7. Promotional Warranty Policies: Analysis and Perspectives

Warranty is a topic that has been studied extensively by different disciplines including engineering, economics, management science, accounting, and marketing researchers [7.1, p. 47]. This chapter aims to provide an overview on warranties, focusing on the cost and benefit perspective of warranty issuers.

After a brief introduction of the current status of warranty research, the second part of this chapter classifies various existing and several new promotional warranty policies to extend the taxonomy initiated by *Blischke* and *Murthy* [7.2].

Focusing on the quantitative modeling perspective of both the cost and benefit analyses of warranties, we summarize five problems that are essential to warranty issuers. These problems are: i) what are the warranty cost factors; ii) how to compare different warranty policies; iii) how to analyze the warranty cost of multi-component systems; iv) how to evaluate the warranty benefits; v) how to determine the optimal warranty policy.

A list of future warranty research topics are presented in the last part of this chapter. We hope that this will stimulate further interest among researchers and practitioners.

Warranty is an obligation attached to products (items or systems) that requires the warranty issuers (manufacturers or suppliers) to provide compensation to consumers according to the warranty terms when the warranted products fail to perform their pre-specified functions under normal usage within the warranty coverage period. Similar definitions can be found in *Blischke* and *Murthy* [7.1, 3], *McGuire* [7.4], and *Singpurwalla* and *Wilson* et al. [7.5]. Based on this definition, a warranty contract should contain at least three characteristics: the coverage period (fixed or random), the method of compensations, and the conditions under which such compensations would be offered. The last characteristic is closely related to warranty execution since it clarifies consumers' rights and protects warranty issuers from excessive false claims. From the costing perspective, the first two characteristics are more important to manufacturers because they determine the depth of the protection against premature failures and the direct cost related to those failures.

Traditionally, warranty serves as a protection instrument attached to products sold to consumers. There are two facets of the protection role: on one hand, it guarantees a properly functioning product for at least a period of w, either financially or physically. On the other hand, it also specifies an upper bound on the liability of the supplier induced by the warranty. In addition to the protection role, warranty has always been one of the most important elements in business marketing strategy. As indicated in [7.4, p.1], manufacturers' primary rationale for offering warranty is to support their products to gain some advantage in the market, either by expressing the company's faith in the product quality, or by competing with other firms. Due to the more than ever fierce competition in the modern economy, the market promotion role of warranty has become even more significant. Manufacturers are fighting with each other through various channels from competitive pricing, improved product reliability, to more comprehensive warranties. Because of technology constraints or time constraint, it is usually difficult to improve product quality in a short time. As a result, warranty has evolved as an essential part of marketing strategy, along with pricing and advertising, which is especially powerful during the introduction period of new, expensive products such as automobiles and complex machinery.

In the last two decades, warranty has been studied extensively among many disciplines such as engineering, economics, statistics, marketing and management science, to name a few. Consequently, the literature on warranty is not only vast, but also disjoint [7.1]. There are three books and hundreds of journal articles that have addressed warranty-related problems within the last ten years. A comprehensive collection of related references up to 1996 can be found in [7.3]. In general, researchers in engineering are interested in quality control and improving product reliability to reduce production and service costs. Some of the major references are *Chen* et al. [7.6], *Djamaludin* et al. [7.7], *Hedge* and *Kubat* [7.8], *Mi* [7.9], *Murthy* and *Hussain* [7.10], *Nguyen* and *Murthy* [7.11], and *Sahin* [7.12]. Economists usually treat warranty as a special type of insurance. Consequently, they developed the economic theory of warranties as one of many applications of microeconomics. We refer readers to *DeCroix* [7.13], *Emons* [7.14, 15], *Lutz* and *Padmanabhan* [7.16], *Padmanabhan* and *Rao* [7.17], *Murthy* and *Asgharizadeh* [7.18] and the references therein. Statisticians mainly focus on warranty claim prediction, statistical inference of warranty cost, and estimation of product reliability or availability. Some of the key references are *Frees* [7.19, 20], *Ja* et al. [7.21], *Kalbfleisch* [7.22], *Kao* and *Smith* [7.23, 24], *Menzefricke* [7.25], *Padmanabhan* and *Worm* [7.26] and *Polatoglu* [7.27]. A long-term trend in warranty study is the focus on various warranty-management aspects. Some recent references are *Chun* and *Tang* [7.28], *Ja* et al. [7.21], *Lam* and *Lam* [7.29], *Wang* and *Sheu* [7.30], and *Yeh* et al. [7.31, 32]. *Blischke* and *Murthy* [7.33] developed a framework for the analytical study of various issues related to warranty. Recently, *Murthy* and *Djamaludin* [7.34] enriched the framework by summarizing the literature since 1992 from an overall business perspective. Another review by *Thomas* and *Rao* [7.35] provided some suggestions for expanding the analysis methods for making warranty decisions.

In this chapter, we briefly review some recent work in warranty literature from the manufacturers' perspective. The objectives of this chapter are to classify various existing and relatively new warranty policies to extend the taxonomy proposed in [7.2], and to summarize and illustrate some fundamental warranty economic problems.

7.1 Classification of Warranty Policies

Numerous warranty policies have been studied in the last several decades. *Blischke* and *Murthy* [7.2] presented a taxonomy of more than 18 warranty policies and provided a precise statement of each of them. In this section, we extend the taxonomy by addressing several recently proposed policies that might be of interests to warranty managers. It should be noted that we mainly focus on type *A* policies [7.2], which, based on the taxonomy, are referred to as policies for single items and not involving product development.

7.1.1 Renewable and Nonrenewable Warranties

One of the basic characteristics of warranties is whether they are renewable or not. For a regular renewable policy with warranty period w, whenever a product fails within w, the buyer is compensated according to the terms of the warranty contract and the warranty policy is renewed for another period w. As a result, a warranty cycle T, starting from the date of sale, ending at the warranty expiration date, is a random variable whose value depends on w, the total number of failures under the warranty, and the actual failure inter-arrival times. Renewable warranties are often offered for inexpensive, nonrepairable consumer electronic products such as microwaves, coffee makers, and so forth, either implicitly or explicitly. One should notice that theoretically the warranty cycle for a renewable policy can be arbitrarily large. For example, consumers can induce the failures so that they keep on getting new warranties indefinitely. Such moral hazard problems might be one of the reasons that renewable policies are not as popular as nonrenewable ones among warranty issuers.

