

## Editors

**Weber, Horst**

Technische Universität Berlin, Optisches Institut, Berlin, Germany

**Herziger, Gerd**

Rheinisch-Westfälische Technische Hochschule, Aachen, Germany

**Poprawe, Reinhart**

Fraunhofer-Institut für Lasertechnik (ILT), Aachen, Germany

## Authors

**Eichler, Hans Joachim**

Technische Universität Berlin, Optisches Institut, Berlin, Germany

**Eppich, Bernd**

Technische Universität Berlin, Optisches Institut, Berlin, Germany

**Fischer, Joachim**

Physikalisch-Technische Bundesanstalt, Abteilung Temperatur und Synchrotronstrahlung, Berlin, Germany

**Güther, Reiner**

Ferdinand-Braun-Institut für Höchstfrequenztechnik, Berlin, Germany

**Gurzadyan, Gagik**

Technische Universität München, Institut für Physikalische und Theoretische Chemie, Garching, Germany

**Hermerschmidt, Andreas**

Technische Universität Berlin, Optisches Institut, Berlin, Germany

**Laubereau, Alfred**

Technische Universität München, Physik Department E11, München, Germany

**Lopota, Vitaliy A.**, member of Russian Academy of Sciences

Central R & D Institute of Robotics and Technical Cybernetics, Saint Petersburg, Russian Federation

**Mehl, Oliver**

Technische Universität Berlin, Optisches Institut, Berlin, Germany

**Vidal, Carl Rudolf**

Max-Planck Institut für Extraterrestrische Physik, Garching, Germany

**Weber, Horst**

Technische Universität Berlin, Optisches Institut, Berlin, Germany

**Wende, Burkhard**

Physikalisch-Technische Bundesanstalt, Abteilung Temperatur und Synchrotronstrahlung, Berlin, Germany

**Landolt-Börnstein****Editorial Office**

Gagernstraße 8  
D-64283 Darmstadt, Germany  
fax: +49 (6151) 171760  
e-mail: lb@springer-sbm.com

**Internet**

<http://www.landolt-boernstein.com>

# Preface

The three volumes VIII/1A, B, C document the state of the art of “Laser Physics and Applications”. Scientific trends and related technological aspects are considered by compiling results and conclusions from phenomenology, observation and experience. Reliable data, physical fundamentals and detailed references are presented.

In the recent decades the laser source matured to a universal tool common to scientific research as well as to industrial use. Today a technical goal is the generation of optical power towards shorter wavelengths, shorter pulses and higher power for application in science and industry. Tailoring the optical energy in wavelength, space and time is a requirement for the investigation of laser-induced processes, i.e. excitation, non-linear amplification, storage of optical energy, etc. According to the actual trends in laser research and development, Vol. VIII/1 is split into three parts: Vol. VIII/1A with its two subvolumes 1A1 and 1A2 covers laser fundamentals, Vol. VIII/1B deals with laser systems and Vol. VIII/1C gives an overview on laser applications.

In Vol. VIII/1A1 the following topics are treated in detail:

## **Part 1: Fundamentals of light-matter interaction**

This part compiles the basic elements of classical electromagnetic wave theory, non-relativistic quantum mechanics of the two-level system and its interaction with the non-quantized radiation field. The relevant relations with their approximations and range of validity are discussed. It starts with Maxwell’s equations, wave equation and SVE-approximations, presents the Schrödinger equations, the field/atom interaction including the Einstein coefficients and cross-sections. The main parameters characterizing the two-level system with typical numbers are given in several tables. Finally, the coherent interaction is briefly discussed. This semiclassical approach is sufficient for most applications in laser technology. The fully quantized theory is offered in Vol. VIII/1A2, Chap. 5.

## **Part 2: Radiometry**

In the first section the definitions of the radiometric quantities and their measurement are summarized. In the second part the main elements of laser beam characterization are compiled with a detailed discussion of the theoretical background. The experimental determination of the essential quantities according to the ISO-normalizations is given.

## **Part 3: Linear optics**

The design of optical resonators and beam handling requires a broad knowledge in optics. In this part the fundamentals of beam propagation, Gaussian beams, diffraction, refraction, lens design and crystal optics are presented. The extensive references give access to detailed information.

