16. Some Statistical Models for the Monitoring of High-Quality Processes

One important application of statistical models in industry is statistical process control. Many control charts have been developed and used in industry. They are easy to use, but have been developed based on statistical principles. However, for today's high-quality processes, traditional control-charting techniques are not applicable in many situations. Research has been going on in the last two decades and new methods have been proposed. This chapter summarizes some of these techniques.

High-quality processes are those with very low defect-occurrence rates. Control charts based on the cumulative count of conforming items are recommended for such processes. The use of such charts has opened up new frontiers in the research and applications of statistical control charts in general. In this chapter, several extended or modified statistical models are described. They are useful when the simple and basic geometric distribution is not appropriate or is insufficient.

In particular, we present some extended Poisson distribution models that can be used for count data with large numbers of zero counts. We also extend the chart to the case of general time-between-event monitoring; such an extension can be useful in service or reliability monitoring.

Traditionally, the exponential distribution is used for the modeling of time-between-events, although other distributions such as the Weibull or gamma distribution can also be used in this context.

Control charting is one of the most widely used statistical techniques in industry for process control and monitoring. It dates back to the 1920s when Walter Shewhart introduced the basic charting techniques in the United States [16.1]. Since then, it has been widely adopted worldwide, mainly in manufacturing and also in service industries. The simplicity of the application procedure allows a non-specialist user to observe the data and plot the control chart for simple decision making. At the same time, it provides sophisticated statistical interpretation in terms of false-alarm probability and average run length, among other important statistical properties associated with decision making based on sample information. The implementation of control charts had helped many companies to focus on important quality issues and problems such as those raised by out-of-control points on a control chart.

However, for high-quality or near-zero-defect processes, traditional Shewhart charts may not be suitable for process monitoring and decision making. This is especially the case for Shewhart attribute charts [16.2]. Many problems such as high false-alarm probability, inability to detect process improvement, unnecessary plotting of many zeros etc., have been identified by various researchers [16.3–6]. To resolve these problems, new models and monitoring techniques have been developed recently.

Traditional charts are all based on the principle of normal distribution and the upper control limit (UCL) and lower control limit (LCL) are routinely computed as the mean plus and minus three times the standard deviation. That is, if the plotted quantity Y has mean μ and standard deviation σ, the control limits are given by

$$\mathrm{UCL} = \mu + 3\sigma \quad \textit{and} \quad \mathrm{LCL} = \mu - 3\sigma \,. \tag{16.1}$$

Generally, when the distribution of Y is skewed, the probability of false alarm, i.e. the probability that a point indicating out-of-control when the process has actually not changed, is different from the nominal value of 0.0027 associated with a truly normal distribution. Note that for attribute charts, the plotted quantities usually follow a binomial or Poisson distribution, and this is far from the normal distribution unless the sample size is very large.

The purpose of this chapter is to review the important models and techniques that can be used to monitor high-quality processes. The procedure based on a general principle of the cumulative count of conforming items is first described; this is then extended to other distributions. The emphasis is on recent developments and also on practical methods that can be used by practitioners.

This chapter is organized as follows. First, the use of probability limits is described. Next, control charts based on monitoring of the cumulative count of conforming items and simple extensions are discussed. Control charts based on the zero-inflated Poisson distribution and generalize Poisson distribution are then presented. These charts are widely discussed in the literature and they are suitable for count or attribute data. For process monitoring, time-between-events monitoring is of growing importance, and we also provide a summary of methods that can be used to monitor process change based on time-between-events data. Typical models are the exponential, Weibull and gamma distribution.

16.1 Use of Exact Probability Limits

For high-quality processes it is important to use probability limits instead of traditional three-sigma limits. This is true when the quality characteristic that is being plotted follows a skewed distribution. For any plotted quality characteristic Y, the probability limits LCL_Y and UCL_Y can be derived as

$$P(X < \mathrm{LCL}_Y) = P(X > \mathrm{UCL}_Y) = \alpha/2 \,, \tag{16.2}$$

where α is the false-alarm probability, i.e., when the process is in control, the probability that the control chart raises an alarm. Assuming that the distribution $F(x)$ is known or has been estimated accurately from the data, the control limits can be computed.