One way to remedy this problem is to modify the regular renewable policy in the following way: instead of offering the original warranty with a period of w repeatedly upon each renewing, warranty issuers could set $w_i = \alpha w_{i-1}$, $\alpha \in (0, 1]$, for $i = 1, 2, \cdots$, where w_i is the warranty length for the i-th renewing, and $w_0 = w$. Actually, this defines a new type of renewable warranty, which we refer to as geometric renewable warranty policies. Clearly, a geometric renewable policy is a generalization of a regular renewable policy, which degenerates to the latter when $\alpha = 1$.

The majority of warranties in the market are nonrenewable; for these the warranty cycle, which is the same as the warranty period, is not random, but predetermined (fixed), since the warranty obligation will be terminated as soon as w units of time pass after sale. This type of policies is also known as a fixed-period warranty.

7.1.2 FRW, FRPW, PRW, CMW, and FSW Policies

According to the methods of compensation specified in a warranty contract upon premature failures, there are three basic types of warranties: free replacement warranty (FRW), free repair warranty (FRPW), and pro-rata warranty (PRW). Combination warranty (CMW) contains both features of FRW/FRPW and PRW. Full-service warranty, (FSW), which is also known as preventive maintenance warranty, is a policy that may be offered for expensive deteriorating complex products such as automobiles. Under this type of policies, consumers not only receive free repairs upon premature failures, but also free (preventive) maintenance.

For nonrepairable products, the failed products under warranty will usually be replaced free of charge to consumers. Such a policy is often referred to as a free replacement warranty or an unlimited warranty. In practice, even if a product is technically repairable, sometimes it will be replaced upon failure since repair may not be economically sound. As a result, for inexpensive repairable products, warranty issuers could simply offer FRW policies. Consequently, these inexpensive repairable products can be treated as nonrepairable. However, for repairable products, if the warranty terms specify that, upon a valid warranty claim, the warranty issuer will repair the failed product to working condition free of charge to buyers, then such a policy is a so-called free repair warranty. In practice, it is not rare that a warranty contract specifies that the warranty issuer would repair or replace a defective product under certain conditions. This is the reason why most researchers do not treat FRW and FRPW separately. Nevertheless, we feel that it is necessary to differentiate these two type of policies based on the following reasoning: first, repair cost is usually much lower than replacement cost except for inexpensive products; secondly, by clearly defining the compensation terms, warranty issuers may establish a better image among consumers, which can surely be helpful for the marketing purpose.

Under a FRW policy, since every failed product within T is replaced by a new one, it is reasonable to model all the subsequent failure times by a single probability distribution. However, under a FRPW, it is necessary to model the repair impact on failure times of a warranted product. If it is assumed that any repair is as-good-as-new (perfect repair), then from the modeling perspective, there is little difference between FRW and FRPW. For deteriorating complex systems, minimal repair is a commonly used assumption. Under this assumption, a repair action restores the system's failure rate to the level at the time epoch when the last failure happened. Minimal repair was first introduced by *Barlow* and *Proschan* [7.36]. Changing a broken fan belt on an engine is a good example of minimal repair since the overall failure rate of the car is nearly unchanged. Perfect repair and minimal repair represent two extremes relating to the degree of repair. Realistically, a repair usually makes a system neither as-good-as-new, nor as-bad-as-old (minimal repair), but to a level in between. This type of repair is often referred to as imperfect repair. In the literature of maintenance and reliability, many researchers have studied various maintenance policies considering imperfect repair. A recent review on imperfect maintenance was given by *Pham* and *Wang* [7.37]. In the warranty literature, the majority of researchers consider repairs as either perfect or minimal. Little has been done on warranty cost analysis considering imperfect repair.

Both FRW and FRPW policies provide full coverage to consumers in case of product failures within T. In contrast, a PRW policy requires that buyers pay a proportion of the warranty service cost upon a failure within T in exchange for the warranty service such as repair or replacement, cash rebate or a discount on purchasing a new product. The amount that a buyer should pay is usually an increasing function of the product age (duration after the sale). As an example, suppose the average repair/replacement cost per failure is c_s, which could be interpreted as the seller's cost per product without warranty, if a linear pro-rata function is used, then the cost for a buyer upon a failure at time t, $t < w$, is $c_s \frac{t}{w}$. The corresponding warranty cost incurred to the manufacturer is $c_s\left(1 - \frac{t}{w}\right)$. PRW policies are usually renewable and are offered for relatively inexpensive products such as tires, batteries, and so forth.

Generally speaking, FRW and FRPW policies are in the favor of buyers since manufacturers take all the responsibility of providing products that function properly during the whole warranty cycle [7.1, p. 221]. In other words, it is the manufacturers that bear all the warranty cost risk. In contrast, for PRW policies manufacturers have the relative advantage with regard to the warranty cost risk. Although they do have to offer cash rebates or discounts to consumers if failures happen during T, they are usually better off no matter what consumers choose to do. If a consumer decides not to file a warranty claim, then the manufacturer saves himself the cash rebate or other type of warranty service. If instead a warranty claim is filed, the manufacturer might enjoy the increase in sales or at least the warranty service cost is shared by the consumer.

To balance the benefits between buyers and sellers, a combination warranty (CMW) that contains both features of FRW/FRPW and PRW policies was created. CMW is a policy that usually includes two warranty periods: a free repair/replacement period w_1 followed by a pro-rata period w_2. This type of warranties is not rare today because it has significant promotional value to sellers while at the same time it provides adequate control over the costs for both buyers and sellers [7.3, p. 12].

