48. Statistical Management and Modeling for Demand of Spare Parts

In recent years increased emphasis has been placed on improving decision making in business and government. A key aspect of decision making is being able to predict the circumstances surrounding individual decision situations. Examining the diversity of requirements in planning and decision-making situations, it is clearly stated that no single forecasting methods or narrow set of methods can meet the needs of all decision-making situations. Moreover, these methods are strongly dependent on factors such as data quantity, pattern and accuracy, that reflect their inherent capabilities and adaptability, such as intuitive appeal, simplicity, ease of application and, not least, cost.

Section 48.1 deals with the placement of the demand-forecasting problem as one of biggest challenge in the repair and overhaul industry; after this brief introduction Sect. 48.2 summarizes the most important categories of forecasting methods; Sects. 48.3–48.4 approach the forecast of spare parts firstly as a theoretical construct, although some industrial applications and results are added from field training, as in many other parts of this chapter.

Section 48.5 undertakes the question of optimal stock level for spare parts, with particular regard to low-turnaround-index (LTI) parts conceived and designed for the satisfaction of a specific customer request, by the application of classical Poisson methods of minimal availability and minimum cost; similar considerations are drawn and compared in Sect. 48.6, which deals with models based on the binomial distribution. An innovative extension of binomial models based on the total cost function is discussed in Sect. 48.7. Finally Sect. 48.8 adds the Weibull failure-rate function to the analysis of the LTI spare-parts stock level in a maintenance system with declared wear conditions.

48.1 The Forecast Problem for Spare Parts

Demand forecasting is one of the most crucial issues for inventory management. Forecasts, which form the basis for the planning of inventory levels, are probably the biggest challenge in the repair and overhaul industry.

An example can be seen in the airline industry, where a common problem is the need to forecast short-term demand with the highest possible degree of accuracy. The high cost of modern aircraft and the expense of repairable spares, such as aircraft engines and avionics, contribute significantly to the considerable total investment of many airline operators. These parts, although required with low demand, are critical to operation and their unavailability can lead to excessive downtime costs.

This problem is absolutely relevant in case of intermittent demand. Demand for an item is classified as *intermittent* when it is irregular and sporadic. This type of demand, typical for a large number of spare parts, is very difficult to predict. This complicates efficient management and control of the inventory system, which requires an acceptable balance between inventory costs on one hand and stock-outs on the other. Inventory management models require accurate forecasts in order to achieve this balance.

We can explicitly consider both the pattern and size of demand as it occurs in order to classify demand patterns into four categories [48.1], as follows:

- *intermittent demand*, which appears to be random, with many time periods having no demand,
- *erratic demand*, which is (highly) variable, the erratic nature relating to the size of demand rather than the demand per unit time period,
- *slow moving* (*smooth*) demand, which also occurs at random, with many time periods having no demand. Demand, when it occurs, is for single or very few units,
- *lumpy demand*, which likewise seems random, with many time periods having no demand. Moreover demand, when it occurs, is (highly) variable. The lumpy concept corresponds to an extremely irregular demand, with great differences between each period's requirements and with a large number of periods with zero requirements.

Traditionally the characteristics of intermittent demand are derived from two parameters: the average inter-demand interval (ADI) and the coefficient of variation (CV). ADI measures the average number of time periods between two successive demands and CV represents the standard deviation of requirements divided by the average requirement over a number of time periods:

$$\mathrm{CV} = \frac{\sqrt{\frac{\sum_{i=1}^{n} (\varepsilon_{\mathrm{r}i} - \varepsilon_{\mathrm{a}})^2}{n}}}{\varepsilon_{\mathrm{a}}}\,, \tag{48.1}$$

where n is the number of periods, and $\varepsilon_{\mathrm{r}i}$ and ε_{a} are the actual and average demand for spare parts in period i, respectively. The four resulting demand categories are represented graphically in Fig. 48.1.

The categorization scheme suggests different ways of treating the resulting categories according to the following characteristics:

- The condition $\mathrm{ADI} \le x$; $\mathrm{CV}^2 \le y$ tests for stock-keeping units (SKUs), which are not very intermittent and erratic (i. e. faster moving parts, or parts whose demand pattern does not raise any significant forecasting or inventory control difficulties);
- The condition $\mathrm{ADI} > x$; $\mathrm{CV}^2 \le y$ tests for low-demand patterns with constant, or more generally not highly variable, demand sizes (i. e. not very erratic);
- The condition $\mathrm{ADI} > x$; $\mathrm{CV}^2 > y$ tests for items with lumpy demand. Lumpy demand may be defined as a demand with large differences between each period's requirements and with a large number of periods having zero requests;
- Finally, the condition $\mathrm{ADI} \le x$; $\mathrm{CV}^2 > y$ tests for items with erratic (irregular) demand with rather frequent demand occurrences (i. e. not very intermittent).

In all cases, x denotes the cutoff value ($\mathrm{ADI} = 1.32$) for ADI, which measures the average number of time periods between two successive demands, and y denotes the corresponding cutoff value ($\mathrm{CV}^2 = 0.49$) for CV^2, which is equal to the square of the standard deviation of the requirements divided by the average requirement over a number of time periods.

Forecasting systems generally depend on the category of part used. Therefore it is important to have two factors in order to indicate deviation from expected val-

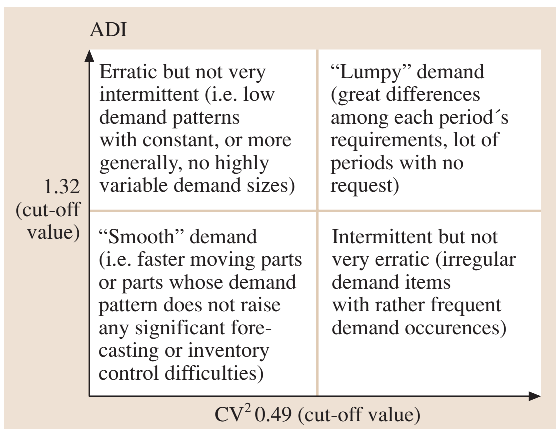

Fig. 48.1 Categorization of demand pattern

Table 48.1 A summary of selected forecasting methods

No.	Method	Abbreviation	Description
1	Additive Winter	AW	Assumes that seasonal effects are of constant size
2	Multiplicative Winter	MW	Assumes that seasonal effects are proportional in size to the local de-seasonalized mean level
3	Seasonal regression model	SRM	Used in time series for modelling data with seasonal effects
4	Component service life (replacement)	MTBR	Estimates of the service-life characteristics of the part (MTBR = mean time between replacement), derived from historical data
5	Croston	Croston	Forecasting in case of low and intermittent demand
6	Single-exponential smoothing	SES	Forecasting in case of low and intermittent demand
7	Exponential weighted moving average	EWMA, Holt	An effective forecasting tool for time series data that exhibits a linear trend
8	Trend-adjusted exponential smoothing	TAES	Forecasting time series data that have a linear trend
9	Weighted moving averages	WMA	A simple variation on the moving average technique that allows for such a weighing to be assigned to the data being averaged
10	Double-exponential smoothing	DES	Forecasting time series data that have a linear trend
11	Adaptive-response-rate single-exponential smoothing	ARRSES	Has an advantage over SES in that it allows the value of α to be modified in a controlled manner as changes in the pattern of data occur
12	Poisson model	POISSON	Models based on the Poisson distribution with the definition of a customer's service level
13	Binomial models	BM	Methods based on the binomial distribution

ues of demand with respect to both demand size and inter-demand interval. The performance of a forecasting method should vary with the level and type of lumpiness. A classification of research on intermittent demand forecasting can be arranged according to Willemain as follows:

1. extension of standard methods [48.2, 3] and variants of the Poisson model [48.4–10];
2. reliability theory and expert systems [48.11];
3. single exponential smoothing, Winter models [48.12–14],
4. Croston's variant of exponential smoothing [48.14–17];
5. bootstrap methods [48.18–21];
6. moving average and variants [48.22, 23];
7. models based on the binomial distribution [48.24–27].

The principle forecasting methods are briefly summarized in Table 48.1.

48.1.1 Exponential Smoothing

Exponential smoothing (ES) methods are widely used time-series methods when reasonably good forecasts are needed over the short term, using historical data to obtain a *smoothed* value for the series. This smoothed value is then extrapolated to become the forecast for the future value of the series. ES methods apply an unequal set of weights that decrease exponentially with time to past data; that is, the more recent the data value, the greater its weighting. In particular, the general form used in computing a forecast by the method of single-exponential smoothing (SES) is given by (48.2), where F_t represents the smoothed estimate, X_t the actual value at time t and α the smoothing constant, which has a value between 0 and 1. SES is best suited to data that exhibits a flat trend.

$$F_{t+1} = \alpha X_t + (1-\alpha) F_t \; . \tag{48.2}$$

When a trend exists, the forecasting technique must consider the trend as well as the series average; ignoring the trend will cause the forecast to underestimate or to overestimate actual demand, depending on whether there is an increasing or decreasing trend. In fact double-exponential smoothing (DES), which is useful when the historic data series are not stationary, applies SES twice and has the general form:

$$F''_{t+1} = \alpha F_{t+1} + (1-\alpha)F_t \; . \quad (48.3)$$

48.1.2 Croston's Method

A little-known alternative to single-exponential smoothing is Croston's method, which forecasts separately the non-zero demand size and the inter-arrival time between successive demands using SES, with forecasts being updated only after demand occurrences. Let F_t and Y_t be the forecasts of the $(t+1)$th demand size and the inter-arrival time respectively, based on data up to demand t, and let Q_t be the inter-arrival time between two successive non-zero demand. Then Croston's method gives:

$$F_t = (1-\alpha)F_{t-1} + \alpha Y_t \; , \quad (48.4)$$

$$Y_t = (1-\alpha)Y_{t-1} + \alpha Q_t \; . \quad (48.5)$$

The predicted demand at time t is the ratio between F_t and Y_t

$$P_t = F_t / Y_t \; . \quad (48.6)$$

The SES and Croston methods are most frequently used for low and intermittent demand forecasting; in particular Croston's method can be useful with intermittent, erratic and slow-moving demand and its use is significantly superior to ES under intermittent demand conditions, according to the categorization scheme of Fig. 48.1. The straight Holt method, exponentially weighted moving average (EWMA), is also only applicable when there are low levels of lumpiness. The widespread use of the SES and mean time between replacement (MTBR) methods for parts with high variation (lumpy demand) are questionable as they consistently lead to poor forecasting performance, which remains poor as the demand variability increases. The only method that fits lumpy demand quite well is the weighted moving average (WMA) method and its superiority to ES methods has been proved: its use could provide tangible benefits to maintenance service organizations forecasting intermittent demand. By WMA we mean a moving average method in which, to compute the average of the most recent n data points, the more recent observations are typically given more weight than older observations.

48.1.3 Holt–Winter Models

Methods based on Winter's models [additive Winter (AW), multiplicative Winter (MW)] consider the seasonal factor and provide the biggest forecasting error when there is high variation (lumpy demand). While computing Holt–Winter filtering of a given time series, unknown parameters are determined by minimizing the squared prediction error; α, β and γ are the parameters of the Holt–Winter filter for the level, trend and seasonal components, respectively; if β is set to 0, the function will perform exponential smoothing, while if the γ parameter is set to 0, a non-seasonal model is fitted.