**Part 4: Nonlinear optics**

Nonlinear effects are widely used in laser technology to generate new wavelengths or to improve beam quality. In four sections the essential nonlinear optical effects are discussed: frequency conversion in crystals, frequency conversion in gases and liquids, stimulated scattering and phase conjugation. In extensive tables the coefficients of the nonlinear processes are compiled.

August 2005

The Editors

# Contents

---

## Part 1 Fundamentals of light-matter interaction

---

<b>1.1</b>	<b>Fundamentals of the semiclassical laser theory</b>	
	V.A. LOPOTA, H. WEBER . . . . .	3
1.1.1	The laser oscillator . . . . .	3
1.1.2	The electromagnetic field . . . . .	5
1.1.2.1	Maxwell's equations . . . . .	5
1.1.2.2	Homogeneous, isotropic, linear dielectrics . . . . .	6
1.1.2.2.1	The plane wave . . . . .	7
1.1.2.2.2	The spherical wave . . . . .	8
1.1.2.2.3	The slowly varying envelope (SVE) approximation . . . . .	9
1.1.2.2.4	The SVE-approximation for diffraction . . . . .	9
1.1.2.3	Propagation in doped media . . . . .	10
1.1.3	Interaction with two-level systems . . . . .	11
1.1.3.1	The two-level system . . . . .	11
1.1.3.2	The dipole approximation . . . . .	12
1.1.3.2.1	Inversion density and polarization . . . . .	12
1.1.3.2.2	The interaction with a monochromatic field . . . . .	14
1.1.3.3	The Maxwell–Bloch equations . . . . .	15
1.1.3.3.1	Decay time $T_1$ of the upper level (energy relaxation) . . . . .	15
1.1.3.3.1.1	Spontaneous emission . . . . .	15
1.1.3.3.1.2	Interaction with the host material . . . . .	15
1.1.3.3.1.3	Pumping process . . . . .	16
1.1.3.3.2	Decay time $T_2$ of the polarization (entropy relaxation) . . . . .	16
1.1.4	Steady-state solutions . . . . .	17
1.1.4.1	Inversion density and polarization . . . . .	17
1.1.4.2	Small-signal solutions . . . . .	19
1.1.4.3	Strong-signal solutions . . . . .	19
1.1.5	Adiabatic equations . . . . .	20
1.1.5.1	Rate equations . . . . .	20
1.1.5.2	Thermodynamic considerations . . . . .	21
1.1.5.3	Pumping schemes and complete rate equations . . . . .	22
1.1.5.3.1	The three-level system . . . . .	23
1.1.5.3.2	The four-level system . . . . .	24
1.1.5.4	Adiabatic pulse amplification . . . . .	25
1.1.5.5	Rate equations for steady-state laser oscillators . . . . .	26
1.1.6	Line shape and line broadening . . . . .	26
1.1.6.1	Normalized shape functions . . . . .	27
1.1.6.1.1	Lorentzian line shape . . . . .	27
1.1.6.1.2	Gaussian line shape . . . . .	27

1.1.6.1.3	Normalization of line shapes . . . . .	27
1.1.6.2	Mechanisms of line broadening . . . . .	28
1.1.6.2.1	Spontaneous emission . . . . .	28
1.1.6.2.2	Doppler broadening . . . . .	28
1.1.6.2.3	Collision or pressure broadening . . . . .	28
1.1.6.2.4	Saturation broadening . . . . .	29
1.1.6.3	Types of broadening . . . . .	29
1.1.6.3.1	Homogeneous broadening . . . . .	29
1.1.6.3.2	Inhomogeneous broadening . . . . .	30
1.1.6.4	Time constants . . . . .	31
1.1.7	Coherent interaction . . . . .	31
1.1.7.1	The Feynman representation of interaction . . . . .	32
1.1.7.2	Constant local electric field . . . . .	33
1.1.7.3	Propagation of resonant coherent pulses . . . . .	34
1.1.7.3.1	Steady-state propagation of $n\pi$ -pulses . . . . .	35
1.1.7.3.1.1	$2\pi$ -pulse in a loss-free medium . . . . .	35
1.1.7.3.1.2	$\pi$ -pulse in an amplifying medium . . . . .	36
1.1.7.3.2	Superradiance . . . . .	37
1.1.8	Notations . . . . .	37
	References for 1.1 . . . . .	40

