Appendix 7.2

Transfer Function Analysis of the MFB-2 circuits

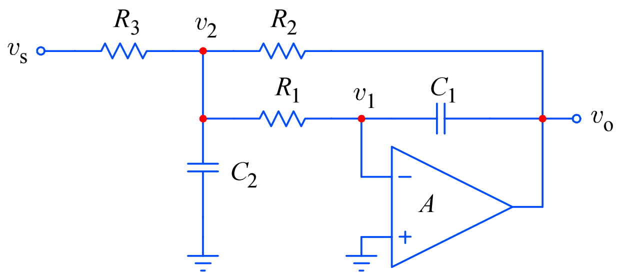

Fig. A7.2.1: The 'Multiple Feedback' 2-pole (MFB-2) low pass filter configurations

The derivation of the transfer function for the second-order stage, shown in Fig. A7.2.1, is a little easier than for the third-order system. Using again the ideal opamp approximation, by summing the currents at v_1 we have:

$$\frac{v_2 - v_1}{R_1} = \frac{v_1 - v_o}{\dfrac{1}{s\,C_1}} \tag{A7.2.1}$$

and by setting $v_1 = 0$:

$$v_2 = -\,v_o\,R_1 s\,C_1 \tag{A7.2.2}$$

By summing the currents at v_2:

$$\frac{v_s - v_2}{R_3} = \frac{v_2}{R_1} + \frac{v_2 - v_o}{R_2} + v_2\,s\,C_2 \tag{A7.2.3}$$

which we solve for v_s:

$$v_s = v_2\left(\frac{R_3}{R_1} + \frac{R_3}{R_2} + 1 + R_3 s\,C_2\right) - v_o\,\frac{R_3}{R_2} \tag{A7.2.4}$$

We substitute v_2 from Eq. A7.2.2:

$$v_s = -\,v_o\,R_1 s\,C_1\left(\frac{R_3}{R_1} + \frac{R_3}{R_2} + 1 + R_3\,s\,C_2\right) - v_o\,\frac{R_3}{R_2} \tag{A7.2.5}$$

and we can rewrite this as:

$$v_s = -\,v_o\left\{\left[R_1 s\,C_1\left(\frac{R_3}{R_1} + \frac{R_3}{R_2} + 1\right) + R_1 R_3\,s^2\,C_1 C_2\right] + \frac{R_3}{R_2}\right\} \tag{A7.2.6}$$

Our only voltage nodes are now the input node v_s and the output node v_o, therefore we can obtain the transfer function by writing the following output to input ratio:

$$\frac{v_{\mathrm{o}}}{v_{\mathrm{s}}} = -\frac{1}{s^2 R_3 R_1 C_1 C_2 + R_1 s\, C_1 \left(\dfrac{R_3}{R_1} + \dfrac{R_3}{R_2} + 1 \right) + \dfrac{R_3}{R_2}} \tag{A7.2.7}$$

We need the normalized canonical form in the denominator:

$$\frac{v_{\mathrm{o}}}{v_{\mathrm{s}}} = -\frac{\dfrac{1}{R_1 R_3 C_1 C_2}}{s^2 + s\left(\dfrac{R_3}{R_1} + \dfrac{R_3}{R_2} + 1 \right) \dfrac{1}{R_3 C_2} + \dfrac{R_3}{R_2} \cdot \dfrac{1}{R_1 R_3 C_1 C_2}} \tag{A7.2.8}$$

But now we must make the numerator equal to the last term of the denominator:

$$\frac{v_{\mathrm{o}}}{v_{\mathrm{s}}} = -\frac{R_2}{R_3} \cdot \frac{\dfrac{R_3}{R_2} \cdot \dfrac{1}{R_1 R_3 C_1 C_2}}{s^2 + s\left(\dfrac{R_3}{R_1} + \dfrac{R_3}{R_2} + 1 \right) \dfrac{1}{R_3 C_2} + \dfrac{R_3}{R_2} \cdot \dfrac{1}{R_1 R_3 C_1 C_2}} \tag{A7.2.9}$$

and this is our transfer function. We see that the system's DC gain A_0 is:

$$A_0 = -\frac{R_2}{R_3} \tag{A7.2.10}$$

and the cut off frequency is:

$$\omega_{\mathrm{h2}} = \sqrt{\frac{R_3}{R_2} \cdot \frac{1}{R_1 R_3 C_1 C_2}} \tag{A7.2.11}$$

By looking at the denominator of the transfer function we see that the circuit is optimized and the capacitance ratio is minimized by making $R_1 = R_3 = R$. We thus obtain:

$$\frac{v_{\mathrm{o}}}{v_{\mathrm{s}}} = -\frac{R_2}{R} \cdot \frac{\dfrac{R}{R_2} \cdot \dfrac{1}{R^2 C_1 C_2}}{s^2 + s\left(2 + \dfrac{R}{R_2} \right) \dfrac{1}{R C_2} + \dfrac{R}{R_2} \cdot \dfrac{1}{R^2 C_1 C_2}} \tag{A7.2.12}$$