The additive Holt–Winter prediction function (for time series with period length p) is

$$\bar{Y}[t+h] = a[t] + h \cdot b[t] + s\,[t+1+(h-1)\,|p|] \quad (48.7)$$

where $a[t]$, $b[t]$ and $s[t]$ are given by

$$a[t] = \alpha(\bar{Y}[t] – s[t-p]) + (1–\alpha)(a[t-1]b[t-1]) \; , \quad (48.8)$$

$$b[t] = \beta(a[t] - a[t-1]) + (1-\beta)b[t-1] \; , \quad (48.9)$$

$$s[t] = \gamma(\bar{Y}[t] - a[t]) + (1-\gamma)s[t-p] \; . \quad (48.10)$$

The multiplicative Holt–Winter prediction function (for time series with period length p) is

$$\bar{Y}[t+h] = (a[t] + hb[t])s[t+1+(h-1)|p|] \; , \quad (48.11)$$

where $a[t]$, $b[t]$ and $s[t]$ are given by

$$a[t] = \alpha\left(\frac{\bar{Y}[t]}{s[t-p]}\right) + (1-\alpha)(a[t-1] + b[t-1]) \; , \quad (48.12)$$

$$b[t] = \beta(a[t] - a[t-1]) + (1-\beta)b[t-1] \; , \quad (48.13)$$

$$s[t] = \gamma\left(\frac{\bar{Y}[t]}{a[t]}\right) + (1-\gamma)s[t-p] \; . \quad (48.14)$$

The function tries to find the optimal values of α and/or β and/or γ by minimizing the squared one-step prediction error, if they are omitted. For seasonal models starting values for a, b and s are detected by performing a simple decomposition in the trend and seasonal components using moving averages on the first period (a simple linear regression on the trend component is used for the starting level and trend). For level/trend models (no seasonal component) starting values for a and b are $X[2]$ and $X[2] - X[1]$, respectively. For level-only models (ordinary exponential smoothing), the starting value for a is $X[1]$.

48.2 Forecasting Methods

Table 48.2 Classification of forecasting methods, corresponding testing ground and applications

Forecasting methods	Testing ground							Main area of business application																			
														Sales													
														aggregate			disaggr.										
	Statistics	Operation research	Economics	Psychology	Long-range planning	Practice	Marketing	Production planning	Production scheduling	Inventories	Material requirements	Personnel scheduling	Personnel planning	Short term	Medium term	Long term	Short term	Medium term	Long term	Pricing	Advertising and promotion	Yearly budgeting	New products	R&D projects	Capital budgeting	Competitive analysis	Strategy
Judgmental																											
New product forecasting	•						•							•	•	•				•	•		•	•	•	•	•
Individual				•		•		•				•		•	•	•				•	•	•	•	•	•	•	•
Group																											
Decision rules				•					•	•	•	•															
Sales force estimates						•	•										•	•				•					
Juries of executive opinion						•	•								•					•	•	•	•	•	•	•	•
Role playing				•																						•	
Aggregate																											
Anticipatory surveys						•	•								•							•					
Market research							•										•	•		•	•		•	•			
Pilot programs, pre-market tests							•										•	•	•	•	•		•				
Quantitative																											
Naive						•				•							•			•							
Smoothing		•				•		•		•	•	•	•				•			•							
Decomposition			•					•	•			•	•		•			•				•					
Autoregressive moving average	•							•	•			•	•	•													
Explanatory																											
Vector autoregression	•		•											•	•					•	•	•					
Regression	•		•											•	•					•	•	•					
Econometrics	•		•											•	•					•	•	•					
Monitoring approaches	•	•				•			•	•	•	•					•										

Table 48.2 (cont.)

Forecasting methods	Testing ground							Main area of business application																				
														Sales aggregate			Sales disaggr.											
	Statistics	Operation research	Economics	Psychology	Long-range planning	Practice	Marketing	Production planning	Production scheduling	Inventories	Material requirements	Personnel scheduling	Personnel planning	Short term	Medium term	Long term	Short term	Medium term	Long term	Pricing	Advertising and promotion	Yearly budgeting	New products	R&D projects	Capital budgeting	Competitive analysis	Strategy
Technological																											
Extrapolative																											
Growth curves	•				•														•					•	•	•	•
Time-independent comparisons						•													•					•	•	•	•
Historical and other analogies						•													•					•	•	•	•
Expert-based																											
Delphi					•													•					•	•	•	•	
Futurists					•													•					•	•	•	•	
Cross-impact matrices					•													•					•	•	•	•	

In our opinion, although many different classification schemes could be used, the most significant classification divides the major approaches to forecasting into three main categories, as summarized in Table 48.2 [48.28]: judgmental, quantitative and technological. Each category includes several types of methods, many individual techniques and variations. *Judgmental methods* are most commonly used in business and government organizations. Such forecasts are most often made as individual judgments or by committee agreements. Nevertheless quantitative methods are better than judgmental ones in determining spare-part inventory levels and we suggest judgmental methods only in extremis.

The second category – *quantitative methods* – is the type on which the majority of the forecasting literature has focused. There are three subcategories of these methods. *Time-series methods* seek to identify historical patterns (using a time reference) and then forecast using a time-based extrapolation of those patterns. *Explanatory methods* seek to identify the relationships that led to observed outcomes in the past and then forecast by applying those relationships to the future. *Monitoring methods*, which are not yet in widespread use, seek to identify changes in patterns and relationships; they are used primarily to indicate when extrapolation of past patterns or relationship is not appropriate.

The third category – *technological methods* – address long-term issues of a technological, societal, economic or political nature. The four subcategories here are extrapolative (using historical patterns and relationships as a basis for forecasts), analogy-based (using historical and other analogies to make forecasts), expert-based and normative-based (using objectives, goals and desired outcomes as a basis for forecasting, thereby influencing future events).

48.2.1 Characterizing Forecasting Methods

In describing forecasting methods there are seven important factors, which reflect their inherent capabilities and adaptability.

1. *Time horizon* – two aspects of the time horizon relate to individual forecasting methods. First is the span of time in the future for which different forecasting methods are best suited. Generally speaking, qualitative methods of forecasting are used more for longer-term forecasts, whereas quantitative methods are used more for intermediate- and shorter-term situations. The second impor-

tant aspect of the time horizon is the number of periods for which a forecast is desired. Some techniques are appropriate for forecasting only one or two periods in advance; others can be used for several periods. There are also approaches for combining forecasting horizons of different lengths.
2. *Pattern of the data* – underlying the majority of forecasting methods is an assumption about the type of pattern(s) found in the data to be forecast: for example, some data series may contain seasonal as well as trend patterns; others may consist simply of an average value with random fluctuations and still others might be cyclical. Because different forecasting methods vary in their ability to predict different types of patterns, it is important to match the presumed pattern(s) in the data with the appropriate technique.
3. *Cost* – generally three direct elements of costs are involved in the application of a forecasting procedure: development, data preparation and operation. The variation in cost obviously affects the attractiveness of different methods for different situations.
4. *Accuracy* – closely related to the level of detail required in a forecast is the desired accuracy. For some decision situations, plus or minus ±10% may be sufficient, whilst in others a small variation of 2% could spell disaster.
5. *Intuitive appeal, simplicity, ease application* – a general principle in the application of scientific methods to management is that only methods that are deeply understood are used by decision makers over time. This is particularly true in the area of forecasting.
6. *Number of data points required from past history* – some methods produce good results without consistent data from the past, because they are less affected by estimation errors in their input parameters.
7. *Availability of computer software* – it is seldom possible to apply a given quantitative forecasting method unless an appropriate computer program exists. Such software must be user-friendly and well conceived.

48.3 The Applicability of Forecasting Methods to Spare-Parts Demands

Companies have to select in advance an appropriate forecasting method matching their cyclical demand for parts. Particular attention has to be paid to the demand for service-part inventories, which is generally irregular and difficult to predict. A summary of the better forecasting methods, related to the categorization scheme in Fig. 48.1, is presented in Table 48.3.

Table 48.3 Summary of the better forecasting methods

	Categorization of the demand			
Forecasting methods	**Intermittent**	**Erratic**	**Slow moving**	**Lumpy**
Additive Winter (AW)		•	•	
Multiplicative Winter (MW)		•	•	
Seasonal regression model (SRM)		•	•	
Component service life (replacement)		•	•	
Croston	•	•	•	
Single-exponential smoothing (SES)		•	•	
Double-exponential smoothing (DES)		•	•	
Exponentially weighted moving average (EWMA)		•	•	
Trend-adjusted exponential smoothing		•	•	
Weighted moving averages	•	•	•	•
Adaptive-response-rate single-exponential smoothing		•	•	
Poisson models	•		•	
Binomial models	•	•	•	•

48.4 Prediction of Aircraft Spare Parts: A Case Study

The technical divisions of airlines are based on total hours flown and on the fleet size. With this data, the purchasing department tries to determine the quantity of stock necessary for a particular operating period. Alternatively, when new types of aircrafts are introduced, the airframe and engine manufacturers normally provide a recommended spares provisioning list, which is based on the projected annual flying hours, and includes forecast usage information for new aircraft. Original equipment manufacturers also provide overhaul manuals for aircraft components that support the assessment of required parts based on reliability information, i. e., on the specified components' operational and life limits. Consequently the forecast of spare parts is practically based on past usage patterns and personal experience.

Before any consideration about lumpiness and aircraft spare-parts forecast a discussion on the selection of the main variables used as the *clock* for spare-parts life evaluation is absolutely necessary. According to Campbell's study on maintenance records of the United State Air Force the demand for spare parts appears to be strongly related to flying hours; but this sometimes does not appear to be the best indicator, e.g., to forecast demand for landing gear, what matters is not how long the aircraft is in the air, but how often it lands, or radar components that work only when the aircraft is on the ground. In conclusion often flying hours are the best clock, but a demonstration of its effectiveness is necessary for each item.

