Academic design with dynamic mixtures

The academic design provides such optimized component weights that the resulting mixture has minimal KL divergence from the user's ideal pdf.

The second order dynamic system is suggested in the example. The two-dimensional data record is generated by the dynamic mixture of three normal components with constant probabilistic component weights α_c:

$$d_t \sim \sum_{c=1}^{3} \alpha_c \, \mathcal{N}_{d_{1,t}}(\theta_{c1}\psi_{1,t}, r_{1c})\mathcal{N}_{d_{2,t}}(\theta_{c2}\psi_{2,t}, r_{2c}),$$

where $\psi_{1,t}$, $\psi_{2,t}$ are regression vectors of respective factors and $\mathcal{N}_d(\mu, R)$ stands for normal distribution with the expectation μ and covariance matrix R. The regression vectors have the form $\psi_{1,t} = [d_{1,t-1}, d_{1,t-2}, d_{2,t}, d_{2,t-1}, d_{2,t-1}, d_{2,t-2}, 1]$ for factors describing d_1 and $\psi_{2,t} = [d_{2,t-1}, d_{1,t-1}, d_{1,t-2}, 1]$ for those describing d_2. In dynamic models the mean value μ depends on the past data.

The introduced mixture model is determined by multivariate parameter consisting of component weights α as well as regression coefficients θ and noise variances r of particular factors in components. The parameters's values are given in the following table:

component, c	factor, i	regression coefficient, θ_{ic}	noise variance, r_{ic}
1	1	[1.81 -0.8189 0 0 0.00438 0.00468 0]	.1
1	2	[0 0 0 0];	.1
2	1	[1.81 -0.8189 0.00438 0.00468 0 0 0];	1
2	2	[-.33 -94 81 0]	.1
3	1	[2.1040 -1.1067 0 0.00468 0.00438 0 0];	.1
3	2	[0 0 0 0]	.1

The specified components, with weights $\alpha = [1/3 \quad 1/3 \quad 1/3]$, have special meaning. The first component represents open-loop dynamic system with a transport delay. The second component describes closed loop system with well designed controller and the third component represents unstable open loop system.

Data simulated by the assumed mixture model are plotted in Fig.1. The first two plots show time

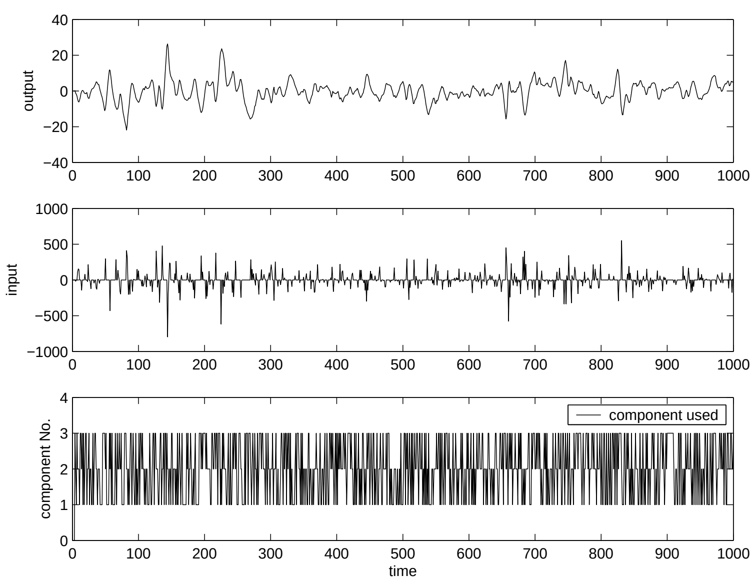

Figure 1: Simulated system data and active components

dependencies of the data record while the third one depicts the component, active at particular time. Notice, that even if the unstable component is active 1/3 of time, the whole process remains stable.

The user's wishes on the system's behavior are represented by the true user's ideal. In particular, parameters of true user's ideal pdf represent the desired values of corresponding data records while noise covariances define the degree of tolerance of these values. When restrictions on particular data records are competitive, the user can express his preferences by changing parameters of user's ideal pdf to reach acceptable results.