Probability limits are very important for attribute charts as the quality characteristics are usually not normally distributed. If this is the case, the false-alarm probability could be much higher than the nominal value ($\alpha = 0.0027$ for traditional three-sigma limits). *Xie* and *Goh* [16.7] studied the exact probability limits calculated from the binomial distribution and the Poisson distribution applied for the np chart and the c chart.

For control-chart monitoring the number of nonconforming items in samples of size n, assuming that the process fraction nonconforming is p, the probability that there are exactly k nonconforming items in the sample is

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \,, \quad k = 0, 1, \ldots n \tag{16.3}$$

and the probability limits can be computed as

$$P(X \leqslant \mathrm{LCL}) = \sum_{i=0}^{\mathrm{LCL}} \binom{n}{i} p^i (1-p)^{n-i} = \frac{\alpha}{2} \tag{16.4}$$

and

$$P(X \leqslant \mathrm{UCL}) = \sum_{i=0}^{\mathrm{UCL}} \binom{n}{i} p^i (1-p)^{n-i} = 1 - \frac{\alpha}{2} \,. \tag{16.5}$$

As discussed, probability limits can be computed for any distributions, and should be used when the distribution is skewed. This will form the basis of the following discussion in this chapter. In some cases, although the solution is analytically intractable, they can be obtained with computer programs. It is advisable that probability limits be used unless the normality test indicates that deviation from normal distribution is not significant.

16.2 Control Charts Based on Cumulative Count of Conforming Items

High-quality processes are usually characterized by low defective rates. In a near-zero-defect manufacturing environment, items are commonly produced and inspected one-by-one, sometimes automatically. We can record and use the cumulative count of conforming items produced before a nonconforming item is detected. This technique has been intensively studied in recent years.

16.2.1 CCC Chart Based on Geometric Distribution

The idea of tracking cumulative count of conforming (CCC) items to detect the onset of assignable causes in an automated (high-quality) manufacturing environment was first introduced in [16.3]. *Goh* [16.4] further developed this idea into what is known as the CCC charting technique. Some related discussions and further studies can be found in [16.8–14], among others. *Xie* et al. [16.15] provided extensive coverage of this charting technique and further analysis of this procedure.

For a process with a defective rate of p, the cumulative count of conforming items before the appearance of a nonconforming item, Y, follows a geometric distribution. This is given by

$$P(Y=n)=(1-p)^{n-1}p, \quad n=1,2,\ldots\,. \tag{16.6}$$

The cumulative probability function of count Y is given by

$$P(Y\leqslant n)=\sum_{i=1}^{n}(1-p)^{i-1}p=1-(1-p)^{n}\,. \tag{16.7}$$

Assuming that the acceptable false-alarm probability is α, the probability limits for the CCC chart are obtained as

$$\mathrm{UCL}=\ln(\alpha/2)/\ln(1-p) \tag{16.8}$$

and

$$\mathrm{LCL}=\ln(1-\alpha/2)/\ln(1-p) \tag{16.9}$$

Usually the center line (CL) is computed as

$$\mathrm{CL}=\ln(1/2)/\ln(1-p)\,. \tag{16.10}$$

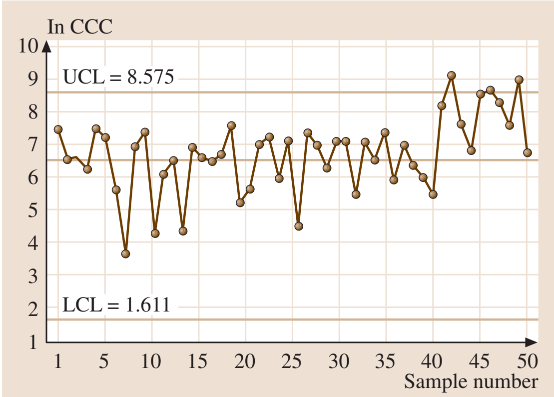

Fig. 16.1 A typical cumulative count of conforming (CCC) items chart

A typical CCC chart is shown in Fig. 16.1 The first 40 data points are simulated with $p=0.001$ and the last one was simulated with $p=0.0002$. The value of α is set to be 0.01 for the calculation of control limits. Note that we have also used a log scale for CCC.

Note that the decision rule is different from that of the traditional p or np chart. If a point is plotted above the UCL, the process is considered to have improved. When a point falls below the LCL, the process is judged to have deteriorated. An important advantage is that the CCC chart can detect not only the increase in the defective rate (process deterioration), but also the decrease in the defective rate (process improvement).