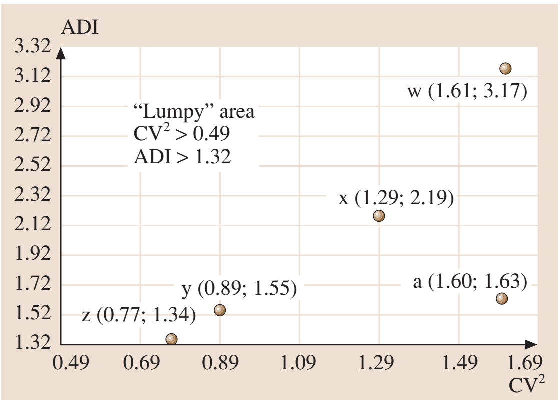

Fig. 48.2 CV^2 and ADI on monthly period for give representative *lumpy* items

In this study different forecasting methods have been considered; briefly:

1. Additive/Multiplicative Winter (AW/MW) For each forecast the optimal combination of level, trend and seasonal parameters is realized. Available values for each variable (level and trend) are 0.2 and 0.01; the seasonal length used is 12 periods.
2. Seasonal regression model (SRM). A multiplicative model with trend and seasonal components. The seasonal length is 12 periods.
3. Single-exponential smoothing (SES). The statistical software applied (Minitab 14.0©) supports the research of the optimal weight of the smoothing constant. The result is then the best forecast with this method.
4. Double-exponential smoothing (DES). Dynamic estimates are calculated for two components: level and trend; the software supports their optimization. In this case the best forecast with DES is also guaranteed.
5. Moving average (on the generic i-period) [MA(i)]. Moving averages (MAs) are calculated with different

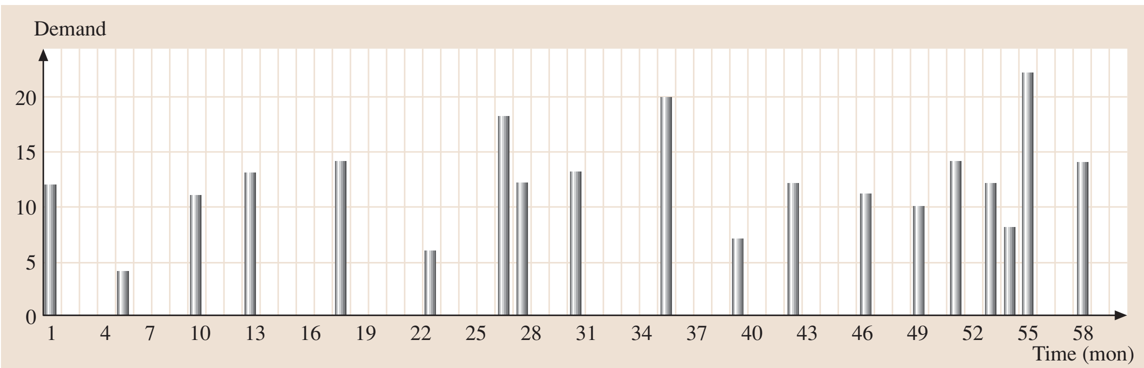

Fig. 48.3 Demand pattern for item z

Table 48.4 Comparison among some methods

Item z	MW	AW	SES	DES	MA(3)	MA(4)	MA(5)	MA(8)	SRM	EWMA (3)	EWMA (4)	EWMA (5)	EWMA (8)
MAD	4.04	3.71	4.54	5.74	5.16	4.80	4.97	4.43	3.53	3.92	3.88	4.06	4.01
MAD/A	0.58	0.53	0.65	0.82	0.74	0.68	0.71	0.63	0.50	0.56	0.55	0.58	0.57

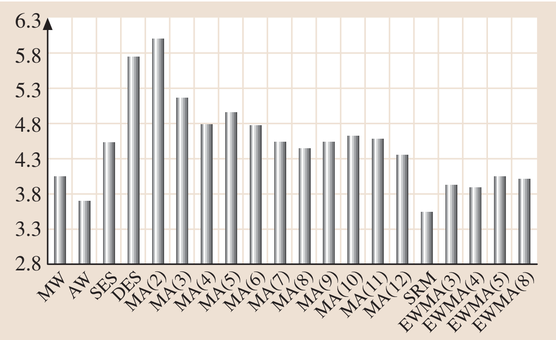

Fig. 48.4 MAD for item z

time horizons (i-period). The notation is MA(i). This analysis employs every period length from 2 to 12.

6. Exponentially weighted moving average [EWMA(i)]. In this case a weight optimization of smoothing coefficient for an MA series has also been realized. EWMA(i) is calculated for $i = 3$, 4, 5 and 8 periods.

Despite their importance in the literature [48.29–31], we do not evaluate and compare methods based on the Poisson approach because they are revealed as inadequate for the prediction of intermittent demand.

The case study deals with more than 3000 different items, with different levels of lumpiness: the Airbus fleet belonging to the Italian national-flag airline. For each component records relate to the daily demand level grouped in monthly interval of item usage, from 1998–2003. In terms of lumpiness these avionic spare parts are classified into five different classes of behavior; for each class a representative item, named a, x, y, z, w for confidentiality, is indicated.

Figure 48.3 presents an exemplifying demand of item z. The five lumpiness levels are reported in Fig. 48.2.

The mean absolute deviation (MAD) of the forecast error is adopted as a performance index

$$\mathrm{MAD} = \frac{\sum_{i=1}^{n} |\varepsilon_{\mathrm{r}i} - \varepsilon_{\mathrm{f}i}|}{n} , \tag{48.15}$$

where $\varepsilon_{\mathrm{f}i}$ is the forecasted demand for period i. Some authors propose the mean absolute percentage error (MAPE) for this comparison, but under lumpy conditions many item demands are zero, and as a consequence MAPE is not defined. For this reason some authors propose a similar index, called MAD/A, also defined when the demand for items is zero:

$$\mathrm{MAD/A} = \frac{\mathrm{MAD}}{\mathrm{AVERAGE}} , \tag{48.16}$$

where AVERAGE is the average value of the historical demand for the item. The tracking signal (TS), as defined by *Brown* [48.32], is used to check if forecasts are in control or not.

$$\mathrm{TS}_t = \left| \frac{\mathrm{CUSUM}_t}{\mathrm{EMAD}_t} \right| , \tag{48.17}$$

where $\mathrm{CUSUM}_t = (\varepsilon_{\mathrm{r}t} - \varepsilon_{\mathrm{f}t}) + \mathrm{CUSUM}_{t-1}$ and $\mathrm{EMAD}_t = \alpha\,|\varepsilon_{\mathrm{r}t} - \varepsilon_{\mathrm{f}t}| + (1-\alpha)\mathrm{EMAD}_{t-1}$.

Limit values of TS and the optimal α value (0.25) are derived from the approach of Alstrom and Madsen [48.33]. For the items analyzed forecasts are in control from the third year (i. e., their tracking signals respect the limits). The different forecasting methods are compared for all items and in particular for the proposed five components.

Table 48.4 and Figs. 48.4 and 48.5 show, respectively, some brief and full results of MAD and MAD/A for item z. Table 48.5 presents, for each representative item, the list of forecasting methods ordered

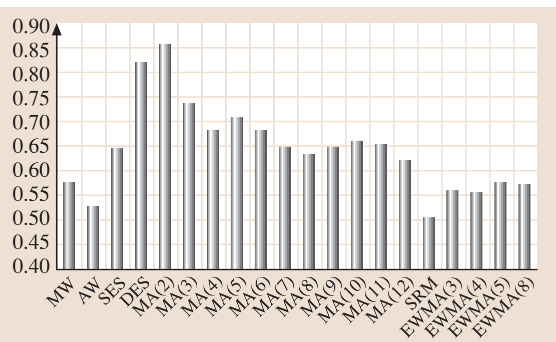

Fig. 48.5 MAD/A for item z

Table 48.5 Ranking based on performance evaluation (MAD)

Weight	*z*	*y*	*x*	*a*	*w*	Method	Total score	Average score
20	SRM	EWMA(3)	SRM	EWMA(4)	SRM	SRM	93	18.6
19	AW	SRM	AW	EWMA(3)	EWMA(5)	EWMA4	89	17.8
18	EWMA(4)	SES	MW	EWMA(5)	EWMA(4)	EWMA3	86	17.2
17	EWMA(3)	EWMA(4)	EWMA(5)	EWMA(8)	EWMA(3)	EWMA5	84	16.8
16	EWMA(8)	EWMA(5)	EWMA(4)	SES	SES	EWMA8	76	15.2
15	MW	EWMA(8)	SES	MW	AW	MW	60	15
14	EWMA(5)	AW	EWMA(8)	SRM	EWMA(8)	SES	75	15
13	MA(12)	MA(10)	EWMA(3)	MA(7)	MA(5)	AW	74	14.8
12	MA(8)	MW	MA(10)	MA(8)	MA(4)	MA12	51	10.2
11	MA(7)	MA(11)	MA(11)	MA(11)	MA(12)	MA11	49	9.8
10	SES	MA(12)	MA(9)	MA(4)	MA(9)	MA9	47	9.4
8	MA(11)	MA(6)	MA(8)	MA(12)	MA(11)	MA7	44	8.8
7	MA(10)	MA(8)	MA(7)	AW	MA(7)	MA8	43	8.6
6	MA(6)	MA(7)	MA(5)	MA(10)	MA(8)	MA4	35	7
5	MA(4)	MA(5)	MA(6)	MA(6)	MA(6)	MA5	32	6.4
4	MA(5)	MA(4)	MA(4)	MA(5)	MA(3)	MA6	29	5.8
3	MA(3)	MA(3)	MA(3)	MA(3)	DES	MA3	16	3.2
2	DES	DES	MA(2)	DES	MA(2)	DES	10	2
1	MA(2)	MA(2)	DES	MA(2)		MA2	7	1.4

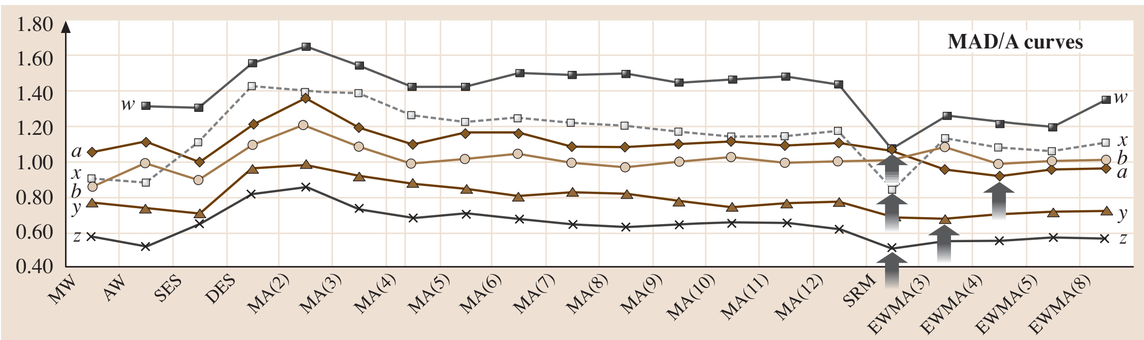

Fig. 48.6 MAD/A data and curves

by decreasing MAD, thus assigning a relative weight related to the ranking position: a simple elaboration of these weights permits a full comparison in terms of total and average scores (MW is not defined for item w, due to its characteristics).

By means of MAD/A it is possible to compare different forecasting methods for different items and their behavior in face of different lumpiness conditions; SRM, EWMA(i) and Winter are the best forecasting methods. This result is not related to the lumpiness level, at least for lumpiness represented by ADI < 3.3 and CV$^2 < 1.8$, which is the typical range for aircraft components. Some interesting observations can be drawn:

- Figure 48.6 clearly attests that item lumpiness is a dominant parameter, whilst the choice of the forecasting method is of secondary relevance; all methods for a slightly lumpy item (e.g. items y and z) generally perform better than the best method for a highly lumpy component (e.g. items x and w). However lumpiness is an independent variable and is not controllable;
- the average value of MAD/A, calculated for all forecasts generated by all methods, is 1.02. The aim of this study is to compare the different forecasting methods, but we can conclude that demand forecasting for lumpy items is very difficult and the results

are not very accurate. Moreover, lumpy demand is often equal to zero or one: all predictions lower than one must be rounded up to one. This phenomenon introduces another source of error;
- for a single component, the average fluctuation (in terms of MAD/A) of the ratio maximum/minimum, among different techniques, is about 1.55 (usually between 1.40 and 1.70); for a single forecasting method, the average fluctuation (in terms of MAD/A) of the ratio maximum/minimum, among different components, is about 2.17 (usually between 1.57 and 2.18). Thus, the demonstrating again the relevance of lumpiness.
- analyzing the effectiveness of a single model, research demonstrates (Tables 48.3 and 48.4) that the seasonal regression model (SRM), the exponentially weighted moving average [EWMA(i)] and the Winter model are the best forecasting methods. It is important to remember that the analyzed items are effectively representative of a population of aircraft spare parts. This result is not related to the lumpiness level, at least for lumpiness represented by ADI < 3.3 and $CV^2 < 1.8$ (the typical range for aircraft components).