For deteriorating complex products, it is essential to perform preventive maintenance to achieve satisfactory reliability performance. Maintenance involves planned and unplanned actions carried out to retain a system at, or restore it to, an acceptable operating condition [7.38]. Planned maintenance is usually referred to as preventive maintenance while unplanned maintenance is labeled as corrective maintenance or repair. The burden of maintenance is usually on the consumers' side. In [7.39], *Bai* and *Pham* proposed a renewable full-service warranty for multi-component systems under which the failed component(s) or subsystem(s) will be replaced; in addition, a (preventive) maintenance action will be performed to reduce the chance of future product failures, both free of charge to consumers. They argue that such a policy is desirable for both consumers and manufacturers since consumers receive better warranty service compared to traditional FRPW policies, while at the same time manufacturers may enjoy cost savings due to the improved product reliability by the maintenance actions. By assuming perfect maintenance, they derived the probability distributions and the first two moments of the warranty cost per warranty cycle for series, parallel, series–parallel, and parallel–series systems.

Many researchers have studied warranty-maintenance problems. Among them Chun [7.40] determined the optimal number of periodic maintenance actions during the warranty period by minimizing the expected warranty cost (EWC). *Jack* and *Dagunar* [7.41] generalized Chun's idea by considering unequal preventive maintenance intervals. *Yeh* [7.32] further extended the work by including the degree of maintenance as one of the decision variables along with the number of maintenance actions and the maintenance schedule. All of these three researches aim to obtain the optimal maintenance warranty to assist manufacturers' decision-making. A related problem is the determination of the optimal maintenance strategy following the expiration of warranty from the consumers' perspective. *Dagpunar* and *Jack* [7.42] studied the problem by assuming minimal repair. Through a general approach, *Sahin* and *Polatoglu* [7.43] discussed both stationary and non-stationary maintenance strategies following the expiration of warranty. They proved the pseudo-convex property of the cost rate function under some mild conditions.

7.1.3 Repair-Limit Warranty

In maintenance literature, many researchers studied maintenance policies set up in such a way that different maintenance actions may take place depending on whether or not some pre-specified limits are met. Three types of limits are usually considered: repair-number-limit, repair-time-limit, and repair-cost-limit. Those maintenance policies are summarized by *Wang* [7.44].

Similarly, three types of repair-limit warranties may be considered by manufacturers: repair-number-limit warranty (RNLW), repair-time-limit warranty (RTLW), and repair-cost-limit warranty (RCLW). Under a RNLW, the manufacturer agrees to repair a warranted product up to m times within a period of w. If there are more than m failures within w, the failed product shall be replaced instead of being repaired again. *Bai* and *Pham* [7.45] recently studied the policy under the imperfect-repair assumption. They derived the analytical expressions for the expected value and the variance of warranty cost per product sold through a truncated quasi-renewal-process approach.

AN RTLW policy specifies that, within a warranty cycle T, any failures shall be repaired by the manufacturer, free of charge to consumers. If a warranty service cannot be completed within τ unit of time, then a penalty cost occurs to the manufacturer to compensate the inconvenience of the consumer. This policy was analyzed by *Murthy* and *Asgharizadeh* [7.18] in the context of maintenance service operation.

For a RCLW policy, there is a repair cost limit τ in addition to an ordinary FRPW policy. That is, upon each failure within the warranty cycle T, if the estimated repair cost is greater than τ, then replacement instead of repair shall be provided to the consumer; otherwise, normal repair will be performed. This policy has been studied by *Nguyen* and *Murthy* [7.46] and others.

It should be noted that various repair limits as well as other warranty characteristics such as renewing may be combined together to define a new complex warranty. For example, it is possible to have a renewable repair-time-limit warranty for complex systems. Such combinations define a large set of new warranty policies that may appear in the market in the near future. Further study is needed to explore the statistical behavior of warranty costs of such policies to assist decisions of both manufacturers and consumers.

7.1.4 One-Attribute Warranty and Two-Attribute Warranty

Most warranties in practice are one-attribute, for which the warranty terms are based on product age or product usage, but not both. Compared to one-attribute warranties, two-attribute warranties are more complex since the warranty obligation depends on both the product age and product usage as well as the potential interaction between them. Two-attribute warranties are often seen in automobile industry. For example, Huyndai, the Korean automobile company, is currently offering 10 years/100 000 miles limited FRPW on the powertrain for most of their new models.

One may classify two-attribute warranties according to the shape of warranty coverage region. *Murthy* et al. defined four types of two-attribute warranties labeled as policy A to policy D (Fig. 1 in [7.47]). The shapes of the warranty regions are rectangular, L-shaped with no limits on age or usage, L-shaped with upper limits on age and usage, and triangular, respectively. Based on the concept of the iso-cost curve, *Chun* and *Tang* [7.28] proposed a set of two-attribute warranty policies for which the expected present values of future repair costs are the same. Some other plausible warranty regions for two-attribute warranty policies were discussed by *Singpurwalla* and *Wilson* [7.5].

In general, there are two approaches in the analysis of two-attribute warranties, namely, the one-dimensional (1-D) approach and the two-dimensional (2-D) approach. The 1-D approach assumes a relationship between product age and usage; therefore it eventually converts a two-attribute warranty into a corresponding one-attribute warranty. This approach is used by *Moskowitz* and *Chun* [7.48], and *Chun* and *Tang* [7.28]. The 2-D approach does not impose a deterministic relationship between age and usage. Instead, a bivariate probability distribution is employed for the two warranty attributes. *Murthy* et al. [7.47] followed the idea and derived the expressions for the expected warranty cost per item sold and for the expected life cycle cost based on a two-dimensional renewal processes. *Kim* and *Rao* [7.49] obtained the analytical expressions for the warranty cost for the policies A and B defined in [7.47] by considering a bivariate exponential distribution. Perhaps the most comprehensive study of two-attribute warranties so far is by *Singpurwalla* and *Wilson* [7.5], in which, through a game-theory set up, they discussed in detail both the optimum price-warranty problem and the warranty reserve determination problem.

7.2 Evaluation of Warranty Policies

Two phenomena make the study of warranties important. First, warranty has become common practice for manufacturers. According to the survey conducted by *McGuire*, nearly 95% percent of producers of industrial products provide warranties on all of their product lines [7.4, p. 1]; secondly, there is a huge amount of money involved in warranty programs. Based on a report by the Society of Mechanical Engineering (www.sme.org), the annual warranty cost is about 6 billion dollars for Ford, General Motors and Chrysler combined in the year 2001.