---

## Part 2 Radiometry

---

<b>2.1</b>	<b>Definition and measurement of radiometric quantities</b>	
	B. WENDE, J. FISCHER . . . . .	45
2.1.1	Introduction . . . . .	45
2.1.2	Definition of radiometric quantities . . . . .	45
2.1.3	Radiometric standards . . . . .	47
2.1.3.1	Primary standards . . . . .	47
2.1.3.2	Secondary standards . . . . .	48
2.1.4	Outlook – State of the art and trends . . . . .	50
	References for 2.1 . . . . .	51
<b>2.2</b>	<b>Beam characterization</b>	
	B. EPPICH . . . . .	53
2.2.1	Introduction . . . . .	53
2.2.2	The Wigner distribution . . . . .	53
2.2.3	The second-order moments of the Wigner distribution . . . . .	55
2.2.4	The second-order moments and related physical properties . . . . .	56
2.2.4.1	Near field . . . . .	56
2.2.4.2	Far field . . . . .	58
2.2.4.3	Phase paraboloid and twist . . . . .	59
2.2.4.4	Invariants . . . . .	60
2.2.4.5	Propagation of beam widths and beam propagation ratios . . . . .	60
2.2.5	Beam classification . . . . .	61
2.2.5.1	Stigmatic beams . . . . .	62
2.2.5.2	Simple astigmatic beams . . . . .	63

2.2.5.3	General astigmatic beams .....	64
2.2.5.4	Pseudo-symmetric beams .....	64
2.2.5.5	Intrinsic astigmatism and beam conversion .....	65
2.2.6	Measurement procedures .....	66
2.2.7	Beam positional stability .....	67
2.2.7.1	Absolute fluctuations .....	67
2.2.7.2	Relative fluctuations .....	69
2.2.7.3	Effective long-term beam widths .....	69
	References for 2.2 .....	70

---

### Part 3 Linear optics

---

<b>3.1</b>	<b>Linear optics</b>	
	R. GÜTHER .....	73
3.1.1	Wave equations .....	73
3.1.2	Polarization .....	75
3.1.3	Solutions of the wave equation in free space .....	78
3.1.3.1	Wave equation .....	78
3.1.3.1.1	Monochromatic plane wave .....	78
3.1.3.1.2	Cylindrical vector wave .....	78
3.1.3.1.3	Spherical vector wave .....	78
3.1.3.2	Helmholtz equation .....	79
3.1.3.2.1	Plane wave .....	79
3.1.3.2.2	Cylindrical wave .....	79
3.1.3.2.3	Spherical wave .....	79
3.1.3.2.4	Diffraction-free beams .....	79
3.1.3.2.4.1	Diffraction-free Bessel beams .....	79
3.1.3.2.4.2	Real Bessel beams .....	80
3.1.3.2.4.3	Vectorial Bessel beams .....	80
3.1.3.3	Solutions of the slowly varying envelope equation .....	80
3.1.3.3.1	Gauss-Hermite beams (rectangular symmetry) .....	81
3.1.3.3.2	Gauss-Laguerre beams (circular symmetry) .....	83
3.1.3.3.3	Cross-sectional shapes of the Gaussian modes .....	83
3.1.4	Diffraction .....	84
3.1.4.1	Vector theory of diffraction .....	85
3.1.4.2	Scalar diffraction theory .....	85
3.1.4.3	Time-dependent diffraction theory .....	89
3.1.4.4	Fraunhofer diffraction patterns .....	89
3.1.4.4.1	Rectangular aperture with dimensions $2a \times 2b$ .....	89
3.1.4.4.2	Circular aperture with radius $a$ .....	90
3.1.4.4.2.1	Applications .....	92
3.1.4.4.3	Gratings .....	92
3.1.4.5	Fresnel's diffraction figures .....	93
3.1.4.5.1	Fresnel's diffraction on a slit .....	93
3.1.4.5.2	Fresnel's diffraction through lens systems (paraxial diffraction) .....	94
3.1.4.6	Fourier optics and diffractive optics .....	94
3.1.5	Optical materials .....	95
3.1.5.1	Dielectric media .....	96
3.1.5.2	Optical glasses .....	97