16.2.2 CCC-*r* Chart Based on Negative Binomial Distribution

A simple idea to generalize a CCC chart is to consider plotting of the cumulative count of items inspected until observing two nonconforming items. This was studied in [16.16] resulting in the CCC-2 control chart. This chart increases the sensitivity of the original CCC chart for the detection of small process shifts in p. The CCC-2 chart has smaller type II error, which is related to chart sensitivity, and steeper OC (Operating Characteristic) curves than the CCC chart with the same type I, error which is the false alarm probability.

A CCC-r chart [16.17, 18] plots the cumulative count of items inspected until r nonconforming items are observed. This will further improve the sensitivity and detect small changes faster. However, it requires more counts to be cumulated in order to generate an alarm signal. The CCC-r charting technique was also studied by *Lu* et al. [16.17].

Let Y be the cumulative count of items inspected until r nonconforming items have been observed. Let the probability of an item to be nonconforming be p. Then Y follows a negative binomial distribution given by

$$P(Y=n)=\binom{n-1}{r-1}p^r(1-p)^{n-r}\,,\quad n=r,r+1,\ldots\,. \tag{16.11}$$

The cumulative distribution function of count Y would be

$$\begin{aligned}F(n,r,p)&=\sum_{i=r}^{n}P(Y=i)\\&=\sum_{i=r}^{n}\binom{i-1}{r-1}p^r(1-p)^{i-r}\,.\end{aligned} \tag{16.12}$$

If the acceptable false-alarm probability is α, then the upper control limit and the lower control limit, UCL_r and LCL_r, respectively, of the CCC$-r$ chart can be obtained as the solution of the following equations:

$$\begin{aligned}F(\mathrm{UCL}_r,r,p)&=\sum_{i=r}^{\mathrm{UCL}_r}\binom{i-1}{r-1}p^r(1-p)^{i-r}\\&=1-\alpha/2\end{aligned} \tag{16.13}$$

and

$$F(\mathrm{LCL}_r,r,p)=\sum_{i=r}^{\mathrm{LCL}_r}\binom{i-1}{r-1}p^r(1-p)^{i-r}=\alpha/2\,. \tag{16.14}$$

Note that this chart is suitable for one-by-one inspection process and so no subjective sample size is needed. On the other hand, the selection of r is a subjective issue if the cost involved is not a consideration. As the value of r increases the sensitivity of the chart may increase, but the user probably needs to wait too long to plot a point. *Ohta* et al. [16.18] addressed this issue from an economic design perspective and proposed a simplified design method to select a suitable value of r based on the economic design method for control charts that monitor discrete quality characteristics.

16.3 Generalization of the c-Chart

The c-chart is based on monitoring of the number of defects in a sample. Traditionally, the number of defect in a sample follows the Poisson distribution. The control limits are computed as

$$\mathrm{UCL}=c+3\sqrt{c}\quad\text{and}\quad\mathrm{LCL}=c-3\sqrt{c}\,, \tag{16.15}$$

where c is the average number of defects in the sample and the LCL is set to be zero when the value computed with (16.15) is negative.

However, for high-quality processes, it has been shown that these limits may not be appropriate. Some extensions of this chart are described in this section.

16.3.1 Charts Based on the Zero-Inflated Poisson Distribution

In a near-zero-defect manufacturing environment, many samples will have no defects. However, for those containing defects, we have observed that there could be many defects in a sample and hence the data has an over-dispersion pattern relative to the Poisson distribution. To overcome this problem, a generalization of Poisson distribution was used in [16.6, 19].

This distribution is commonly called the zero-inflated Poisson distribution. Let Y be the number of defects in a sample; the probability mass function is given by

$$\begin{cases}P(Y=0)=(1-p)+p\,\mathrm{e}^{-\lambda}\\ P(Y=d)=p\frac{\lambda^d\,\mathrm{e}^{-\lambda}}{d!}\qquad d=1,2,\ldots\,.\end{cases} \tag{16.16}$$

This has an interesting interpretation. The process is basically zero-defect although it is affected by causes that lead to one or more defects. If the occurrence of these causes is p, and the severity is λ, then the number of defects in the sample will follow a zero-inflated Poisson distribution.