Let, for the example considered, the user's aim is to have quantity $d_{1,t}$ as small as possible. Regarding the second quantity $d_{2,t}$, two possibilities will be inspected: i)to keep $d_{2,t}$ also small; and ii) to let be $d_{2,t}$ large. Thus, the true user's ideal pdf ${}^{\lfloor U}f(d(\mathring{t}))$ can be one static component:

$$ {}^{\lfloor U}d_t \sim \mathcal{N}_{d_{1,t}}({}^{\lfloor U}\theta_1 \psi_{1,t}, {}^{\lfloor U}r_1)\mathcal{N}_{d_{2,t}}({}^{\lfloor U}\theta_2 \psi_{2,t}, {}^{\lfloor U}r_2) \qquad (1) $$

with parameters

A: ${}^{\lfloor U}\theta_1 = 0,\ {}^{\lfloor U}\theta_2 = 0,\ {}^{\lfloor U}r_1 = .1,\ {}^{\lfloor U}r_{12} = 0,\ {}^{\lfloor U}r_2 = 1$ in the case when $d_{2,t}$ should be small, and

B: ${}^{\lfloor U}\theta_1 = 0,\ {}^{\lfloor U}\theta_2 = 0,\ {}^{\lfloor U}r_1 = .1,\ {}^{\lfloor U}r_{12} = 0,\ {}^{\lfloor U}r_2 = 10000$ in the case when $d_{2,t}$ can be large

Additional requirements on the advising strategy, specified at design stage, (for example, quality of advises) can influence user's ideal on recommended pointers to components ${}^{\lfloor U}f(c)$ (default value is set to uniform, i.e. ${}^{\lfloor U}f(c) = [1/3 \quad 1/3 \quad 1/3]$). Normally, this optional design parameter cannot be *directly* influenced by the user. However the present illustrative example allows its change. Reader is advised to set different values ${}^{\lfloor U}f(c)$ to imitate an exclusion some of the components, or to make it less desirable. For example, as the third component is unstable, it can be avoided completely by specifying: ${}^{\lfloor U}f(c) = [1/2 \quad 1/2 \quad 0]$.

The optimization is performed according to the Algorithm 9.2.3{348}. The result is substantially influenced by the chosen optimization horizon $1 \leq hor < \infty$. Special attention deserve the choice optimization horizon, which is determined by two-element vector, i.e. $hor = [hor_1 hor_2]$, where hor_2 determines the period (in number of steps) over which each component is considered separately (as the only that component is active) and hor_1 is a number of such periods. Setting $hor_2 = 1$ causes all the components be considered together at each step. Notice that the sufficiently high choice of hor_2 can remove pointer to the unstable component from the recommended pointers.

Data generated by the advised optimized system for the variant **A** of the true user's ideal (1) with ${}^{\lfloor U}f(c) = [1/3 \quad 1/3 \quad 1/3]$, hor=10, are shown in Fig.2

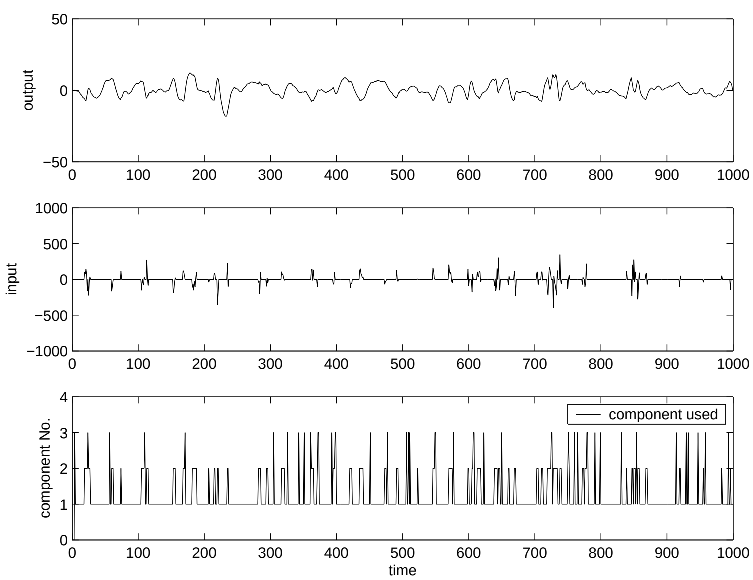

Figure 2: Optimized data and advised components

Data generated by the advised optimized system for the variant **A** of the true user's ideal (1) with ${}^{\lfloor U}f(c) = [1/3 \quad 1/3 \quad 1/3]$, hor=10, are shown in Fig.3

