Academic design with static mixtures

The academic design provides such optimized component weights that the resulting mixture has minimal KL divergence from the user's ideal pdf.

The two-dimensional data record is generated by the static mixture of three normal components with constant probabilistic component weights α_c:

$$d_t \sim \sum_{c=1}^{3} \alpha_c \mathcal{N}_{d_t} \left(\begin{bmatrix} \theta_{c1} \\ \theta_{c2} \end{bmatrix}, \begin{bmatrix} r_{11c} & r_{12c} \\ r_{12c} & r_{22c} \end{bmatrix} \right), \text{ where}$$

$\mathcal{N}_d(\mu, R)$ stands for normal distribution with the expectation μ and covariance matrix R. The use of static mixture as a system model yields simple and transparent demonstration of academic design.

The introduced mixture model is determined by multivariate parameter consisting of component weights α as well as regression coefficients θ and noise variances r of particular factors in components. The parameters's values are given in the following table:

component, c	weight, α_c	θ_{c1}	θ_{c2}	r_{11c}	r_{12c}	r_{22c}
1	1/3	1	2	1	0	1
2	1/3	4	3	1	0	1
3	1/3	3	1	1	0	2

The system mixture and data generated by it are plotted in Fig.1 and Fig.2, respectively.

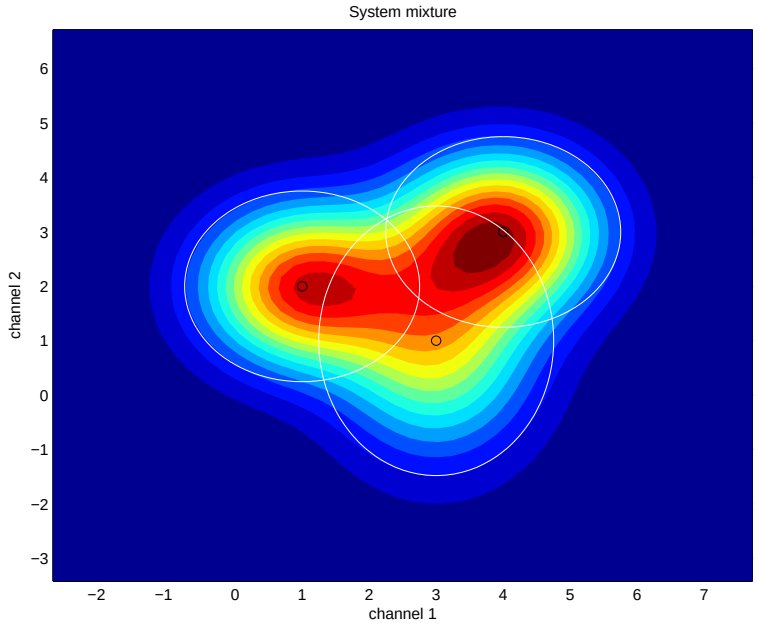

Figure 1: System mixture

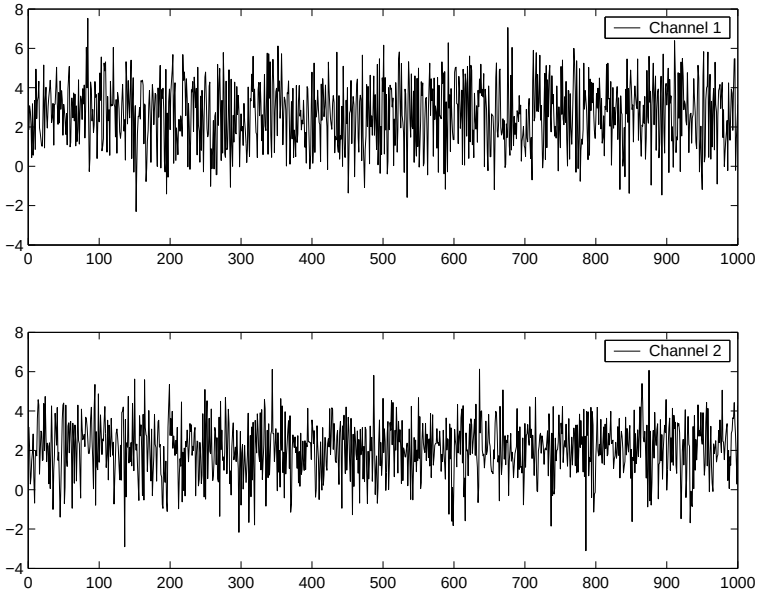

Figure 2: Data generated by system mixture

The true user's ideal pdf represents user's wishes on the system's behavior. Its parameters represent the desired values of corresponding data records while noise covariances define the degree of tolerance of these values. When demands on particular data records are competitive, the user can change the degree of compromise by changing parameters of user's ideal pdf to reach acceptable results.