When the zero-inflated Poisson distribution provides a good fit to the data, two types of control charts can be applied. One is the exact probability limits control chart, and the other is the CCC chart. When implementing the exact probability limits chart, *Xie* and *Goh* [16.6] suggested that only the upper control limit n_u should be considered, since the process is in a near-zero-defect manufacturing environment and the probability of zero is usually very large. The upper control limit can be determined by:

$$\sum_{d=n_\mathrm{u}}^{\infty} p \frac{\lambda^d \mathrm{e}^{-\lambda}}{d!} \leq \alpha \,, \tag{16.17}$$

where α is the probability of the type I error. It should be noticed that n_u could easily be solved because it takes only discrete values.

Control charts based on the zero-inflated Poisson distribution commonly have better performance in the near-zero-defect manufacturing environment. However, the control procedure is more complicated than the traditional methods since more effort is required to test the suitability of this model with more parameters.

For the zero-inflation Poisson distribution we have that [16.20]

$$E(Y) = p\lambda \tag{16.18}$$

and

$$\mathrm{Var}(Y) = p\lambda + p\lambda(\mu - p\lambda) \,. \tag{16.19}$$

It should be pointed out that the zero-inflation Poisson model is very easy to use, as the mean and variance are of close form. For example, the moment estimates can be obtained straightforward. On the other hand, the maximum-likelihood estimates can also be obtained.

The maximum-likelihood estimates can be obtained by solving

$$\begin{cases} p = \dfrac{1 - n_0/n}{1 - \exp(-\lambda)} \\ \lambda = \bar{y}/p \end{cases} , \tag{16.20}$$

where $\bar{y} = \sum_{i=1}^{n} y_i/n$, [16.20].

When the count data can be fitted by a zero-inflation Poisson model, statistical process control procedures can be modified. Usually, the lower control limit for zero-inflation Poisson model will not exist, because the probability of zero is larger than the predetermined type I error level. This is common for the attribute control chart. In the following section, the upper control limit will be studied.

The upper control limit n_u for a control chart based on the number of nonconformities can be obtained as the smallest integer solution of the following equation:

$$P(n_\mathrm{u} \text{ or more nonconformities in a sample}) \leqslant \alpha_\mathrm{L} \,, \tag{16.21}$$

where α_L is the predetermined false-alarm probability for the upper control limit n_u.

Here our focus is on data modeling with appropriate distribution. It can be noted that the model contains two parameters. To be able to monitor the change in each parameter, a single chart may no be appropriate. *Xie* and *Goh* [16.6] developed a procedure for the monitoring of individual parameter. First, a CCC chart is used for data with zero count. Second, a c-chart is used for those with one or more non-zero count.

Note that a useful model should have practical interpretations. In this case, p is the occurrence probability of problem in the process, and λ measures the severity of the problem when it occurs. Hence it is a useful model and important to be able to monitor each of these parameters, so that any change from normal behavior can be identified.

Example 1

An example is used here for illustration [16.2]. The data set used in Table 16.1 is the read–write errors discovered in a computer hard disk in a manufacturing process.

For the data set in Table 16.1, it can be seen that it contains many samples with no nonconformities. From the data set, the maximum-likelihood estimates are $\hat{p} = 0.1346$ and $\hat{\mu} = 8.6413$. The overall zero-inflation

Table 16.1 A set of defect count data

0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	6	0	9
11	0	1	2	0	0	0	0	0	0	0	0	3	3	0	0	5	0	15	6
0	0	0	4	2	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
75	0	0	0	0	75	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0
0	2	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0
0	0	1	0	0	0	0	0												

Poisson model for the data set is

$$f(y) = \begin{cases} 1 - 0.1346 + 0.1346 \exp(-8.6413)\,, \\ \quad \text{if } y = 0\,, \\ 0.1346 \frac{8.6413^y \exp(-8.6413)}{y!}\,, \\ \quad \text{if } y > 0\,. \end{cases} \tag{16.22}$$

For the data set in Table 16.1, it can be calculated that the upper control limit is 14 at an acceptable false-alarm rate of 0.01. This means that there should not be any alarm for values less than or equal to 14 when the underlying distribution model is a zero-inflated Poisson distribution.