Among many issues related to warranty, there are two fundamental questions that must be answered, especially for warranty issuers: (1) how much a warranty will cost; (2) how much benefit can be earned from a certain warranty. This section summarizes some ideas and discussions appeared in the literature that are closely related to these two questions.

7.2.1 Warranty Cost Factors

Due to the random nature of many warranty cost factors such as product failure times, warranty cost is also a random variable whose statistical behavior can be determined by establishing mathematical links between warranty factors and warranty cost. There are numerous factors that may be considered in warranty studies. Among them, we believe that the followings are of great importance: the characteristics of warranty policies; warranty service cost per failure; product failure mechanism; impact of warranty service on product reliability; warranty service time; and warranty-claim-related factors.

Different warranty policies may require different mathematical models for warranty cost. One way to model the warranty cost per item sold is through a stochastic counting process $[N(t), t \geq 0]$, which represents the number of failures over time of a warranted product. Let $S_1, S_2, \cdots$ be the subsequent failure times, and denote by C_{S_i} the warranty cost associated with the i-th failure. Assuming that all product failures are claimed, that all claims are valid, and instant warranty service, then the total warranty cost per item sold, $C(w)$, can be expressed as

$$C(w) = \begin{cases} \sum_{i=0}^{N[T(w)]} C_{S_i}, & \text{for } N[T(w)] = 1, 2, \cdots \\ 0, & \text{for } N[T(w)] = 0\,. \end{cases} \tag{7.1}$$

From (7.1), it is clear that the probabilistic behavior of $C(w)$ solely depends on $N[T(w)]$ (the number of failures within a warranty cycle T) and C_{S_i}, as well as the potential interaction between them. In general it is very difficult to determine the distribution of $C(w)$. However, it is possible to obtain the moments of $C(w)$ using modern stochastic process theory and probability theory.

For nonrepairable products or repairable products with a single component, warranty service cost per failure is often assumed to be constant. However, for repairable multi-component products, warranty service cost per failure in general is a random variable whose distribution is related to the product (system) structure and the warranty service cost for each component.

Product (system) failure mechanism can be described by the distributions of subsequent system failure times. This involves the consideration of system structure, the reliability of components and the impact of repair on components' reliability and system reliability. System structure is essential in determining system reliability. Extensive research on reliability modeling has been done for different systems such as series–parallel systems, parallel–series systems, standby systems, k-out-of-n systems, and so forth, in the literature of reliability [7.50]. Unfortunately, to our knowledge, there is no complete theory or methodology in warranty that incorporates the consideration of various system structure.

If a warranted product is nonrepairable or the as-good-as-new repair assumption is used for repairable products, then a single failure-time distribution can be adopted to describe the subsequent product failure times under warranty. However, if a warranted product is repairable and repairs are not as-good-as-new, then the failure time distribution(s) of repaired products differ(s) from that of a new product. This situation may be modeled by considering a failure-time distribution for all repaired products different from that of new products [7.1]. Strictly speaking, distributions of subsequent failure times of a repairable product are distinct, therefore, such an approach can be only viewed as an approximation.

As mentioned in Sect. 7.1, warranty compensation includes free replacement, free repair or cash rebate. For the case of free replacement, warranty service cost per failure for manufacturers is simply a constant that does not depend on the product failure times. In the case of cash rebate (pro-rata policy), warranty cost per failure usually relies on product failure time as well as the rebate function. When repair, especially the not as-good-as-new repair, is involved in warranty service, one has to model the repair impact on product reliability, which in turn has a great impact on warranty cost per failure. One way to model subsequent failure times under this situation is to consider them as a stochastic process. Consequently, modern stochastic theory of renewal processes, nonhomogeneous Poisson processes, quasi-renewal processes [7.38] and general point processes could be applied.

To our knowledge, most warranty literature assumes that warranty service is instant. This may be justified when the warranty service time is small compared to the warranty period or the warranty cycle. A better model is to incorporate explicitly the service times into warranty cost modeling. One recent attempt to include non-zero service time in warranty analysis is by *Murthy* and *Asgharizadeh* [7.18]. In this chapter, they developed a game-theoretic formulation to obtain the optimal decision in a maintenance service operation.

Warranty claims-related factors include the response of consumers to product failures and the validation of warranty claims by warranty issuers. It is no secret that not all consumers will make warranty claims even if they are entitled to do so. It is also true that warranty issuers, to serve their own benefits, usually have a formal procedure to validate warranty claims before honoring them. Such situations may be modeled by assigning two new parameters α and β, where α is the probability that a consumer will file a claim upon a failure within T, and β is the proportion of the rejected claims [7.51].

There are other factors that may be of importance in warranty cost evaluation such as nonconforming product quality [7.6], multiple modes of failure, censored observations [7.20], and etc. Unfortunately, it is impossible to consider all the factors in one warranty cost model. Even if such a model exists, it would be too complicated to be applied.

7.2.2 Criteria for Comparison of Warranties

Warranty managers usually have several choices among various warranty policies that might be applied to a certain type of products. This requires some basic measures as the criteria to make the comparison among these policies.

There are several measures available, including expected warranty cost (EWC) per product sold, expected discounted warranty cost (EDWC) per warranty cycle, monetary utility function and weighted objective function. EWC and EDWC are more popular than the others since they are easy to understand and can be estimated relatively easily. The key difference between them is that the latter one considers the value of time, an important factor for warranty cost accounting and financial managers.

To our opinion, monetary utility function, $U(x)$, is a better candidate for the purpose of comparing warranty policies. The functional form of $U(x)$ reflects the manufacturer's risk attitude. If a manufacturer is risk-neutral, then $U(x)$ is linear in x. This implies that maximizing $\mathrm{E}[U(x)]$ is the same as maximizing $U[\mathrm{E}(x)]$. However, manufacturers may be risk-averse if they are concerned about the variations in profit or in warranty cost. For example, a particular manufacturer may prefer a warranty with less cost variation than another with much larger variation in warranty cost if the difference between the EWCs is small. If this is the case, then it can be shown that the corresponding utility function is concave [7.52]. The main difficulty of the utility theory approach is that utility functions are subjective.