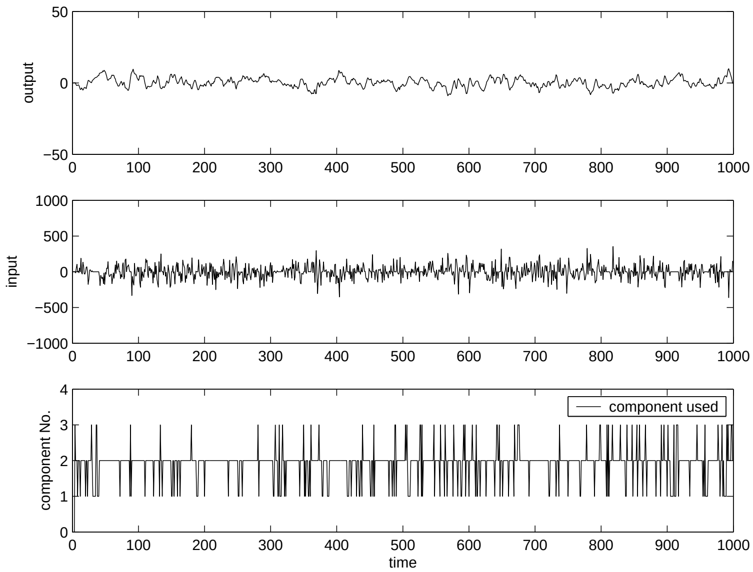

Figure 3: Optimized data and advised components

There are two ways how to exclude the unstable component from the further considerations:

- explicit choice of ${}^{\lfloor U}f(c) = [1/2 \quad 1/2 \quad 0]$. Recall that in the real case, user cannot set ${}^{\lfloor U}f(c)$ directly, but only via some algorithm that will qualify the components according to user's specifications (for example, instability criterion).

  Data generated by the optimized system for the variant **A** of the true user's ideal (1), *hor*=10 and with this artificial choice ${}^{\lfloor U}f(c) = [1/2 \quad 1/2 \quad 0]$ are shown in Fig.4

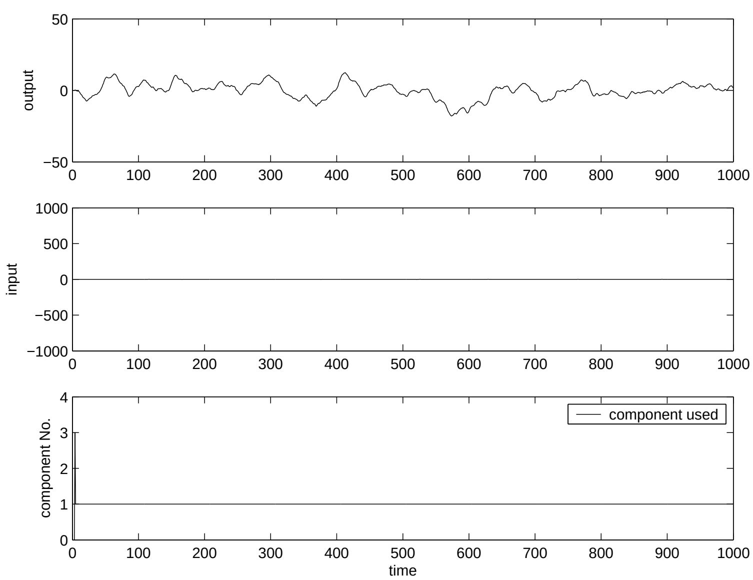

Figure 4: Optimized data and advised components

  Data generated by the optimized system for the variant **B** of the true user's ideal (1), *hor*=10 and with this artificial choice ${}^{\lfloor U}f(c) = [1/2 \quad 1/2 \quad 0]$ are shown in Fig.5

- choice of such optimization strategy that the instability of particular component will be more apparent. It is expected that individual components will be acting for longer time successively and optimization horizon set to *hor*= [1 50].