16.3.2 Chart Based on the Generalized Poisson Distribution

The generalized Poisson distribution is another useful model that extends the traditional Poisson distribution, which only has one parameter. A two-parameter model is usually much more flexible and able to model different types of data sets. Since in the situation of over-dispersion or under-dispersion the Poisson distribution is no longer preferable as it must have equal mean and variance, the generalized Poisson distribution [16.21] can be used.

This distribution has two parameters (θ, λ) and the probability mass function is defined as

$$P_X(\theta, \lambda) = \frac{\theta(\theta + x\lambda)^{x-1}\,\mathrm{e}^{-\theta - x\lambda}}{x!}, \quad x = 0, 1, 2 \ldots\,, \tag{16.23}$$

where $\lambda, \theta > 0$.

For the generalized Poisson distribution we have that [16.21]

$$E(X) = \theta(1 - \lambda)^{-1} \tag{16.24}$$

and

$$\mathrm{Var}(X) = \theta(1 - \lambda)^{-3}\,. \tag{16.25}$$

It should be pointed out that the generalized Poisson distribution model is very easy to use as both the mean and variance are of closed form. For example, the moment estimates can easily be calculated. On the other hand, the maximum-likelihood estimates can also be obtained straightforwardly. Consider a set of observations $\{X_1, X_2, \ldots, X_n\}$ with sample size n, the maximum-likelihood estimation $(\hat{\theta}, \hat{\lambda})$ can be obtained by solving

$$\begin{cases} \sum_{i=1}^{n} \frac{x_i(x_i - 1)}{\bar{x} + (x_i - \bar{x})\hat{\lambda}} - n\bar{x} = 0\,, \\ \hat{\theta} = \bar{x}(1 - \hat{\lambda})\,. \end{cases} \tag{16.26}$$

Here a similar approach as for the zero-inflated Poisson model can be used. One could also developed two charts for practical monitoring. One chart can be used to monitor the severity and another to monitor the dispersion or variability in terms of the occurrence of defects.

Example 2

The data in Table 16.1 can also be modeled with a generalized Poisson distribution. Based on the data, the maximum-likelihood estimates can be computed as $\hat{\theta} = 0.144297$ and $\hat{\lambda} = 0.875977$. The overall generalized Poisson distribution model for the data set is

$$f(x) = \frac{0.144297(0.144297 + 0.875977x)^{x-1}}{x!} \times \frac{\mathrm{e}^{-0.144297 - 0.875977x}}{x!}, \quad x = 0, 1, 2 \ldots\,. \tag{16.27}$$

With this model, it can be calculated that the upper control limit is 26 at a false-alarm rate of 0.01. This means that there should not be any alarm for the values less than or equal to 26 when the underlying distribution model is the generalized Poisson distribution. It should be mentioned here that, for this data set, both models can fit the data well, and the traditional Poisson distribution is rejected by statistical tests.

16.4 Control Charts for the Monitoring of Time-Between-Events

Chan et al. [16.22] proposed a charting method called the cumulative quantity control chart (CQC chart). Suppose that defects in a process are observed according to a Poisson process with mean rate of occurrence equal to λ (>0). Then the number of units Q required to observe exactly one defect is an exponential random variable. The control chart for Q can be constructed to monitor possible shifts of λ in the process, which is the CQC chart.

The CQC chart has several advantages. It can be used for low-defective-rate processes as well as moderate-defective-rate processes. When the process defect rate is low or moderate, the CQC chart does not have the shortcoming of showing up frequent false alarms. Furthermore, the CQC chart does not require rational grouping of samples. The data required is the time between defects or defective items. This type of data is commonly available in equipment and process monitoring for production and maintenance.

When process failures can be described by a Poisson process, the time between failures will be exponential and the same procedure can be used in reliability monitoring. Here we briefly describe the procedure for this type of monitoring. Since time is our preliminary concern, the control chart will be termed a t-chart in this paper. This is in line with the traditional c-chart or u-chart, to which our t-chart may be a more suitable alternative. In fact, the notation also makes it easier for the extension to be discussed later.