Weighted objective functions could also be used for the purpose of comparing warranties for manufacturers. One commonly used weighted objective function is $\mathrm{E}[\pi(x)] - \rho\mathrm{V}[\pi(x)]$, where ρ is a positive parameter representing the subjective relative importance of the risk (variance or standard deviation) against the expectation and $\pi(x)$ is the manufacturers profit for a given warranty policy x. Interestingly, such an objective function coincides to a special case of the utility theory approach when the manufacturer's subjective utility function is assumed to only depend on the first two centered moments of $\pi(x)$ [7.53, 54].

In the above discussion, the term *warranty cost* refers to the manufacturer's cost per warranted product. In our opinion, this is the fundamental measure for the purpose of evaluating any warranty for manufacturers since it provides precise information on the additional cost incurred to manufacturers due to warranty. An equally useful measure is the discounted warranty cost (DWC) per cycle. This measure incorporates the value of time, therefore it is useful when warranty managers are interested in determining warranty reserve level. It is also of importance to financial managers performing warranty cost analysis.

Some researchers have proposed warranty cost per unit time, or warranty cost rate, as the primary warranty cost measure. As indicated by *Blischke* and *Murthy* [7.3], warranty cost rate is useful in managing warranty servicing resources, such as parts inventory over time with dynamic sales.

Another related measure is warranty cost over a product life cycle. *Blischke* and *Murthy* named this cost as *life cycle cost-II* (LCC-II) [7.1]. A product life cycle begins with the launch of the product onto the market and ends when it is withdrawn.

For consumers, warranty cost analysis is usually conducted over the life time of a product. In [7.1], this cost is labeled as *life cycle cost-I* (LCC-I). LCC-I is a consumer-oriented cost measure and it includes elements such as purchase cost, maintenance and repair costs following expiration of a warranty, operating costs as well as disposal costs.

7.2.3 Warranty Cost Evaluation for Complex Systems

Most products (systems), especially expensive ones, are composed of several nonrepairable components. Upon a failure, the common repair practice is to replace failed components instead of replacing the whole system. For such products, warranty may be offered for each of the individual components, or for the whole system. For the former case, the warranty cost modeling and analysis for single-component products can be applied readily. In fact, most warranty literature focuses on the analysis of warranty for single-component systems via a black-box approach. However, for the latter case, it is necessary to investigate warranty with explicit consideration of system structure because evidently system structure has a huge impact on product reliability, therefore it is a crucial factor in warranty cost study. Unfortunately, as indicated by *Chukova* and *Dimitrov* [7.55, pp. 544], so far there has been only limited study on this topic.

Some researchers have discussed the warranty cost modeling for parallel systems. For example, *Ritchken* [7.56] provided an example of a two-component parallel system under a two-dimensional warranty. *Hussain* and *Murthy* [7.57] also discussed warranty cost estimation for parallel systems under the setting that uncertain quality of new products may be a concern for the design of warranty programs. *Chukova* and *Dimitrov* [7.55] presented a two-component parallel system under a FRPW policy. Actually, for nonrepairable parallel systems, the modeling techniques of warranty cost is essentially the same as that of black-box systems unless the system is considered as repairable.

To our knowledge, the only published work about warranty study on series systems is by *Chukova* and *Dimitrov* [7.55, p. 579–580]. They derived the EWC per system sold for a two-component series system under a FRPW policy which offers free replacement of the failed component if any system failure happens within the warranty period w. Recently, *Bai* and *Pham* [7.39] obtained the first two moments of a renewable FSW policy for series, parallel, series–parallel and parallel–series systems. The derivation of the first two moments of the DWC of nonrenewable FRPW and PRW policies for minimally repaired series systems can be found in [7.58].

It is possible to use a Markovian model to analyze warranty cost for complex systems. *Balachandran* et al. [7.59] dealt with the problem of determining warranty service cost of a three-component system using the Markovian approach. A similar discussion can be seen in [7.55] and the references therein. Although this approach is a powerful tool in the literature of reliability, queuing systems, and supply-chain management, there are some limitations in the applications of warranty. First of all, it is difficult to determine the appropriate state space and the corresponding transition matrix for the applications in warranty. Secondly, most Markovian models only provide the analysis of measures in the steady states by assuming infinite horizon. In other words, the statistical behavior of those measures in finite horizon (short-run) is either too difficult to obtain or not of practical interest. However, in warranty study, it is crucial to understand the finite-horizon statistical behavior of warranty cost. Thirdly, warranty claim data as well as reliability data are scarce and costly. Markovian models usually require more data since they contain more parameters than ordinary probability models that could be applied to warranty cost study.

7.2.4 Assessing Warranty Benefits

As mentioned in the introduction, warranty is increasingly used as a promotional device for marketing purposes. Consequently, it is necessary to predict and assess quantitatively the benefit that a manufacturer might generate from a specific warranty [7.35, p. 189]. For promotional warranties, such benefit is usually realized through the demand side. Manufacturers generally expect that the increase in profit as a result of the increase in sale, which is boosted by warranty, should cover the future warranty cost.

A simple way to quantify the benefit is to model it as a function of the parameter(s) of a warranty policy, for example, w, the warranty period. A linear form and a quadratic form of w were employed by *Thomas* [7.35, 60] for this purpose. As he acknowledged, both forms were not well-founded and shared the problem of oversimplification [7.35, p. 193]. Another approach is to estimate the demand function empirically. *Menezes* and *Currim* [7.61] posited a general demand function where the quantity sold by a firm offering a warranty with period w is a function of its price, warranty length, advertising, distribution, quality, product feature, and the corresponding values for the firm's competitor. Based on the data from *Ward's Automotive Yearbook, Consumer Reports, Advertising Age, Leading National Advertisers*, and other sources during the period 1981–1987, they obtained the price elasticity and the warranty elasticity, which enabled them to obtain the optimal warranty length through maximizing the present value of cumulative future profit over a finite planning horizon. One of the limitations of this approach, as pointed out by the authors, is that it requires the support of historical sales data. As a result, it cannot be applied to new products or existing products without such historical data [7.61, p. 188].