In conclusion, intermittent demand for, usually highly priced, service parts is a very critical issue, especially for the prediction of lumpy demand, as is typical for avionic spare parts. In the literature forecasting for lumpy demand has not been investigated deeply, apart from Ghobbar's interesting research, and conflicting results are sometimes recovered. The introduction of the economic question is the final development: it is absolutely necessary to check the impact of stocking costs and out-of-stock components on the forecasting methods; an aircraft operator can incur costs of more than \$30 000 per hour if a plane is on the ground.

48.5 Poisson Models

For builders of high-technology products, such as automatic packaging machines, the supply of spare parts creates a strategic advantage with respect to their competitors, with particular regards to low-turnaround-index (LTI) parts conceived and designed for the satisfaction of a specific customer request. The strategic problem to solve is to determine the minimum number of spare parts required to avoid downtimes of the customer's plants for a specific period, called the covering period, which coincides with the time between two consecutive consignments.

The procedure actually used by a great number of manufacturers, called *recommended parts*, consists of the creation, at the design stage, of different groups of replaceable parts with different covering times for every functional machine group. This methodology is very qualitative and depends strongly on the opinion of the designer; moreover, it does not consider information feedback from customers, and usually overestimates the number of spare parts with respect to the real demands of customers. Even though this avoids plant downtimes, which are absolutely forbidden due to the high costs of production loss, it normally creates excessive and expensive stocks, with undesirably high risks of damage and obsolescence. For LTI items the usual economic batch or safety stock methods are not suitable to forecast the amount of spare parts required. For such a situation a lot of different approaches have been developed in recent years, usually based on the Poisson distribution; of these, conditioning of the stock level to minimal availability or to minimum total cost (Fig. 48.7) are considered the most interesting.

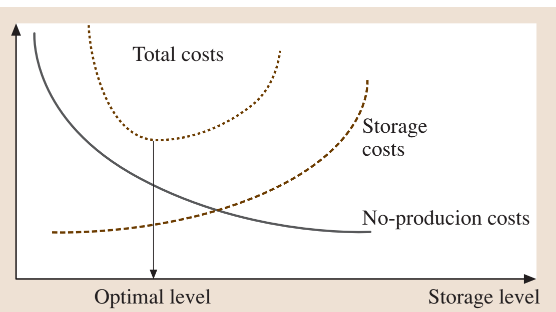

Fig. 48.7 Economic approach

Every study reported in the literature [48.34, 35] assumes that an item's failure time (for spare-parts demand) is exponentially distributed and, as a consequence, the failure rate $\lambda(t)$ is independent of time; this simplifying hypothesis is due to the difficulties in estimating real values of mean time before failure (MTBF). Finally it is important to underline that the quantity of spare parts and its temporal distribution also represent

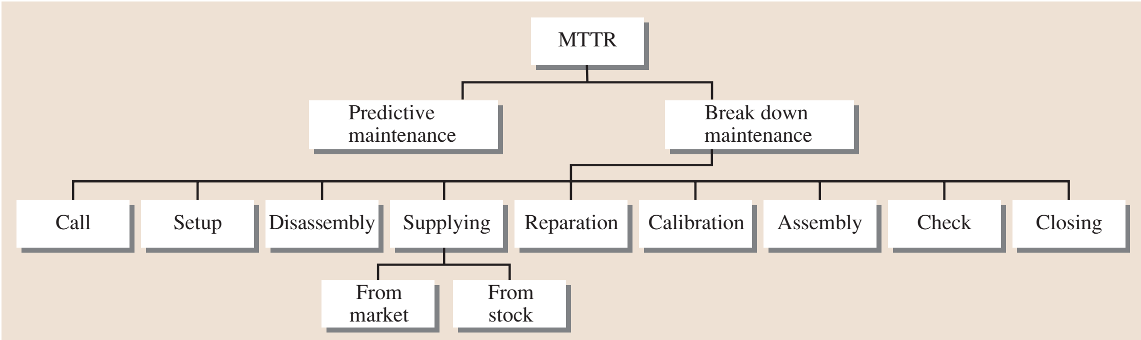

Fig. 48.8 MTTR structure

strategic information during negotiations with customers for the purchase of plants and the quantification of related costs.

48.5.1 Stock Level Conditioned to Minimal Availability

This method firstly needs to calculate the asymptotic availability A by the known formula:

$$A = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} \,. \tag{48.18}$$

The mean time to repair (MTTR) term is derived from different factors, as shown in Fig. 48.8.

Its value depends strongly on the spare part being on consignment, i. e. on hand, or not, and can be calculated by the formula

$$\text{MTTR} = T_1 + \int_0^{T_S} (T_S - T_x) f(T_x)\, \mathrm{d}T_x = \text{MTTR}(N) \,, \tag{48.19}$$

where T_1 is the amount of time due to factors except supply time (for instance, disassembling), T_S is the supply lead time for unavailable components, N is the number of spare parts available in stock at time zero, T_x is the time interval between the instant when the consumption of the part reaches the value N (empty stock situation) and the consignment of the spare part, and $f(t)$ is the failure density distribution.

It is worth noting that, for increasing N, we get decreasing MTTR, increasing availability A and the falling downtime costs. Secondly the method affords the quantitative definition of the storage cost, which requires the definition of the average number of parts stored during the time of supply T_S. If the warehouse contains N parts at time zero, the probability P_N of N failures in T_S can be described by the Poisson formula

$$P_N = \frac{(\lambda T_S)^N \cdot \mathrm{e}^{(-\lambda T_S)}}{N!} \,. \tag{48.20}$$

In the same way it is possible to calculate the probability of one, two, or N failures.

Let R indicate the cost of each spare part, and s be the stocking cost index per year; the annual stock costs C can be evaluated by the formula $C = Rs[NP_0 + (N-1)P_1 + (N-2)P_2 + \ldots P_{N-1}]$, which can be used in an iterative manner to find the optimum level N that leads to a minimum for the cost C, while allowing the minimum level of availability $A_{\min}(N)$ to guarantee on-time technical requests to be satisfied (for example, safety questions or productivity level)

$$\begin{cases} \min \left\{ C = Rs\left[NP_0 + (N-1)P_1 \right.\right. \\ \qquad \left.\left. + (N-2)P_2 + \cdots + P_{N-1} \right] \right\} \\ \text{subject to } A(N) = \frac{\text{MTBF}}{\text{MTBF}+\text{MTTR}(N)} \geq A_{\min} \,. \end{cases} \tag{48.21}$$

48.5.2 Stock Level Conditioned to Minimum Total Cost

The aim of this method is to determinate the total amount N of replaceable parts that minimizes the total cost function C_{tot} defined by

$$C_{\text{tot}}(N) = C_1 + C_2 \,. \tag{48.22}$$

The warehousing cost term C_1 can be estimated as in (48.21), while for the cost C_2 it is necessary to quantify the probability of stock-out situations. During the time T_S production losses could occur if the number of failures exceeds the number N of parts supplied at the consignment time, assumed to be zero. The cumulative

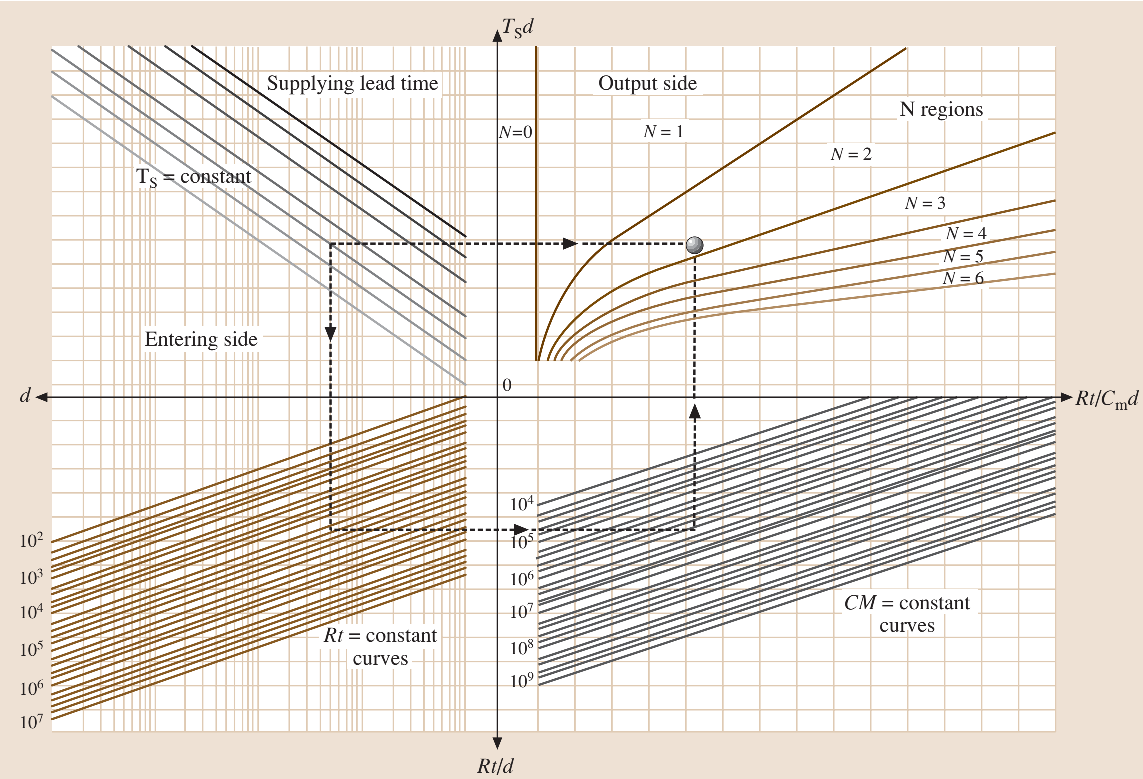

Fig. 48.9 Graphical abacus

probability, calculated by the Poisson distribution, is

$$P = P_{N+1} + P_{N+2} + P_{N+3} + \ldots . \quad (48.23)$$

Let d indicate the annual part consumption of a customer and CM the cost corresponding to a loss of production; the term due to stock-out is

$$C_2 = CMdP . \quad (48.24)$$

For a rapid choice it is possible to employ a user-friendly abacus (Fig. 48.9).

48.6 Models Based on the Binomial Distribution

Industrial applications show that methodologies based on the Poisson formula usually overestimate the actual replacement consumption. To overcome this problem we present a new quantitative procedure that, in contrast to the Poisson methods, does not assume that requests for parts are linear over time.

The innovative approach calculates the requirement for components, for a given covering period T, by the addition of two addenda x_1 and x_2: the first is related to the wear damage of the replaceable component and can be deduced from the MTBF value, while the second refers to the randomness of breakdowns and covers the possibility of failures in advance of the average situation. The optimal number of replacements is $N = x_1 + x_2$.