3.1.5.3	Dispersion characteristics for short-pulse propagation . . . . .	97
3.1.5.4	Optics of metals and semiconductors . . . . .	98
3.1.5.5	Fresnel's formulae . . . . .	98
3.1.5.6	Special cases of refraction . . . . .	101
3.1.5.6.1	Two dielectric isotropic homogeneous media ( $\hat{n}$ and $\hat{n}'$ are real) . . . . .	101
3.1.5.6.2	Variation of the angle of incidence . . . . .	101
3.1.5.6.2.1	External reflection ( $n < n'$ ) . . . . .	101
3.1.5.6.2.2	Internal reflection ( $n > n'$ ) . . . . .	101
3.1.5.6.3	Reflection at media with complex refractive index (Case $\hat{n} = 1$ and $\hat{n}' = n' + i k'$ ) . . . . .	103
3.1.5.7	Crystal optics . . . . .	104
3.1.5.7.1	Classification . . . . .	104
3.1.5.7.2	Birefringence (example: uniaxial crystals) . . . . .	106
3.1.5.8	Photonic crystals . . . . .	107
3.1.5.9	Negative-refractive-index materials . . . . .	108
3.1.5.10	References to data of linear optics . . . . .	108
3.1.6	Geometrical optics . . . . .	108
3.1.6.1	Gaussian imaging (paraxial range) . . . . .	108
3.1.6.1.1	Single spherical interface . . . . .	109
3.1.6.1.2	Imaging with a thick lens . . . . .	110
3.1.6.2	Gaussian matrix (ABCD-matrix, ray-transfer matrix) formalism for paraxial optics . . . . .	111
3.1.6.2.1	Simple interfaces and optical elements with rotational symmetry . . . . .	112
3.1.6.2.2	Non-symmetrical optical systems . . . . .	112
3.1.6.2.3	Properties of a system . . . . .	112
3.1.6.2.4	General parabolic systems without rotational symmetry . . . . .	112
3.1.6.2.5	General astigmatic system . . . . .	116
3.1.6.2.6	Symplectic optical system . . . . .	116
3.1.6.2.7	Misalignments . . . . .	116
3.1.6.3	Lens aberrations . . . . .	117
3.1.7	Beam propagation in optical systems . . . . .	120
3.1.7.1	Beam classification . . . . .	120
3.1.7.2	Gaussian beam: complex $q$ -parameter and its ABCD-transformation . . . . .	120
3.1.7.2.1	Stigmatic and simple astigmatic beams . . . . .	120
3.1.7.2.1.1	Fundamental Mode . . . . .	120
3.1.7.2.1.2	Higher-order Hermite-Gaussian beams in simple astigmatic beams . . . . .	123
3.1.7.2.2	General astigmatic beam . . . . .	123
3.1.7.3	Waist transformation . . . . .	124
3.1.7.3.1	General system (fundamental mode) . . . . .	124
3.1.7.3.2	Thin lens (fundamental mode) . . . . .	124
3.1.7.4	Collins integral . . . . .	126
3.1.7.4.1	Two-dimensional propagation . . . . .	126
3.1.7.4.2	Three-dimensional propagation . . . . .	127
3.1.7.5	Gaussian beams in optical systems with stops, aberrations, and waveguide coupling . . . . .	127
3.1.7.5.1	Field distributions in the waist region of Gaussian beams including stops and wave aberrations by optical system . . . . .	127
3.1.7.5.2	Mode matching for beam coupling into waveguides . . . . .	128
3.1.7.5.3	Free-space coupling of Gaussian modes . . . . .	128
3.1.7.5.4	Laser fiber coupling . . . . .	129
	References for 3.1 . . . . .	131