A related problem of the demand side of warranty is the modeling of sales over time. Mahajan et al. presented several variant diffusion models that may be appropriate for consumer durables [7.62]. *Ja* et al. obtained the first two moments of warranty cost in a product life cycle by assuming a nonhomogeneous Poisson sale process [7.21]. It seems that such models do not allow the interaction between warranty and sales, therefore, they may not be used in estimating warranty benefit.

There is some research (*Emons* [7.15], *Lutz* and *Padmanabhan* [7.16], and *Padmanabhan* and *Rao* [7.17], etc.) on the demand side of warranty concerning moral hazard, advertising, consumers satisfaction, and so forth. However, compared to the vast warranty literature on estimating total warranty cost, the study on the demand side of warranty is far behind. Hopefully we will see more studies on this aspect in the future.

7.2.5 On the Optimal Warranty Policy

One of the most important objectives of warranty study is to assist warranty management. In particular, in the design phase of a warranty program, there are often a set of warranties that might be appropriate for a specific type of products. The problem faced by warranty managers therefore is how to determine the optimal warranty policy.

An early attempt to address the warranty design problem is based on the concept of life-cycle costing (*Blischke* [7.63], *Mamer* [7.64]). It is assumed that a consumer requires the product over a certain time period or life cycle from the same producer repeatedly upon each product failure no matter whether under warranty or not. Under this idealized producer–consumer relationship, the producer's life-cycle profit and the consumer's life-cycle cost can be calculated. Consequently, a consumer indifference price may be determined by comparing consumer's life-cycle costs with or without warranty. Similarly, the producer's indifference price may be calculated based on the comparison of the life-cycle profits with or without warranty.

An alternative approach is to set up an optimization problem to determine the optimal price and warranty length combination jointly through a game-theoretic perspective. In general, two parties, a warranty issuer and a representative consumer, participate in the game. The latter acts as a follower who responses rationally to each potential warranty offer by maximizing his/her utility. The former, as a leader, makes the decision on the optimal warranty strategy, which maximizes the expected profit, based on the anticipated rational response by the consumer. *Singpurwalla* and *Wilson* [7.5] studied two-attribute warranties through this approach. Some others references are *Chun* and *Tang* [7.65], *DeCroix* [7.13], *Glickman* and *Berger* [7.66], *Ritchken* [7.67], *Thomas* [7.60] and the references therein. In the context of production planning and marketing, *Mitra* and *Patankar* [7.68] presented a multi-criteria model that could be used in warranty design.

Now, we present a general formulation of the warranty design problem with some discussion, which may raise more interest among researchers and practitioners for further study.

Let $\Psi = \{\psi_1, \psi_2, \cdots, \psi_n\}$ represent the set of appropriate warranty policies for a given type of products. Policy ψ_i may contain more than one parameter. Denote by $\boldsymbol{w}_i$ the set of warranty parameters for ψ_i; then we can represent ψ_i by $\psi(\boldsymbol{w}_i)$ or $\boldsymbol{w}_i$. If $\boldsymbol{w}_i$ contains only one parameter, say, w_i, the warranty period, then $\boldsymbol{w}_i = \{w_i\}$. Denote by $p(\boldsymbol{w}_i)$ the selling price under the policy ψ_i, and by $C_j(\boldsymbol{w}_i)$ the random warranty cost for the j-th product sold under the policy ψ_i. Let p_0 be the production cost per unit (not including the warranty cost), then the optimal warranty policy $\psi(\boldsymbol{w}^*)$ may be obtained by solving

$$\max_{\{\boldsymbol{w}_i, \forall i, i=1,2,\cdots,n\}} \mathrm{E}\{\mathrm{U}[\pi(\boldsymbol{w}_i)]\}$$

$$\text{s.t.} \quad w_i^l \le w_i \le w_i^u, \forall i, i = 1, 2, \cdots, n$$

$$\mathrm{P}\left[\sum_{j=1}^{d(\boldsymbol{w}_i)} C_j(\boldsymbol{w}_i) \ge R_0\right] \le \alpha, \forall i, i = 1, 2, \cdots, n ,$$

where $U(\cdot)$ is the monetary utility function that reflects the risk attitude of the manufacturer. It is a linear function if the manufacturer is risk-neutral and a concave function in the case of a risk-averse manufacturer; $\pi(\boldsymbol{w}_i) = \sum_{j=1}^{d(\boldsymbol{w}_i)} [p(\boldsymbol{w}_i) - p_0 - C_j(\boldsymbol{w}_i)]$; w_i^l, w_i^u are some lower and upper bounds of w_i; $d(\boldsymbol{w}_i)$ represents the demand function for $\psi(\boldsymbol{w}_i)$; R_0 is the predetermined warranty budget level; and α is the risk-tolerance level of the manufacturer with regard to R_0.

One should note that the second set of constraints is actually related to *value at risk* (VaR), a concept widely used in risk management, which indicates the maximum percentage value of an asset that could be lost during a fixed period within a certain confidence level [7.69]. It is reasonable to assume that manufacturers want to control VaR such that the probability that the total warranty cost is over the budget is within the accepted level α.

Solving the optimization problem might be a challenge. First of all, it is difficult to determine the demand function $d(\boldsymbol{w}_i)$, although it is possible to estimate it through marketing surveys or historical data. Secondly, it is required that warranty managers have complete knowledge of the selling price $p(\boldsymbol{w}_i)$. This requires a pricing strategy in the design phase of warranty. It should be noted that we could have considered $p(\boldsymbol{w}_i)$ as one of the decision variables, but this makes the problem more complicated. Besides, it is not rare in practice that the price is simply set by adding a fixed margin over the estimated production cost with warranty. Thirdly, it is required that the probability distribution of warranty cost should be known. Little research has been done with regard to this issue except *Polatoglu* and *Sahin* [7.27] and *Sahin* and *Polatoglu* [7.70]. In general, numerical methods are required for this purpose. Fourthly, the problem is formulated as a nonlinear optimization problem with some constraints, which may be solved by nonlinear optimization software such as GAMS. However, in general there is no guarantee of the existence of a global optimal solution.