Let n be the number of different employments of a component in several machines owned by the customer and let T be the covering period; x_1 can be expressed by:

$$x_1 = \text{int}\left(\frac{T}{\text{MTBF}}\right) n . \quad (48.25)$$

This average term assumes interesting values only in the presence of high consumption of the component, in par-

Table 48.6 Example of N evaluation for a specific item (code 0X931: pin for fork gear levers)

Input	
MTBF (h)	3945
Positions (n)	26
Confidence level	97%
Supplying time (h)	2300

Output	
Failure ratio λ	2.53×10^{-4}
X_1	0
Tresidual (h)	2300
P (Tresidual)	0.442
X_2	See table
$N = X_1 + X_2$	16

ticular in the rare situations when a LTI part has a lot of applications, indicated by n. Anyway this term x_1 represents scant information; we have to consider the second term, which corresponds to the number of parts needed to obtain the required value of the customer service level (LS) in the residual time T_{residual}, defined as the residue of the ratio between T and MTBF. The customer service level is the probability that the customer finds the parts during the remaining period, and can be fixed separately according to strategic and economic assessments.

The value of x_2 is obtained as follows

$$T_{\text{residual}} = T - \text{int}\left(\frac{T}{\text{MTBF}}\right)\text{MTBF}\,, \tag{48.26}$$

$$p = Q(T_{\text{residual}}) = 1 - \mathrm{e}^{-\left(\frac{T_{\text{residual}}}{\text{MTBF}}\right)}\,, \tag{48.27}$$

where p represents the failure probability during T_{residual}. Using p and the binomial distribution it is easy to calculate the probability that a component (with n applications) requires fewer than x_2 replacements in T_{residual}:

$$P\left[x \le x_2;\, n;\, Q(T_{\text{residual}})\right] = \sum_{i=0}^{x_2} \binom{n}{i} (1-p)^{n-i} p^i\,. \tag{48.28}$$

As a consequence it is possible to quantify the confidence level for no stock-outs to compare with the customer satisfaction as

$$\text{LS}(x_2) = 1 - P\left[x \le x_2;\, n;\, Q(T_{\text{residual}})\right]\,. \tag{48.29}$$

The main innovative result is that the procedure, in contrast to other methods, does not consider the total requests for spare parts to be linear with time; it tries to set the best moment for supply in order to maximize the customer service level without increasing the average number of spare parts. In fact the new method respects the average consumption through the term x_1 and increases the levels of customer service by planning requirements for spare parts in the residual time through the term x_2.

48.6.1 An Industrial Application

This procedure is successfully running on PC systems in an Italian company that is a leader in manufacturing

Table 48.6 (cont.)

X_2 (items)	LS	(X_1+X_2) (items)	X_2 (items)	LS	(X_1+X_2) (items)
0	0.000	0	14	0.883	14
1	0.000	1	15	0.943	15
2	0.000	2	16	0.976	16
3	0.000	3	17	0.991	17
4	0.002	4	18	0.997	18
5	0.007	5	19	0.999	19
6	0.022	6	20	1.000	20
7	0.055	7	21	1.000	21
8	0.118	8	22	1.000	22
9	0.218	9	23	1.000	23
10	0.351	10	24	1.000	24
11	0.505	11	25	1.000	25
12	0.658	12	26	1.000	26
13	0.787	13			

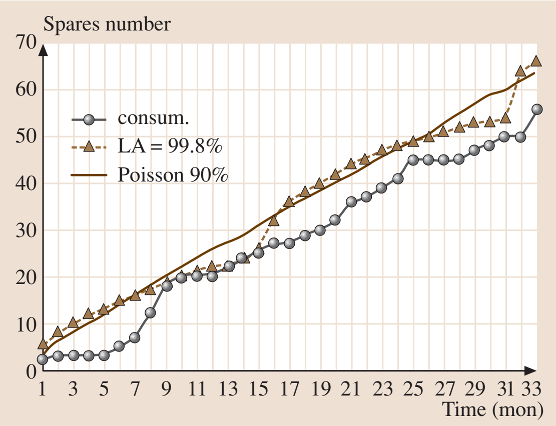

Fig. 48.10 Forecast and applications for the pin

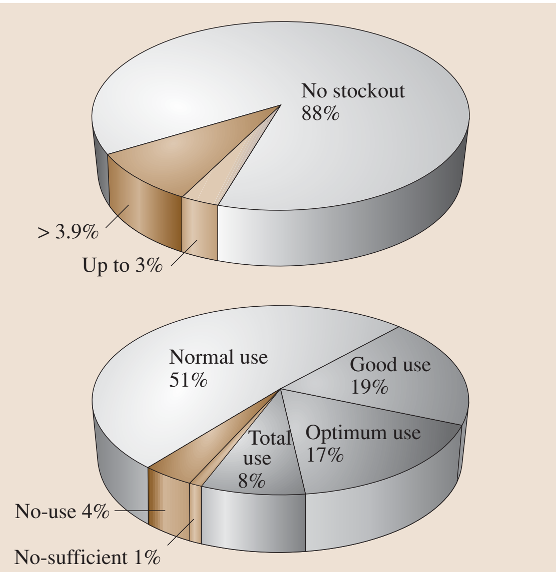

Fig. 48.11 Simulated stock-out periods (months) and real utilization of dispatched replacements

of packaging machines. The supply of spare parts creates a strategic advantage over competitors, because the automatic packaging machines usually present a long life cycle and contain a lot of functional groups, often conceived and designed ad hoc. The economic impact of replacement activity is not negligible: it usually amounts to 15% of global business volume. A good forecast of spares parts can surely simplify manufacturer production planning. Before its industrial real-time application the innovative procedure was tested to forecast the consumption of 190 different spare

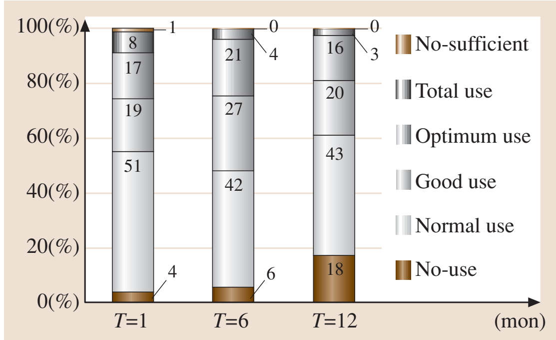

Fig. 48.12 Real use compared to supply time T

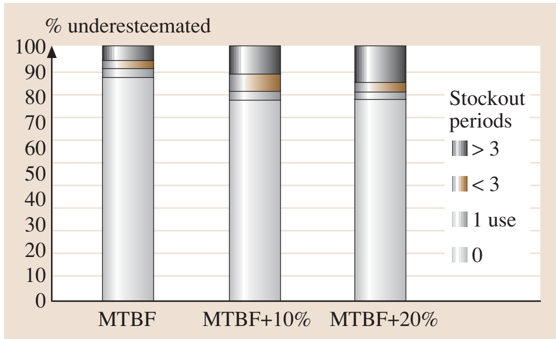

Fig. 48.13 Sensitivity to MTBF evaluation

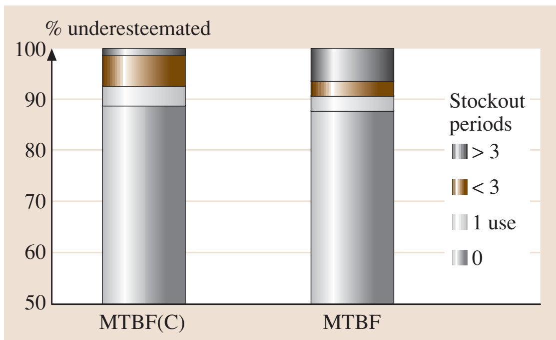

Fig. 48.14 MTBF correction effect on stock-out

parts indicated by several customers over a 33-month period.

The experimental results of these procedure were evaluated by two performance indexes:

1. percentage of spare parts without stock-out periods,
2. effective utilization of replacements by the customer.

As a first test the new procedure was applied with the restrictive hypothesis of a covering period of one month: using an LS value of 99.8% the method performed well for 166 replaceable parts and there was very good correlation between predicted and actual customer consumption. This is shown in the comparison in Table 48.6 between actual consumption and the Poisson linear forecast (LS = 90%) for a pin for fork gear levers with 26 applications in the customer's machine park.

88% of the components investigated did not suffer from stock-out in any month; 3% presented less than three months of under-evaluation and 9% had more than three months in stock-out. On the other hand the utilization index showed that 87% of the components had normal, good or optimal customer use: in other words they did not remain in the spare-parts warehouse for more than 15 days before installation (Fig. 48.11).

Moreover, parts with bad forecasts were investigated to understand the reasons, with very encouraging results; errors were usually caused by preventive maintenance operations, machine revisions, changes of suppliers or changes in the application of the component not pointed out by the customer. The effect of the extension of the covering period was analyzed by testing the new method with different values of T. For increasing T we obtain an increasing stock of spare parts but the number of stock breakages strongly decreases: in fact for $T = 1$ month the method performed well for 88% of the parts investigated, and this percentage increased to 90% for $T = 3$ months, 95% for $T = 6$ months and to 98% for $T = 1$ year. Therefore it was possible to study the optimum extension of the covering period, that for the 190 components investigated was found to be equal to three periods (Fig. 48.12).

Some simulations with different values of MTBF show the influence of its approximate evaluation: values of MTBF overestimated by 10% and 20% reduce the performance of the method respectively by 6% and 9% in terms of the percentage of spare parts without stock-out periods (Fig. 48.13).

In spite of this important conclusion, it is important to remember that MTBF values have to be updated, starting from the initial value of $\text{MTBF}_{\text{initial}}$, by feedback information from the customers; the most suitable control parameter is the component quantity Y employed by the customer during the covering period T and the relation (48.30) that gave the best results, as shown in Figs. 48.14 and 48.15:

$$\text{MTBF}_{\text{updated}} = \frac{\text{MTBF}_{\text{initial}}\,(\text{MTBF}_{\text{initial}}) + \frac{nT}{Y}(nT)}{\text{MTBF}_{\text{initial}} + nT}\,, \tag{48.30}$$

where $\text{MTBF}_{\text{updated}}$ is the weighted average of $\text{MTBF}_{\text{initial}}$, and $\frac{nT}{Y}$ (weights in round brackets).

48.7 Extension of the Binomial Model Based on the Total Cost Function

The proposed model required the assumption of a specific spare-part LS defined as the probability of finding the part in case of breakdown. Some simulations with different values of LS show that it is important to assume LS $\le$ 80% and to reserve LS $\ge$ 90% for particular situations, e.g. customers placed in a distant country or without skilled workers. It is possible to determine the number N of replaceable parts needed and therefore the LS value capable of minimizing a total cost function defined by the sum of costs due to storage and production losses.