For the example considered, the true user's ideal pdf ${}^{\lfloor U}f(d(\mathring{t}))$ is defined by:

$$ {}^{\lfloor U}d_t \sim \mathcal{N}_{d_t}\left(\left[\begin{array}{c} {}^{\lfloor U}\theta_1 \\ {}^{\lfloor U}\theta_2 \end{array}\right], \left[\begin{array}{cc} {}^{\lfloor U}r_{11} & {}^{\lfloor U}r_{12} \\ {}^{\lfloor U}r_{12} & {}^{\lfloor U}r_{22} \end{array}\right]\right), $$

with the parameters ${}^{\lfloor U}\theta_1 = 2$, ${}^{\lfloor U}\theta_2 = 2$, ${}^{\lfloor U}r_1 = 1$, ${}^{\lfloor U}r_{12} = 0$, ${}^{\lfloor U}r_2 = 1$. The resulting density is depicted in Fig.3

Additional requirements on the advising strategy, specified at design stage, (for example, quality of advises) can influence user's ideal on recommended pointers to components ${}^{\lfloor U}f(c)$. Normally, this optional design parameter cannot be *directly* influenced by the user. However this illustration example allows its change. Reader is advised to set different values to imitate exclusion some of the component, or to make it less desirable. The default value of this parameter is set to uniform, i.e. ${}^{\lfloor U}f(c) = [1/3 \quad 1/3 \quad 1/3]$.

Horizon of the optimization is set to $hor = 1$. Using higher horizon for this case (static mixtures) has no sense as the result of optimization does not depend on the data.

The optimization is performed according to the Algorithm 9.2.3{348} and gives the following optimized components weights determining advising strategy:

$$ {}^{\lfloor I}f(c) = [0.6040 \quad 0.0817 \quad 0.3142] $$

The optimized mixture and data generated by it are shown in Fig.4 and Fig.5, respectively.

