared with the transmission of the input signal. The ratio is called far-field fidelity, because the aperture is placed in the focal plane of a focusing lens.

4.4.5 Applications of SBS phase conjugation

Phase conjugation generates a wave which retraces the incoming wave in a time-reversed way. Thereby it is possible to eliminate phase distortions in optical systems. For example, in a solid-state laser amplifier, the incoming beam is not only amplified but suffers also from phase distortions due to thermal refractive-index changes in the laser crystal. After passing this amplifier crystal, the beam is reflected by a phase conjugator and passes the crystal a second time. As the wavefronts are inverted with respect to the propagation direction, the refractive-index changes reduce the phase distortions and after the second passage, these distortions disappear so that the beam quality of the incoming wave is reproduced. In Fig. 4.4.5 a double-pass scheme with phase-conjugate mirror to compensate for phase distortions is shown.

Typically, phase conjugators are applied in *Master Oscillator Power Amplifier* (*MOPA*) setups, where a nearly diffraction-limited master oscillator beam is increased in power within an amplifier

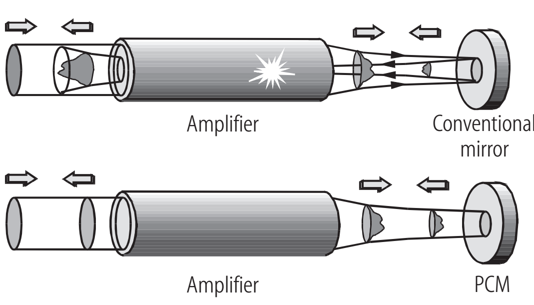

Fig. 4.4.5. Double-pass scheme with phase-conjugate mirror to compensate for phase distortions.

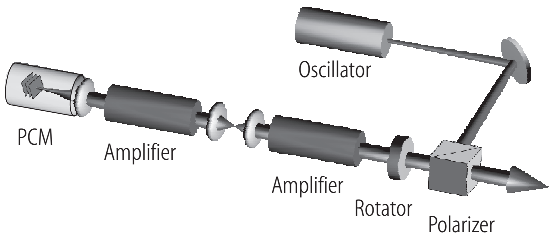

Fig. 4.4.6. Master oscillator power amplifier (MOPA) setup with phase-conjugate mirror.

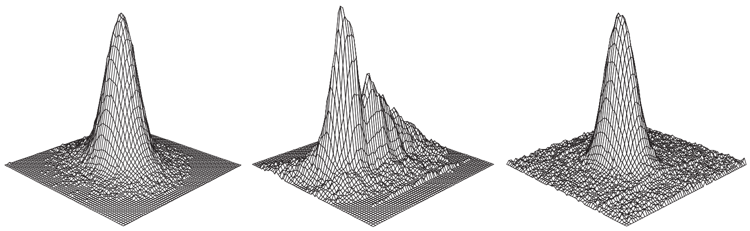

Fig. 4.4.7. Far-field intensity distributions of the oscillator beam, the distorted beam after single-pass amplification, and the highly amplified beam after double-pass amplification with phase conjugation.

arrangement, see Fig. 4.4.6. After the first amplification pass the beam quality is reduced due to thermally induced phase distortions. The spatial-distorted beam enters the SBS mirror and becomes phase-conjugated. The initial beam quality of the master oscillator can be roughly reproduced after the second amplification pass. The amplified beam is extracted with an optical isolation, which consists in this case of a Faraday rotator and a polarizer.

Figure 4.4.6 shows a MOPA system producing up to 210 W average output power at 2 kHz average repetition rate (1.08 μm wavelength). The system is part of an advanced setup yielding up to 520 W average output power [99Eic]. The oscillator beam has a nearly diffraction-limited beam quality ($M^2 < 1.2$) which is already reduced in front of the first amplifier ($M^2 \cong 1.5$). This results from optical components between oscillator and amplifier which introduce phase distortions. After single-pass amplification the beam quality decreases to $M^2 \cong 5$ due to phase distortions introduced by both pumped amplifier rods at 6.5 kW pumping power for each amplifier. After phase conjugation and double-pass amplification the initial beam quality can be nearly reproduced ($M^2 < 1.9$). Differences between the initial and final beam quality are caused by a fidelity smaller than unity and diffraction at several apertures in the amplifier chain.

The performance of the phase-conjugate mirror can be illustrated by far-field intensity profiles recorded at different positions in the setup. In Fig. 4.4.7 the oscillator output beam exhibits a smooth Gaussian profile corresponding to the nearly diffraction-limited beam quality. After single-pass amplification the reduction of beam quality is confirmed by a strongly aberrated far-field profile. After phase conjugation and double-pass amplification the initial intensity distribution can be nearly reproduced. In this example the average power of the master oscillator beam (approx. 1 W) was increased to 130 W after double-pass amplification.