48.7.1 Service-Level Optimization: Minimum Total Cost Method

The aim of this paragraph is to determine the requirement N of replaceable parts capable of minimizing a total cost function defined by the sum of production losses costs C_1 and storage costs C_2. During the time T_S production losses could occur if the number of failures exceeds the number N of supplied parts that are available after the consignment at time zero. The corresponding cumulative probability can be calculated by formula:

$$\begin{aligned} P &= P(N+1) + P(N+2) + P(N+3) + \dots \\ &= 1 - \text{LS}(x_2) = 1 - \text{LS}(N - x_1)\,, \end{aligned} \tag{48.31}$$

$$\text{LS}(N - x_1) = P[X \le N - x_1,\ n,\ Q(T_{\text{residual}})]\,. \tag{48.32}$$

If d and CM represent, respectively, the customer annual part consumption and the cost for a production lack, the total cost C_1 due to stock-out is

$$C_1 = CMdP\,. \tag{48.33}$$

The storage cost C_2 requires the definition of the average number of parts stored during the supplying time T_S. Two different situations are possible related to the spare-part MTBF and T_S:

Table 48.7 LS % and minimum cost related to $T_s d$ and $Rt/(C_m d)$– no. of employments $n = 5$

LS % ($n = 5$)	$T_s d$							
$Rt/(C_m d)$	5	4	2	1	0.8	0.2	0.02	0.02
9	0.7	1.8	13.5	37	44.9	81.9	98	99.8
3	6.5	13.1	13.5	37	44.9	81.9	98	99.8
1.5	26.4	40.6	46.8	37	44.9	81.9	98	99.8
0.9	60.5	40.6	46.8	78	44.9	81.9	98	99.8
0.3	89.9	74.3	79.6	78	83.9	81.9	98	99.8
0.1	100	94.9	95.7	95	97.4	81.9	98	99.8
0.03	100	100	99.6	100	97.4	98.6	98	99.8
0.003	100	100	100	100	99.8	99.9	100	99.8
0.0003	100	100	100	100	100	100	100	100

Case a. MTBF $< T_S$; in this case $N = x_1 + x_2 = x_2$ because $x_1 = 0$.

If the warehouse contains N parts at time zero, the probability P_N of N failures in T_S can be calculated using (48.21). Let R indicates the cost of each spare part, and t the annual stocking cost index; the global annual storage cost C can be evaluated by

$$C = Rt \sum_{k=0}^{N} \left[\binom{N}{k} (1-p)^{N-k} p^k \right] (N-k) \,. \qquad (48.34)$$

Case b. MTBF $\geq T_S$

In this case the definition of the average number of parts stored during the supply time T_S has to take in account both contributions, in terms of average stock $S(x_1)$ and $S(x_2)$, of x_1 and x_2. LS is connected to x_2, and the minimum real value of N is therefore x_1. We can get the contributions in (48.36) by the previously defined values of x_1 and x_2. The annual stock cost C can be evaluated by

$$C = Rt\,[S(x_1) + S(x_2)] \,, \qquad (48.35)$$

which can be used in an iterative process to find the optimum level N according to the minimization of previous cost C.

$$S(x_1) = \frac{1}{2}\left(\frac{x_1 \mathrm{MTBF}}{T_S}\right) ;$$

$$S(x_2) = \frac{x_2 \mathrm{MTBF}}{T_S} + \frac{\sum_{k=0}^{N-x_1} \binom{N-x_1}{k} (1-p)^{N-x_1-k} p^k}{T_S} \times (N - x_1 - k)\,(T_S - \mathrm{MTBF}) \,. \qquad (48.36)$$

48.7.2 Simulation and Results

A simulation model has been designed in order to find the optimum value of LS for different values of the parameters. Input data are the MTBF, number of employments n, time for supply T_S, cost of each spare part R, annual stocking cost index t, downtime cost per hour C_m, MTTR, total hours per year of uptimes plus

Table 48.8 LS % and minimum cost related to $T_s d$ and $Rt/(C_m d)$– no. of employments $n = 15$

LS % ($n = 5$)	$T_s d$							
$Rt/(C_m d)$	5	4	2	1	0.8	0.2	0.02	0.02
9	4.7	10.2	13.5	37	45.0	81.9	98	99.8
3	34.8	28.2	42.5	75	45.0	81.9	98	99.8
1.5	57.3	52	71.4	75	81.9	81.9	98	99.8
0.9	76.9	73.8	71.4	75	81.9	81.9	98	99.8
0.3	89.8	88.5	89.3	93	96	98.4	98	99.8
0.1	96.4	95.9	96.9	99	99.4	98.4	98	99.8
0.03	99	98.8	99.3	100	99.4	99.9	98	99.8
0.003	100	100	100	100	99.9	99.9	100	100
0.0003	100	100	100	100	100	100	100	100

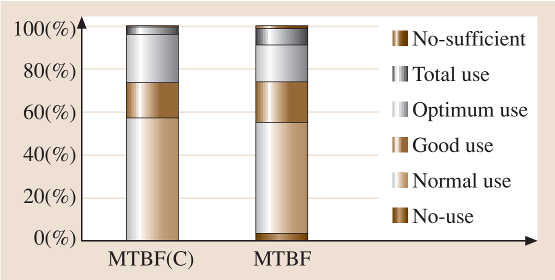

Fig. 48.15 MTBF correction effect on items' utilization

downtimes H, and the customer annual part consumption d. Assuming for instance $\mathrm{MTBF} = 10\,000\,\mathrm{h}$ and $n = 5$, the optimum value of LS versus the two variables $(Rt)/(C_{\mathrm{m}}d)$ and $T_{\mathrm{S}}d$ is reported in Tables 48.7 and 48.8 (for $n = 5$ and $n = 15$, respectively). It is worth stating that LS must be close to 100% when $C_{\mathrm{m}} \gg R$, while in the opposite case the optimum LS is a function of $T_{\mathrm{S}}d$, and always tends upwards for decreasing n.

Figure 48.16 shows how to employ the abacus.

Table 48.9 Optimization of T_{s} for fixed number of spare parts N

Component	$N = x_1 + x_2$	$T_{\mathrm{s}}(d)$	LS(%)
Support grid	3	400	99
Clamp	4	90	98
Special gasket	5	90	95

48.7.3 An Industrial Application

This case study is related to an important producer of steam boiler systems that actually manufacturing components for internal use and for replacement ordered by customers according to a fixed economic order quantity (EOQ). The application deals with the optimization of the supply time in order to reduce the total management costs of spare parts at the assigned EOQ [48.36]; that is, the aim is to define the time between consignments capable of reducing total costs for the same value of EOQ. Three components (a support grid, the clamp and a special gasket) are considered, with a downtime calculated

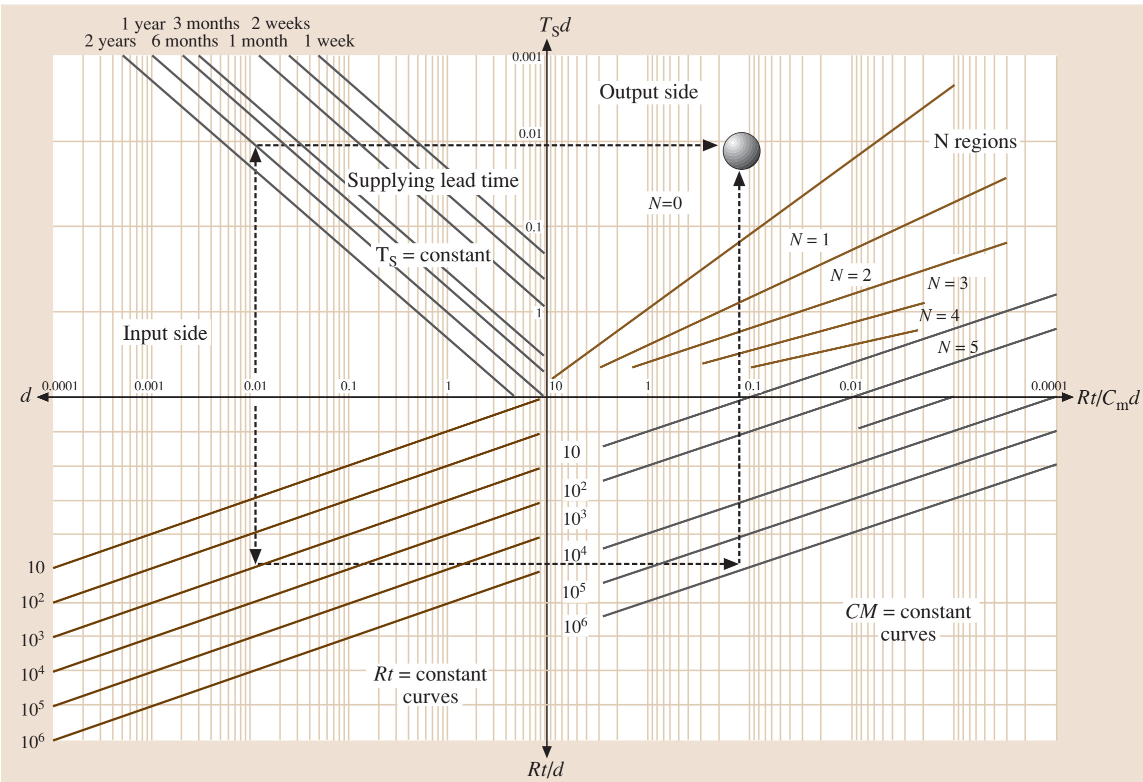

Fig. 48.16 Graphical solution for this methodology

as

$$\text{Downtime} = \text{MTTR} \times dn \ . \quad (48.37)$$

In particular the support grid has $n = 5$ employments, $H = 1760$ total hours per year, $d = 0.5$ annual part consumption [unit/year] on the whole, $\text{MTTR} = 10\,\text{h}$, $\text{MTBF} = 8800\,\text{h}$ downtime cost per hour $C_m = 1000\,€/\text{h}$ and, as a consequence, $CM = C_m\text{MTTR} = 10\,000\,€$ cost for a production stock-out. This component was supplied in fixed EOQ with $N = 3$ elements, at a cost per unit of $R = 100\,€/\text{unit}$ ($R \ll C_m$); the relative risk of damage and obsolescence suggests $t = 0.1$ for the annual stocking cost index. Entering the abacus with these values for N, CM and Rt (Fig. 48.15) we obtain as a result the optimal time for supply $T_S = 400\,\text{d}$; by means of (48.31) and (48.32) with $N = x_1 + x_2 = 0 + 3$ ($x_1 = 0$ because MTBF $< T_S$) we determine LS $\approx 99\%$; this very high level is due to the low values of t and the rate R/C_m.

The results following the use of the graphical abacus are shown in Table 48.9 for the whole set of components.

The results are summarized in Fig. 48.17 and compared with the output of an existent minimum-cost method based on the Poisson distribution.

48.8 Weibull Extension

This innovative methodology can be extended to the whole lifetime by implementing the Weibull failure-rate function to the stock level of LTI spare parts level in maintenance systems with declared wear conditions. The Weibull distribution is one of the most commonly used lifetime distributions and is flexible in modeling failure-time data, as the corresponding failure-rate function can vary or be assumed to be constant. The literature offers a lot of papers dealing with models for bath-tub-shaped failure rates. For example *Hjorth* [48.37] proposed a three-parameter distribution; *Mudholkar* and *Srivastava* [48.38] introduced an exponential Weibull distribution; *Chen* [48.39] spoke about a two-parameter lifetime distribution with a bath-tub shape or an increasing failure-rate function; *Xie* [48.40] wrote a very interesting paper about a model that can be seen as a generalization of the Weibull distribution and tries to improve the procedure for estimation of the parameters.