16.4.1 CQC Chart Based on the Exponential Distribution

The distribution function of the exponential distribution with parameter λ is given by

$$F(t;\lambda) = 1 - \mathrm{e}^{-\lambda t}, \quad t \geqslant 0\,. \tag{16.28}$$

The control limits for t-chart are defined in such a manner that the process is considered to be out of control when the time to observe exactly one failure is less than the lower control limit (LCL), T_{L}, or greater than the upper control limit (UCL), T_{U}. When the behavior of the process is normal, there is a chance for this to happen and it is commonly known as a false alarm. The traditional false-alarm probability is set to be 0.27%, although any other false-alarm probability can be used. The actual acceptable false-alarm probability should in fact depend on the actual product or process. Assuming an acceptable probability for false alarms of α, the control limits can be obtained from the exponential distribution as:

$$T_{\mathrm{L}} = \lambda^{-1} \ln \frac{1}{1-\alpha/2} \tag{16.29}$$

and

$$T_{\mathrm{U}} = \lambda^{-1} \ln \frac{2}{\alpha}\,. \tag{16.30}$$

The median of the distribution is the center line (CL), T_{C}, and it can be computed as

$$T_{\mathrm{C}} = \lambda^{-1} \ln 2 = 0.693\lambda^{-1}\,. \tag{16.31}$$

These control limits can then be utilized to monitor the failure times of components. After each failure the time can be plotted on the chart. If the plotted point falls between the calculated control limits, this indicates that the process is in the state of statistical control and no action is warranted. If the point falls above the upper control limit, this indicates that the process average, or the failure occurrence rate, may have decreased, resulting in an increase in the time between failures. This is an important indication of possible process improvement. If this happens the management should look for possible causes for this improvement and if the causes are discovered then action should be taken to maintain them. If the plotted point falls below the lower control limit, this indicates that the process average, or the failure occurrence rate, may have increased, resulting in a decrease in the failure time. This means that the process may have deteriorated and thus actions should be taken to identify and remove them.

In either case the people involved can know when the reliability of the system has changed and by a proper follow-up they can maintain and improve the reliability. Another advantage of using the control chart is that it informs the maintenance crew when to leave the process alone, thus saving time and resources.

16.4.2 Chart Based on the Weibull Distribution

It is well known that the lifetime distribution of many components follows a Weibull distribution [16.23]. Hence when monitoring reliability or equipment failure, this distribution has been shown to be very useful. The Weibull distribution function is given as

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right], \quad t \geq 0\,, \tag{16.32}$$

where $\theta > 0$ and $\beta > 0$ are the so called scale parameter and shape parameter, respectively.

The Weibull distribution is a generalization of exponential distribution, which is recovered for $\beta = 1$. Although the exponential distribution has been widely used for times-between-event, Weibull distribution is more suitable as it is more flexible and is able to deal with different types of aging phenomenon in reliability. Hence in reliability monitoring of equipment failures, the Weibull distribution is a good alternative.

A process can be monitored with a control chart and the time-between-events can be used. For the Weibull distribution, the control limits can be calculated as:

$$\mathrm{UCL} = \theta_0 \left[\ln\left(\frac{2}{\alpha}\right)\right]^{1/\beta_0} \tag{16.33}$$

and

$$\mathrm{LCL} = \theta_0 \left[\ln \left(\frac{2}{2-\alpha} \right) \right]^{1/\beta_0} , \tag{16.34}$$

where α is the acceptable false-alarm probability, and β_0 and θ_0 are the in-control shape and scale parameter, respectively. Generally, the false-alarm probability is fixed at $\alpha = 0.0027$, which is equivalent to the three-sigma limits for an X-bar chart under the normal-distribution assumption.

The center line can be defined as

$$\mathrm{CL} = \theta_0 \, [\ln 2]^{1/\beta_0} . \tag{16.35}$$

Xie et al. [16.24] carried out some detailed analysis of this procedure. Since this model has two parameters, a single chart may not be able to identify changes in a parameter. However, since in a reliability context, it is unlikely that the shape parameter will change and it is the scale parameter that could be affected by ageing or wear, a control chart as shown in Fig. 16.2 can be useful in reliability monitoring.

16.4.3 General *t*-Chart

In general, to model time-between-events, any distribution for positive random variables could be used. Which distribution is used should depend on the actual data, with the exponential, Weibull and Gamma being the most common distributions. However, these distributions are usually very skewed. The best approach is to use probability limits. It is also possible to use a transformation so that the data is transformed to near-normality, so that traditional chart for individual data can be used; such charting procedure is commonly available in statistical process control (SPC) software.