Presently, phase distortion elimination in double or multipass laser amplifiers is the most often application of phase conjugation. In addition phase conjugators are useful as mirrors in laser oscillators replacing one of the conventional mirrors. Again, the phase conjugator eliminates phase distortions in the laser medium induced by optical or discharge pumping. For recent advances and applications of SBS-phase-conjugation see [02Eic, 03Rie, 04Rie].

4.4.6 Photorefraction

The photorefractive effect belongs to the nonlinear optical effects with the highest sensitivity for operation at low optical intensity levels. Photorefractive phase conjugators are able to operate using intensities of only mW/cm^2. The price paid of the low intensity performance is diminished speed. The response times of recent photorefractive phase conjugators span in the range of milliseconds to several minutes.

The photorefractive effect describes light-induced refractive-index changes in the material when the incident light is spatially nonuniform [88Gue, 93Yeh, 95Nol, 96Sol]. The spatial nonuniformity distinguishes the photorefractive effect from other common nonlinear optical effects that occur under spatial uniform intensity. The maximum refractive-index change induced in a photorefractive material does not occur necessarily locally where the light intensity is a maximum. The nonlocal response occurs because electric charges move and are stored inside the material. In case of classical nonorganic bulk photorefractive materials, such as ferroelectric oxides ($BaTiO_3$, $LiNbO_3$, $KNbO_3$), sillenites ($Bi_{12}SiO_{20}$, $Bi_{12}TiO_{20}$, $Bi_{12}GeO_{20}$) or semi-insulating semiconductors (GaAs, InP, CdTe), electrons (or holes) are photoexcited from localized impurity centers or defect sites, which are energetically located deep in the band gap of the material, into the conduction (or valence) band.

The energy of the exciting photons is smaller than the band-gap energy. Free carriers excited in bright crystal regions move due to diffusion and drift into dark crystal regions where they are trapped by empty defect sites, see Fig. 4.4.8. As a consequence of separated and trapped electric charges the formation of space-charge electric fields occurs. These electric fields change the refractive index of the material by electrooptics effects, usually the Pockels effect.

Nonuniform illumination occurs when two coherent laser beams interfere in the crystal. The intersecting beams create a periodical interference pattern. The formation of a photorefractive index grating due to a sinusoidal intensity pattern is shown in Fig. 4.4.9. When diffusion is the main effect for the transport of the excited charge carriers (there is no external electric field applied on the crystal) the electric-field maxima are shifted by a quarter fringe spacing relative to the intensity maxima. This $\pi/2$ phase shift of the induced index grating plays a fundamental role in photorefractive non-linear optical wave mixing. It allows for an energy transfer between the two beams writing the grating in a process called two-wave mixing. One of the beams (called signal beam) is amplified at the expense of the other beam (called pump beam).

A phase-conjugate beam can be created by four-wave mixing processes. In this case the two-wave mixing arrangement is extended with a second pump beam which counterpropagates with respect to the first pump beam, see Fig. 4.4.3. In case of external pump beams, the phase-conjugation

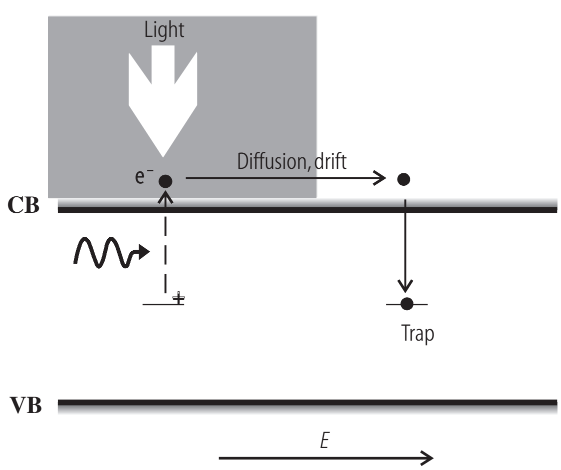

Fig. 4.4.8. Band transport model of photorefraction.