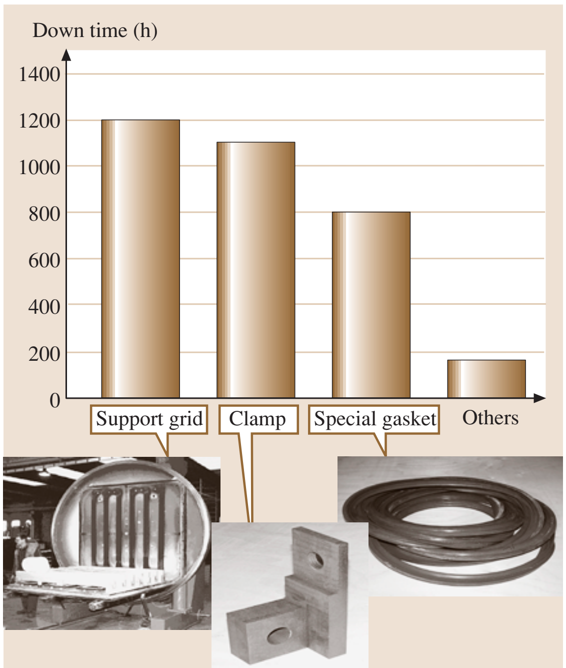

Fig. 48.17 Downtimes for grid, clamp, gasket and all others components (cumulative)

Estimation of the well-known parameters η (scale) and β (shape) in a Weibull distribution can be performed graphically but this is not accurate unless there is a large sample size, which is not always the case for LTI spare parts; anyway we focus our attention to the final zone in the traditional *bath-tub* wear model, and our aim is to understand whether the hypothesis of constant failure rate in our previous works increases the spare-parts costs, in comparison with more sophisticated distributions. For this reason we developed our model using the traditional Weibull distribution, but this could be extended to any of the models mentioned above.

48.8.1 The Extension of the Modified Model Using the Weibull Distribution

Using historical data it is possible to determine the cumulative percentage of component failures related to their lifetime. The graphical approach of Fig. 48.18 permits the definition of the Weibull distribution parameters η (scale parameter) and β (shape parameter). This is possible by Plait transformation (Weibull transformation); starting from the failure rate calculated as in (48.38),

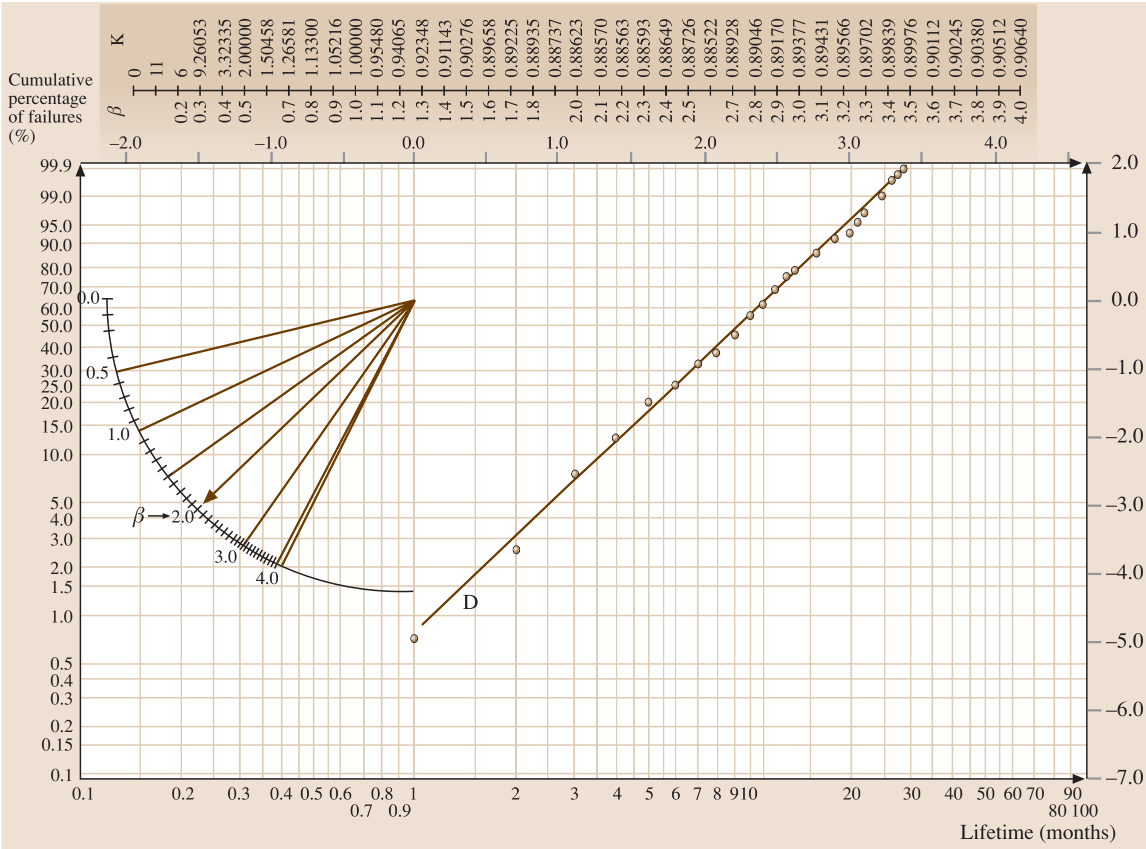

Fig. 48.18 Graphical estimation of the β value

the reliability in (48.39), the cumulative distribution functions in (48.40), and the definition of a normalized parameter x using (48.41), find a linear correlation between the parameter x and the cumulative distribution function (48.42), represented in Fig. 48.18

$$\lambda(t) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{t}{\eta}\right)^{\beta} = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} = \frac{\beta}{\eta^{\beta}} t^{\beta-1}\,, \tag{48.38}$$

$$R(t) = \mathrm{e}^{-\left(\frac{t}{\eta}\right)\beta}\,, \tag{48.39}$$

$$F(t) = 1 - R(t) = \mathrm{e}^{\left(-\frac{t}{\eta}\right)^{\beta}}\,, \tag{48.40}$$

$$x = -\frac{t}{\eta}\,, \tag{48.41}$$

$$\beta \ln(x) = \ln\left[\ln\left(\frac{1}{1-F(x)}\right)\right]. \tag{48.42}$$

The optimal replacement number of LTI spare parts is also given by relation (48.25), whilst the relations (48.27) and (48.28) are modified by the Weibull parameters η and β:

$$p = Q\left(T_{\text{residual}}\right) = 1 - \mathrm{e}^{-\left(\frac{t}{\eta}\right)^{\beta}}\,, \tag{48.43}$$

$$P\left[x \le x_2;\, n;\, Q\left(T_{\text{residual}}\right)\right] = \sum_{i=0}^{x_2} \binom{n}{i} (1-p)^{n-i} p^{i}\,. \tag{48.44}$$

It is possible to quantify the no-stock-out confidence level to compare with customer satisfaction as $\mathrm{LS}(x_2) = 1 - P[x \le x_2;\, n;\, Q(T_{\text{residual}})]$. As previously stated LS must be close to 100% when $C_{\mathrm{m}} \gg R$, while in the opposite case the optimum LS is a function of $T_{\mathrm{S}}d$; in this case, for fixed MTBF, the optimum LS increases with the number of employments n.

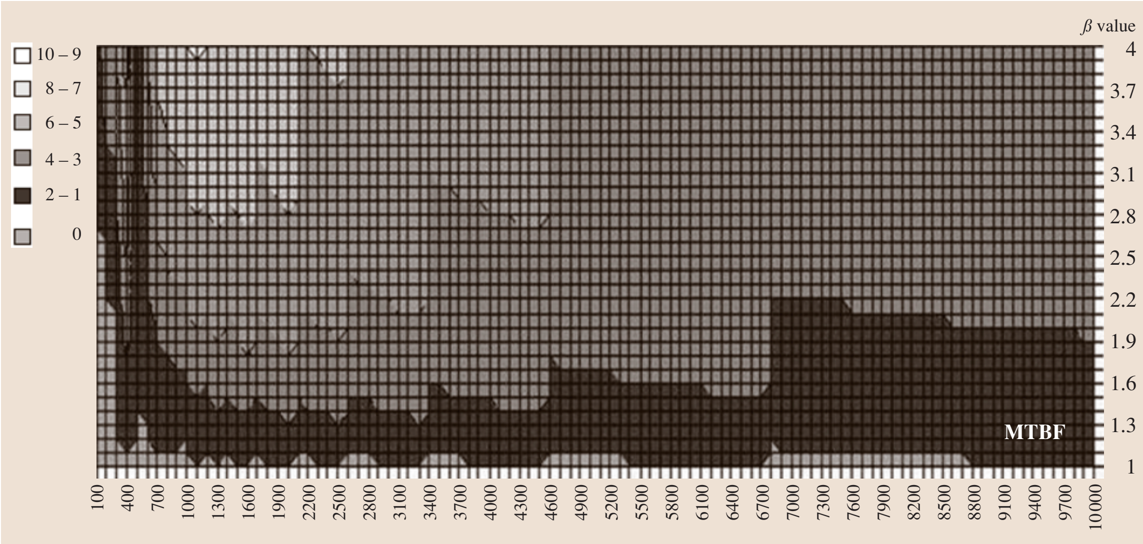

Fig. 48.19 Difference between optimal numbers of replacements calculated by (48.29) or (48.44) with respect to the average life of the component MTBF and β values

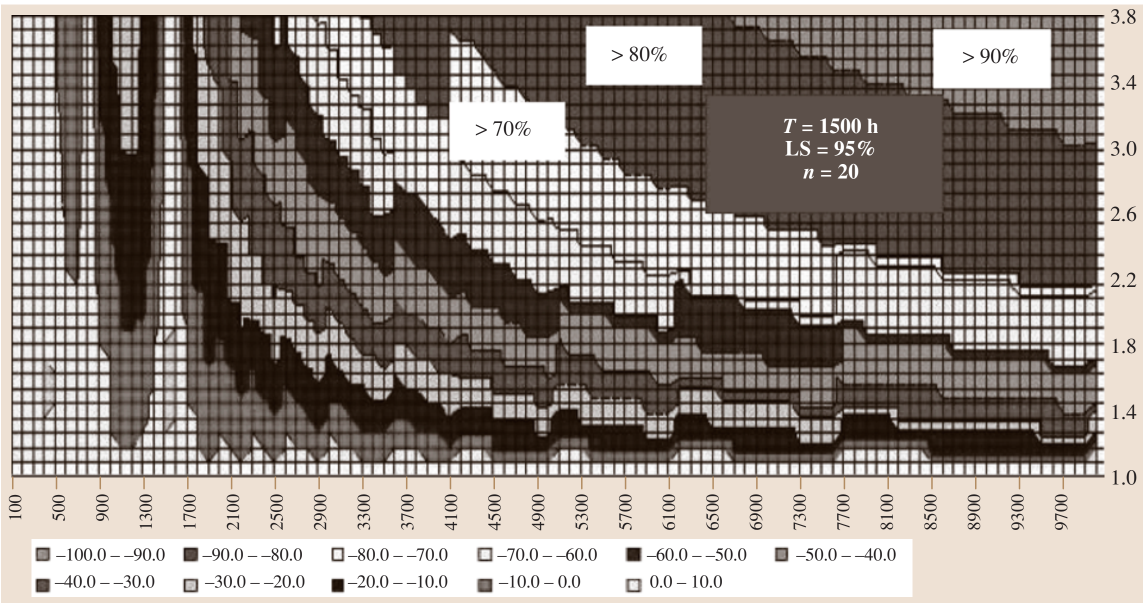

Fig. 48.20 Number of spare parts saved

48.8.2 Simulation and Results

This extended model is compared with previously proposed models for different values of the parameters involved. The relation between the parameters MTBF, T_S and β appears very interesting. In fact the first two parameters are fundamental to finding the quantity x_2, see (48.29) and (48.44), while β indicates the gap from the hypothesis of constant failure rate. The surface of Fig. 48.19 (with T_S equal to 500 h and a customer service level of 95%) relates the difference between optimal replacement numbers calculated