7.3 Concluding Remarks

A warranty problem, by its nature, is a multi-disciplinary research topic. Many researchers ranging from the industry engineer, economist, statistician, to marketing researchers have contributed greatly to warranty literature. In this chapter, we present an overview of warranty policies, focusing on the cost and benefit analysis from warranty issuers' perspective. Although we have successfully addressed several problems in this area, there are still a lot of opportunities for future research, a few of which are listed below:

- To advance warranty optimization models and perform empirical study based on the new developed models.
- To develop and apply efficient algorithms to solve warranty optimization problems.
- To propose and analyze new warranty policies appropriate for complex systems.
- To Study the distribution and the moments of discounted warranty cost for various policies.
- Warranty cost modeling for systems with more complex structures, including standby systems, bridge systems and network systems, etc.
- Develop warranty models considering failure dependency between components due to environmental impact.

References

7.1 W. R. Blischke, D. N. P. Murthy: *Warranty Cost Analysis* (Marcel Dekker, New York 1994)
7.2 W. R. Blischke, D. N. P. Murthy: Product warranty management–I: a taxonomy for warranty policies, Eur. J. Oper. Res. **62**, 127–148 (1993)
7.3 W. R. Blischke, D. N. P. Murthy (Eds.): *Product Warranty Handbook* (Marcel Dekker, New York 1996)
7.4 E. P. McGuire: *Industrial Product Warranties: Policies and Practices* (The Conference Board, New York 1980)
7.5 N. D. Singpurwalla, S. Wilson: The warranty problem: Its statistical, game theoretic aspects, SIAM Rev. **35**, 17–42 (1993)
7.6 J. Chen, D. D. Yao, S. Zheng: Quality control for products supplied with warranty, Oper. Res. **46**, 107–115 (1988)
7.7 I. Djamaludin, D. N. P. Murthy: Quality control through lot sizing for items sold with warranty, Int. J. Prod. Econ. **33**, 97–107 (1994)
7.8 G. G. Hegde, P. Kubat: Diagnosic design: A product support strategy, Eur. J. Oper. Res. **38**, 35–43 (1989)
7.9 Jie Mi: Warranty, burn-in, Naval Res. Logist. **44**, 199–210 (1996)
7.10 D. N. P. Murthy, A. Z. M. O. Hussain: Warranty, optimal redundancy design, Eng. Optim. **23**, 301–314 (1993)
7.11 D. G. Nguyen, D. N. P. Murthy: Optimal reliability allocation for products sold under warranty, Eng. Optim. **13**, 35–45 (1988)
7.12 I. Sahin: Conformance quality, replacement costs under warranty, Prod. Oper. Man. **2**, 242–261 (1993)
7.13 G. A. DeCroix: Optimal warranties, reliabilities, prices for durable goods in an oligopoly, Eur. J. Oper. Res. **112**, 554–569 (1999)
7.14 W. Emons: Warranties, moral hazard, the lemons problem, J. Econ. Theory **46**, 16–33 (1988)
7.15 W. Emons: On the limitation of warranty duration, J. Ind. Econ. **37**, 287–301 (1989)
7.16 M. A. Lutz, V. Padmanabhan: Warranties, extended warranties, product quality, Int. J. Ind. Organ. **16**, 463–493 (1998)
7.17 V. Padmanabhan, R. C. Rao: Warranty policy, extended service contracts: theory, an application to automobiles, Market. Sci **12**, 97–117 (1993)
7.18 D. N. P. Murthy, E. Asgharizadeh: Optimal decision making in a maintenance service operation, Eur. J. Oper. Res. **116**, 259–273 (1999)
7.19 E. W. Frees: Warranty analysis, renewal function estimation, Naval Res. Logist. Quart. **33**, 361–372 (1986)
7.20 E. W. Frees: Estimating the cost of a warranty, J. Bus. Econ. Stat. **6**, 79–86 (1988)