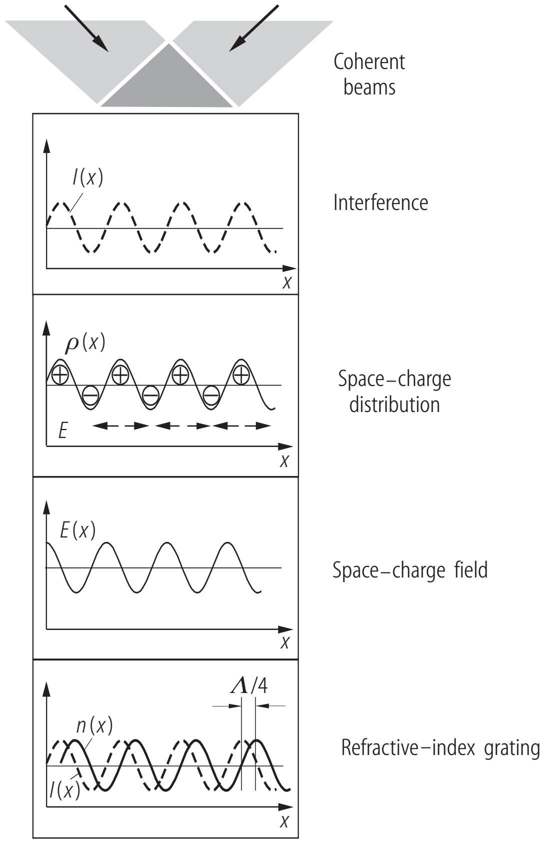

Fig. 4.4.9. Formation of a photorefractive index grating due to a sinusoidal intensity pattern.

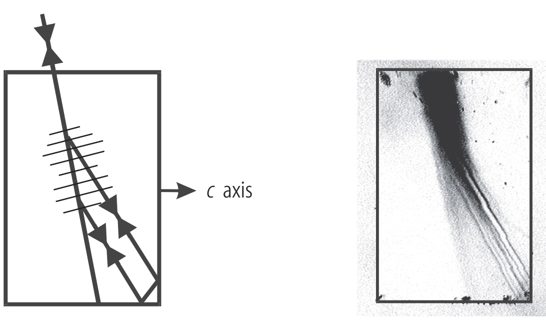

Fig. 4.4.10. Scheme of photorefractive total-internal-reflection phase conjugator (cat conjugator), left. The light propagation in the crystal can be seen due to scattering, right.

process may be highly efficient leading to large reflectivities well above 100 % in relation to the incoming power.

Self-pumped phase conjugators require only a single incident beam and because of their simplicity they are more advantageous for practical applications. The operation of photorefractive self-pumped phase conjugators is based on a non-linear optical process called beam fanning. When a single beam is incident on a photorefractive crystal, some light is scattered inside the crystal. This scattered light forms a set of gratings with the incident light and is amplified by two-wave mixing. This process was named fanning because a broad fan of scattered amplified light is generated emerging from the crystal.

Perhaps the most commonly used photorefractive self-pumped phase conjugator type is the so-called cat conjugator [82Fei]. In this case the first pump beam is generated from the incident beam by fanning, the second pump beam by backreflection on the crystal corner. Figure 4.4.10 shows a rhodium-doped barium titanate crystal which acts as a cat conjugator for an incident beam of 5 mW

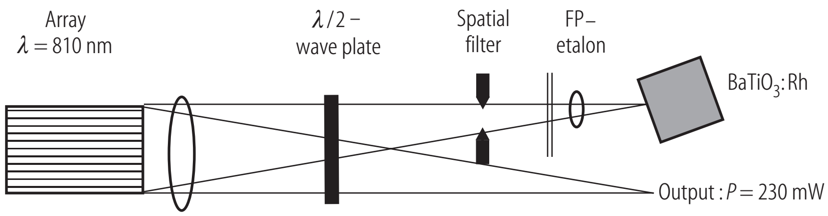

Fig. 4.4.11. Coherent diode laser array coupled by a phase-conjugating $BaTiO_3$:Rh crystal [98Lob].

optical power at 808 nm wavelength. The formed internal phase-conjugation loops can be observed in the lower right-hand corner of the crystal. Self-pumped phase-conjugate reflectivities as high as 60–80 % have been reported for visible and near-infrared wavelengths by numerous investigators using photorefractive crystals in various arrangements [85Gue, 86Pep, 95Mu, 94Wec, 97Huo].

The efficient operation of photorefractive phase conjugators at low and moderate power levels makes this type of device attractive especially for diode-laser applications. Free-running high-power diode laser arrays emit laser beams of poor spatial and spectral quality. Optical phase-conjugate feedback can increase both the spatial and the temporal coherence of the radiation. Figure 4.4.11 shows an external-cavity diode laser system comprising a photorefractive $BaTiO_3$ crystal as phase conjugator, a Fabry-Perot etalon, and a spatial filter forcing the laser diode array to operate in a single spatial and a single longitudinal mode [98Lob]. The coherence length of the phase-conjugate laser system has been increased by a factor of 70 and the output has become almost diffraction-limited. The output power is reduced from 440 mW to 230 mW.