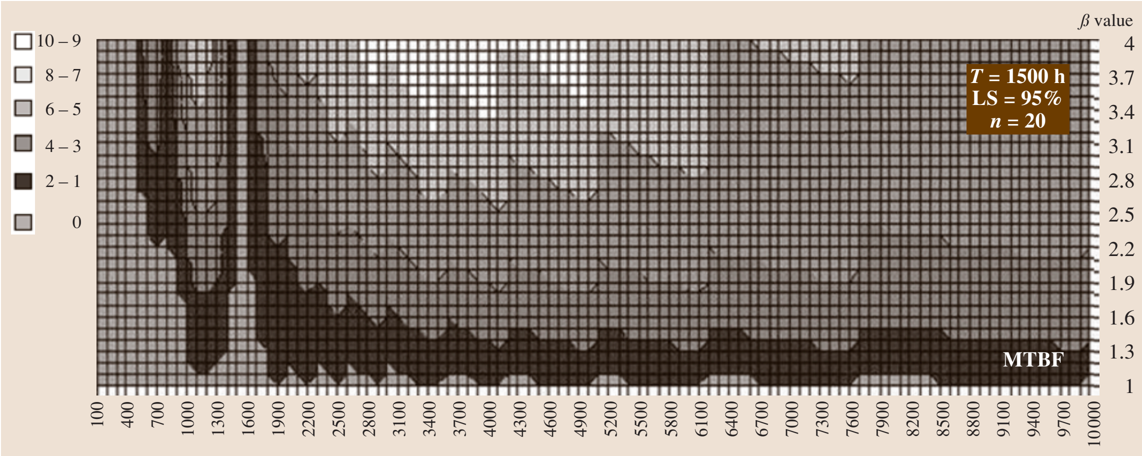

Fig. 48.21 Share of saving with respect to hypothesis of constant failure rate

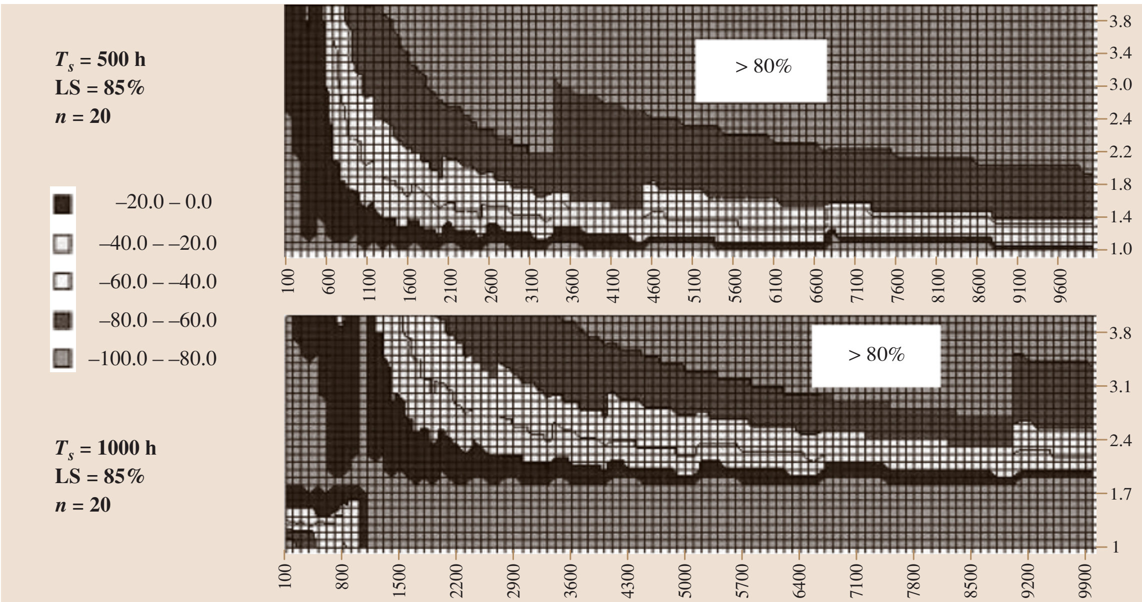

Fig. 48.22 Share of saving with respect to hypothesis of constant failure rate for different parameter values

by (48.29) or (48.44) respectively for given MTBF and β values. For a fixed value of MTBF the saving increases with greater values of β, because the use of the Weibull distribution takes into account that failures are grouped in a specific time region where part breakdowns occur with high probability. The MTBF value of 500 h is very important because it defines T_{residual} equal to zero and so x_2 equals zero for any approach. Values of MTBF lower than 500 h mean that the optimal replacement number is influenced by the quantity x_1, while values grater than 500 h are just defined by the use of quantity x_2 (x_1 being equal to zero). Considering a MTBF range starting from the T_S value for a specific value of β; the saving of spare parts needed decreases with greater values of MTBF, as shown in Fig. 48.23 where two different values of the parameter T_S (500 and 1000 h) are compared. Figure 48.19, as Fig. 48.20, relates the difference between the optimal replacement number calculated by (48.29) and (48.44) for T_S equal to 1500 h.

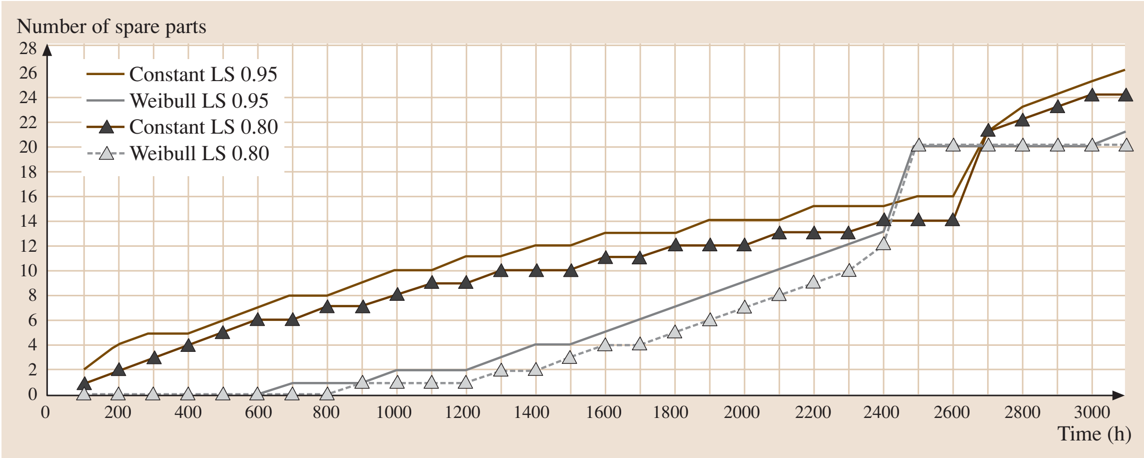

Fig. 48.23 Number of spare parts for component A (number of employments $n = 20$, and $\beta = 4$)

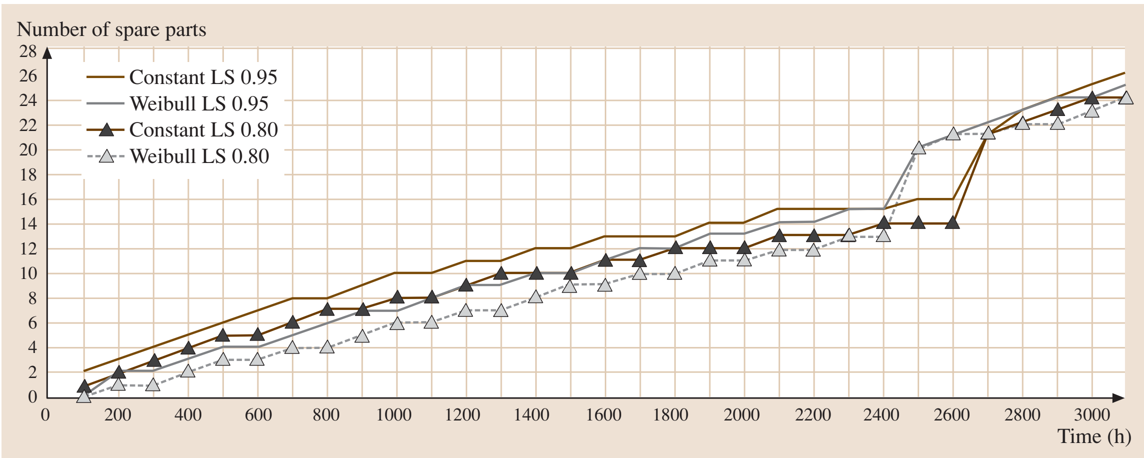

Fig. 48.24 Number of spare parts for component B (number of employments $n = 20$, and $\beta = 1.5$)

Obviously a low frequency of consignments creates a greater requirement for spare parts. Also in this case it is important to notice that methods behave as one when the MTBF is equal to T_S.

48.8.3 Case Study: An Industrial Application

This industrial application deals with an iron metallurgy plant, a European leader in the manufacture of merchant bars. We are interested in production machines characterized by electric iron steel furnaces, rolled sections and the final steps of finishing mills: straightening, cutting to length and packaging.

The extended method was applied with very interesting results, with an average saving of more than 20% in spare parts management. To focus the aim of this study we consider two different components (conic couples), called A and B, that have similar values of MTBF and number of employments n (about 20) but very different values of the parameter β: $\beta = 4$ for part A and $\beta = 1.5$ for part B. The graph in Fig. 48.23 compares the number of spare parts for part A forecasted by methods based on the hypothesis of a time-independent failure rate $\lambda(t)$ or based on the Weibull distribution.

Two different LS values are investigated. It is clear that the use of the Weibull extension reduces the stocks of parts by creating a time delay in the consignments.

The impact will not be significant for values of the β parameter close to one, as stated in Fig. 48.24 for part B, where forecasts are very similar for the methods investigated.

The subject of this study is the evaluation of the spare-parts stock level in maintenance systems in the presence of LTI parts. This paragraph deepens understanding of the fundamental features of a new approach that, in contrast to existing methods, does not consider total spares request to be linear and presents a lower sensitivity to MTBF errors. The proposed model assumes a specific spare-part service level LS defined as the probability of finding the part in case of breakdown: the best values for LS are suggested in Sect. 48.7.1. The aim of the study is to understand whether the hypothesis of a constant failure rate leads to increasing costs or not, compared to more sophisticated distributions. Model extension by the use of the traditional Weibull distribution shows interesting savings in spare parts for values of the β parameter greater than 3 and in the presence of longer times between consignments. The Weibull distribution appears to be a very interesting failure-rate function, but the literature reports some other valid functions; the extended model can easily be extended to any suggested failure-rate function.

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