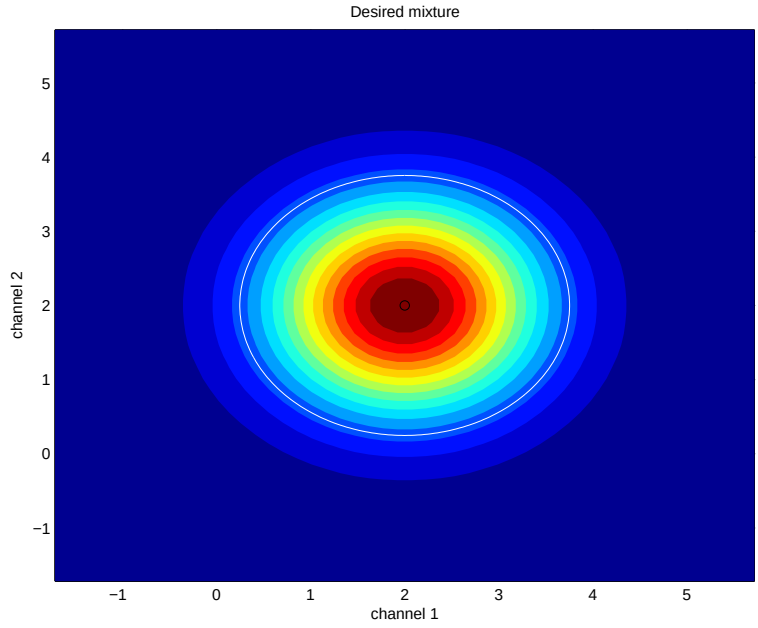

Figure 3: True user"s ideal

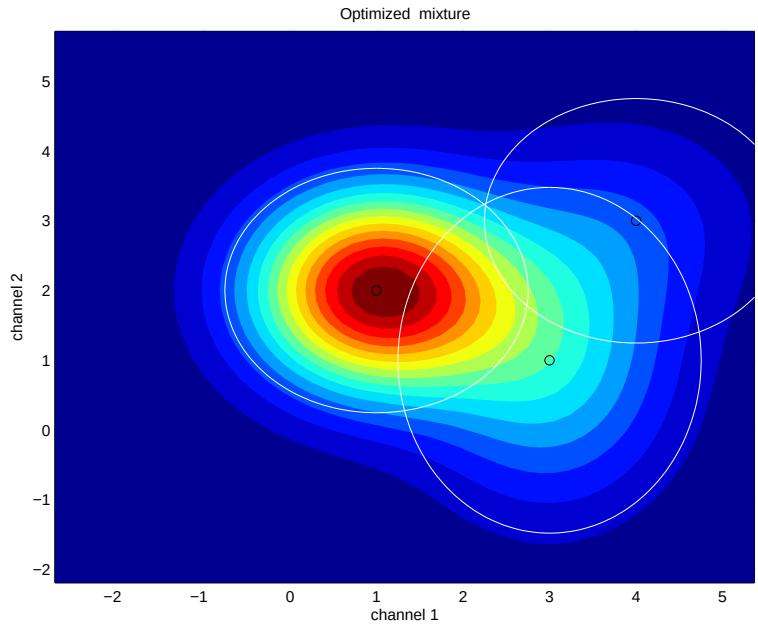

Figure 4: Optimized mixture

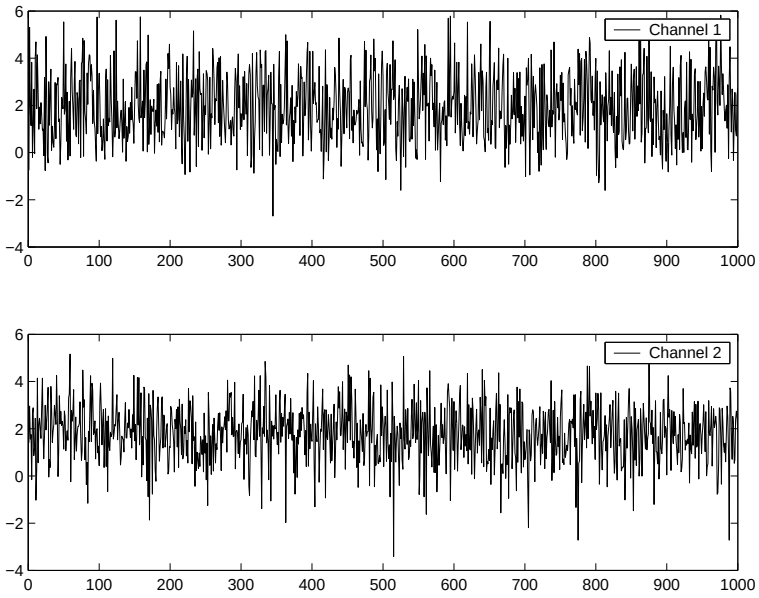

Figure 5: Data generated by optimized mixture

The following Table summarizes computed mean values and variances of particular data records for the original and optimized mixtures.

		original mixture	true user's ideal	optimized mixture
d_1	mean value	2.65	2	1.93
d_1	variance	2.72	1	2.09
d_2	mean value	2.06	2	1.79
d_2	variance	1.97	1	1.49

Recall, that in the academic design the system components are fixed. To influence them the components weights can be changed.

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