  Data generated by the optimized system for the variant **A** of the true user's ideal (1), with default ${}^{\lfloor U}f(c) = [1/3 \quad 1/3 \quad 1/3]$ and optimization horizon set to *hor*= [1 50] are shown in Fig.6

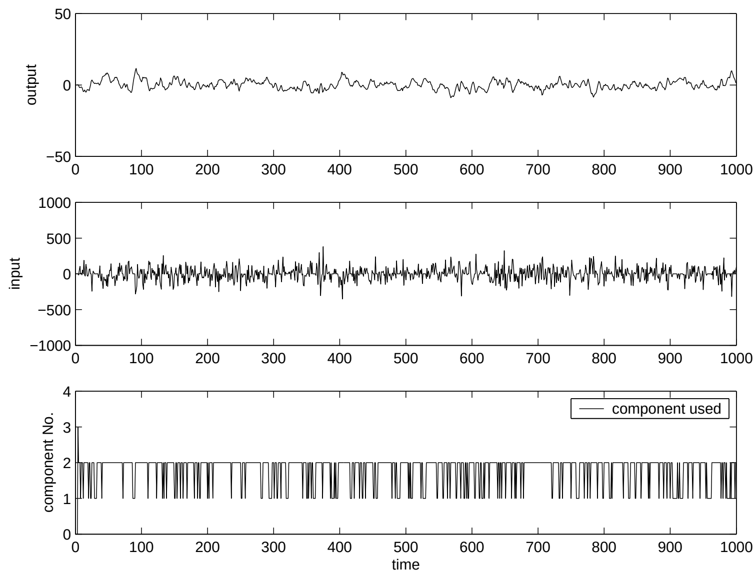

Figure 5: Optimized data and advised components

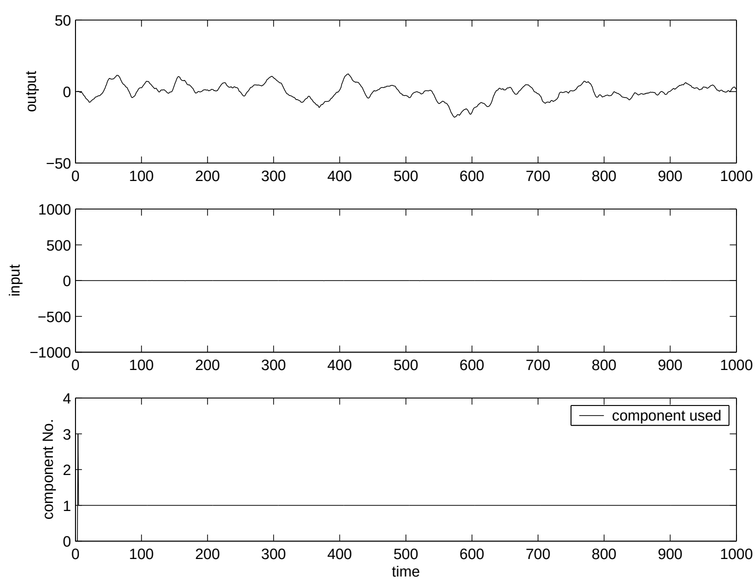

Figure 6: Optimized data

Data generated by the optimized system for the variant **A** of the true user's ideal (1), with default ${}^{\lfloor U}f(c) = [1/3 \quad 1/3 \quad 1/3]$ and optimization horizon set to *hor*$= [1\ 50]$ are shown in Fig.7

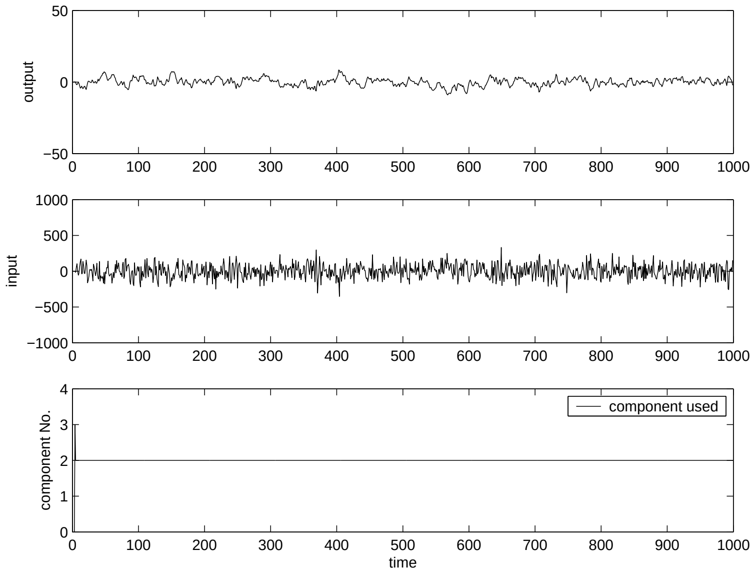

Figure 7: Optimized data

Example demonstrates how the user's ideal on pointers to components and the choice of *hor* influence results of academic design.

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