In general, if the variable Y follows the distribution $F(t)$, the probability limits can be computed as usual, that is:

$$F(\mathrm{LCL}_Y) = 1 - F(\mathrm{UCL}_Y) = \alpha/2 , \tag{16.36}$$

where α is the fixed false-alarm rate. This is an approach that summarizes the specific cases described earlier. However, it is important to be able to identify the distribution to be used.

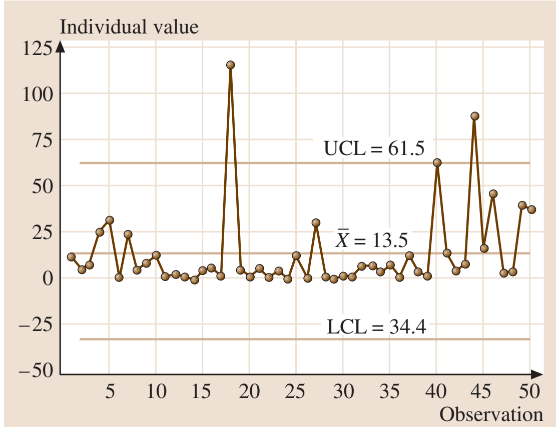

Fig. 16.2 A set of Weibull data and the plot

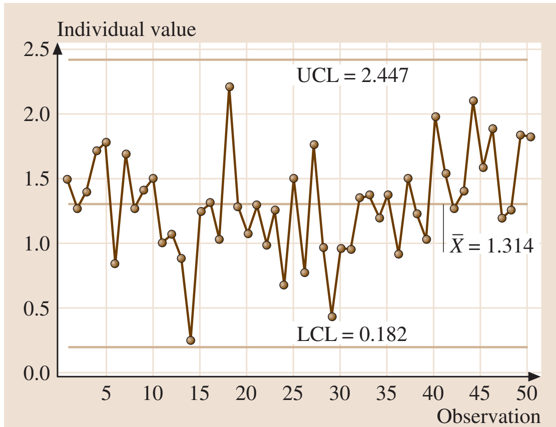

Fig. 16.3 The same data set as in Fig. 16.2 with the plot of the Box–Cox transformation

Furthermore, to make better use of the traditional monitoring approach, we could use a simple normality transformation. The most common ones are the Box–Cox transformation and the log or power transformations. They can be easily realized in software such as MINITAB. Figure 16.2 shows a chart for a Weibull-distributed process characteristic and Fig. 16.3 shows the individual chart with a Box–Cox transformation.

16.5 Discussion

In this chapter, some effective control-charting techniques are described. the statistical monitoring technique should be tailored to the specific distribution of the data that are collected from the process. Perfunctory use of the traditional chart will not help much in today's manufacturing environment towards near-zero-defect process. For high-quality processes, it is more appropriate to monitor items inspected between two nonconforming items or the time between two events.

The focus in this article is to highlight some common statistical distributions for process monitoring. Several statistical models such as the geometric, negative binomial, zero-inflated Poisson, and generalized Poisson can be used for count-data monitoring in this context. The exponential, Weibull and Gamma distributions can be used to monitor time-between-events data, which is common in reliability or equipment failure monitoring. Other general distributions of time-between-events can also be used when appropriate. The approach is still simple: by computing the probability limits for a fixed false-alarm probability, any distribution can be used in a similar way. The simple procedure is summarized below:

Step 1. Study the process and identify the statistical distribution for the process characteristic;

Step 2. Collect data and estimate the parameters (and validate the model, if needed);

Step 3. Compute the probability limits or use an appropriate normality transformation with an individual chart;

Step 4. Identify any assignable cause and take appropriate action.

The distributions presented in this paper open the door to further implementation of statistical process control techniques in a near-zero-defect era. Several research issues remain. For example, the problem with correlated data and the estimation problem has to be studied. In a high-quality environment, failure or defect data is rare, and the estimation problem becomes serious. In the case of continuous production and measurement, data correlation also becomes an important issue. It is possible to extend the approach to consider the exponentially weighted moving-average (EWMA) or cumulative-sum (CUSUM) charts that are widely advocated by statisticians. A further area of importance is multivariate quality characteristics. However, a good balance between statistical performance and ease of implementation and understanding by practitioners is essential.

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