7.21 S. Ja, V. Kulkarni, A. Mitra, G. Patankar: Warranty reserves for non-stationary sales processes, Naval Res. Logist. **49**, 499–513 (2002)
7.22 J. D. Kalbfleisch, J. F. Lawless, J. A. Robinson: Methods for the analysis, prediction of warranty claims, Technometrics **33**, 273–285 (1991)
7.23 E. P. C. Kao, M. S. Smith: Computational approximations of renewal process relating to a warranty problem: the case of phase-type lifetimes, Eur. J. Oper. Res. **90**, 156–170 (1996)
7.24 E. P. C. Kao, M. S. Smith: On excess, current, total life distributions of phase-type renewal processes, Naval Res. Logist. **39**, 789–799 (1992)
7.25 U. Menzefricke: On the variance of total warranty claims, Comm. Statist. Theory Methods **21**, 779–790 (1992)
7.26 J. G. Patankar, G. H. Worm: Prediction intervals for warranty reserves, cash flows, Man. Sci. **27**, 237–241 (1981)
7.27 H. Polatoglu, I. Sahin: Probability distribution of cost, revenue, profit over a warranty cycle, Eur. J. Oper. Res. **108**, 170–183 (1998)
7.28 Y. H. Chun, K. Tang: Cost analysis of two-attribute warranty policies based on the product usage rate, IEEE Trans. Eng. Man. **46**, 201–209 (1999)
7.29 Y. Lam, P. K. W. Lam: An extended warranty policy with options open to consumers, Eur. J. Oper. Res. **131**, 514–529 (2001)
7.30 C. Wang, S. Sheu: Optimal lot sizing for products sold under free-repair warranty, Eur. J. Oper. Res. **164**, 367–377 (2005)
7.31 R. H. Yeh, W. T. Ho, S. T. Tseng: Optimal production run length for products sold with warranty, Eur. J. Oper. Res. **120**, 575–582 (2000)
7.32 R. H. Yeh, H. Lo: Optimal preventive-maintenance warranty policy for repairable products, Eur. J. Oper. Res. **134**, 59–69 (2001)
7.33 D. N. P. Murthy: Product warranty management—III: A review of mathematical models, Eur. J. Oper. Res. **62**, 1–34 (1992)
7.34 D. N. P. Murthy, I. Djamaludin: New product warranty: a literature review, Int. J. Prod. Econ. **79**, 231–260 (2002)
7.35 M. U. Thomas, S. S. Rao: Warranty economic decision models: A summary, some suggested directions for future research, Oper. Res. **47**, 807–820 (1999)
7.36 R. E. Barlow, F. Proschan: *Mathematical Theory of Reliability* (Wiley, New York 1965)
7.37 H. Pham, H. Wang: Imperfect maintenance, Eur. J. Oper. Res. **94**, 425–438 (1996)
7.38 H. Wang: Reliability and maintenance modeling for systems with imperfect maintenance and dependence. Ph.D. Thesis (Rutgers University, Piscataway 1997) (unpublished)
7.39 J. Bai, H. Pham: Cost analysis on renewable full-service warranties for multi-component systems, Eur. J. Oper. Res. **168**, 492–508 (2006)
7.40 Y. H. Chun: Optimal number of periodic preventive maintenance operations under warranty, Reliab. Eng. Sys. Saf. **37**, 223–225 (1992)
7.41 N. Jack, J. S. Dagpunar: An optimal imperfect maintenance policy over a warranty period, Microelectron. Reliab. **34**, 529–534 (1994)
7.42 J. S. Dagpunar, N. Jack: Optimal repair-cost limit for a consumer following expiry of a warranty, IMA J. Math. Appl. Bus. Ind. **4**, 155–161 (1992)
7.43 I. Sahin, H. Polatoglu: Maintenance strategies following the expiration of warranty, IEEE Trans. Reliab. **45**, 221–228 (1996)
7.44 H. Wang: A survey of maintenance policies of deteriorating systems, Eur. J. Oper. Res. **139**, 469–489 (2002)
7.45 J. Bai, H. Pham: RLRF warranty policies with imperfect repair: A censored quasirenewal process approach, working paper, Department of Industrial and Systems Engineering, Rutgers University (2003)
7.46 D. G. Nguyen, D. N. P. Murthy: Optimal replacement–repair strategy for servicing products sold under warranty, Eur. J. Oper. Res. **39**, 206–212 (1989)
7.47 D. N. P. Murthy, B. P. Iskandar, R. J. Wilson: Two dimensional failure-free warranty policies: two-dimensional point process models, Oper. Res. **43**, 356–366 (1995)
7.48 H. Moskowitz, Y. H Chun: A Poisson regression model for two-attribute warranty policies, Naval Res. Logist. **41**, 355–376 (1994)
7.49 H. G. Kim, B. M. Rao: Expected warranty cost of two-attribute free-replacement warranties based on a bivariate exponential distribution, Comput. Ind. Eng. **38**, 425–434 (2000)
7.50 E. A. Elsayed: *Reliability Engineering* (Addison Wesley Longman, Reading 1996)
7.51 V. Lee Hill, C. W. Beall, W. R. Blischke: A simulation model for warranty analysis, Int. J. Prod. Econ. **16**, 463–491 (1998)
7.52 D. M. Kreps: *A Course in Microeconomic Theory* (Princeton Univ. Press, Princeton 1990)
7.53 H. Markowitz: *Portfolio Selection* (Yale Univ. Press, Yale 1959)
7.54 P. H. Ritchken, C. S. Tapiero: Warranty design under buyer, seller risk aversion, Naval Res. Logist. Quart. **33**, 657–671 (1986)
7.55 S. Chukova, B. Dimitrov: Warranty analysis for complex systems. In: *Product Warranty Handbook*, ed. by W. R. Blischke, D. N. P. Murthy (Marcel Dekker, New York 1996) pp. 543–584
7.56 P. H. Ritchken: Optimal replacement policies for irreparable warranted item, IEEE Trans. Reliab. **35**, 621–624 (1986)
7.57 A. Z. M. O. Hussain, D. N. P. Murthy: Warranty, redundancy design with uncertain quality, IEEE Trans. **30**, 1191–1199 (1998)

7.58 J. Bai, H. Pham: Discounted warranty cost for minimally repaired series systems., IEEE Trans. Reliab. **53**(1), 37–42 (2004)
7.59 K. R. Balachandran, R. A. Maschmeyer, J. L. Livingstone: Product warranty period: A Markovian approach to estimation, analysis of repair, replacement costs, Acc. Rev. **1**, 115–124 (1981)
7.60 M. U. Thomas: Optimum warranty policies for nonreparable items, IEEE Trans. Reliab. **32**, 283–288 (1983)
7.61 M. Menezes: An approach for determination of warranty length, Int. J. Res. Market. **9**, 177–195 (1992)
7.62 V. Mahajan, E. Muller, Y. Wind: *New-product diffusion models* (Kluwer Academic, Dordrecht 2000)
7.63 W. R. Blischke, E. M. Scheuer: Calculating the cost of warranty policies as a function of estimated life distributions, Naval Res. Logist. Quart. **28**, 193–205 (1975)
7.64 J. W. Mamer: Discounted, per unit costs of product warranty, Man. Sci. **33**, 916–930 (1987)
7.65 Y. H. Chun, K. Tang: Determining the optimal warranty price based on the producer's, customers' risk preferences, Eur. J. Oper. Res. **85**, 97–110 (1995)
7.66 T. S. Glickman, P. D. Berger: Optimal price, protection period decisions for a product under warranty, Man. Sci. **22**, 1381–1390 (1976)
7.67 P. H. Ritchken: Warranty policies for non-repairable items under risk aversion, IEEE Trans. Reliab. **34**, 147–150 (1985)
7.68 A. Mitra, J. G. Patankar: An integrated multicriteria model for warranty cost estimation, production, IEEE Trans. Eng. Man. **40**, 300–311 (1993)
7.69 P. Jorion: *Value-at-Risk: The New Benchmark for Managing Financial Risk* (McGraw-Hill, New York 2000)
7.70 I. Sahin, H. Polatoglu: Distributions of manufacturer's, user's replacement costs under warranty, Naval Res. Logist. **42**, 1233–1250 (1995)