---

**Part 4 Nonlinear optics**


---

<b>4.1</b>	<b>Frequency conversion in crystals</b>	
	G.G. GURZADYAN . . . . .	141
4.1.1	Introduction . . . . .	141
4.1.1.1	Symbols and abbreviations . . . . .	141
4.1.1.1.1	Symbols . . . . .	141
4.1.1.1.2	Abbreviations . . . . .	142
4.1.1.1.3	Crystals . . . . .	142
4.1.1.2	Historical layout . . . . .	143
4.1.2	Fundamentals . . . . .	144
4.1.2.1	Three-wave interactions . . . . .	144
4.1.2.2	Uniaxial crystals . . . . .	145
4.1.2.3	Biaxial crystals . . . . .	145
4.1.2.4	Effective nonlinearity . . . . .	147
4.1.2.5	Frequency conversion efficiency . . . . .	151
4.1.2.5.1	General approach . . . . .	151
4.1.2.5.2	Plane-wave fixed-field approximation . . . . .	152
4.1.2.5.3	SHG in “nonlinear regime” (fundamental wave depletion) . . . . .	154
4.1.3	Selection of data . . . . .	154
4.1.4	Harmonic generation (second, third, fourth, fifth, and sixth) . . . . .	156
4.1.5	Sum frequency generation . . . . .	167
4.1.6	Difference frequency generation . . . . .	172
4.1.7	Optical parametric oscillation . . . . .	176
4.1.8	Picosecond continuum generation . . . . .	186
	References for 4.1 . . . . .	187
<b>4.2</b>	<b>Frequency conversion in gases and liquids</b>	
	C.R. VIDAL . . . . .	205
4.2.1	Fundamentals of nonlinear optics in gases and liquids . . . . .	205
4.2.1.1	Linear and nonlinear susceptibilities . . . . .	205
4.2.1.2	Third-order nonlinear susceptibilities . . . . .	206
4.2.1.3	Fundamental equations of nonlinear optics . . . . .	207
4.2.1.4	Small-signal limit . . . . .	207
4.2.1.5	Phase-matching condition . . . . .	208
4.2.2	Frequency conversion in gases . . . . .	209
4.2.2.1	Metal-vapor inert gas mixtures . . . . .	209
4.2.2.2	Mixtures of different metal vapors . . . . .	209
4.2.2.3	Mixtures of gaseous media . . . . .	209
	References for 4.2 . . . . .	212
<b>4.3</b>	<b>Stimulated scattering</b>	
	A. LAUBEREAU . . . . .	217
4.3.1	Introduction . . . . .	217
4.3.1.1	Spontaneous scattering processes . . . . .	217
4.3.1.2	Relationship between stimulated Stokes scattering and spontaneous scattering . . . . .	219

4.3.2	General properties of stimulated scattering .....	219
4.3.2.1	Exponential gain by stimulated Stokes scattering .....	219
4.3.2.2	Experimental observation .....	220
4.3.2.2.1	Generator setup .....	220
4.3.2.2.2	Oscillator setup .....	220
4.3.2.2.3	Stimulated amplification setup .....	221
4.3.2.3	Four-wave interactions .....	221
4.3.2.3.1	Third-order nonlinear susceptibility .....	221
4.3.2.3.2	Stokes-anti-Stokes coupling .....	222
4.3.2.3.3	Higher-order Stokes and anti-Stokes emission .....	222
4.3.2.4	Transient stimulated scattering .....	222
4.3.3	Individual scattering processes .....	223
4.3.3.1	Stimulated Raman scattering (SRS) .....	223
4.3.3.2	Stimulated Brillouin scattering (SBS) and stimulated thermal Brillouin scattering (STBS) .....	227
4.3.3.3	Stimulated Rayleigh scattering processes, SRLS, STRS, and SRWS .....	228
	References for 4.3 .....	232
<b>4.4</b>	<b>Phase conjugation</b>	
	H.J. EICHLER, A. HERMERSCHMIDT, O. MEHL .....	235
4.4.1	Introduction .....	235
4.4.2	Basic mathematical description .....	236
4.4.3	Phase conjugation by degenerate four-wave mixing .....	236
4.4.4	Self-pumped phase conjugation .....	237
4.4.5	Applications of SBS phase conjugation .....	240
4.4.6	Photorefraction .....	242
	References for 4.4 .....	245
<b>Index</b>	.....	247