Simultaneous design with dynamic mixtures

Simultaneous academic and industrial design represents the most powerful type of design. It combines features of academic and industrial designs and *simultaneously* optimizes recommended pointers to components and recommended recognizable actions. The simultaneous design searches for the such advising strategy that the resulting optimized mixture has minimal KL divergence from the user's ideal pdf. The recognizable actions coincide with system inputs in the present example.

The second order dynamic system is suggested. The two-dimensional data record, consisting of system input and output, is generated by the dynamic mixture of three normal components with constant probabilistic component weights α_c:

$$d_t \sim \sum_{c=1}^{3} \alpha_c \, \mathcal{N}_{d_{1,t}}(\theta_{c1}\psi_{1,t}, r_{1c})\mathcal{N}_{d_{2,t}}(\theta_{c2}\psi_{2,t}, r_{2c}),$$

where $\psi_{1,t}$, $\psi_{2,t}$ are regression vectors of respective factors and $\mathcal{N}_d(\mu, R)$ stands for normal distribution with the expectation μ and covariance matrix R. The regression vectors have the form $\psi_{1,t} = [d_{1,t-1}, d_{1,t-2}, d_{2,t}, d_{2,t-1}, d_{2,t-1}, d_{2,t-2}, 1]$ for factors describing d_1 and $\psi_{2,t} = [d_{2,t-1}, d_{1,t-1}, d_{1,t-2}, 1]$ for those describing d_2. In dynamic models the mean value μ depends on the past data.

The introduced mixture model is determined by multivariate parameter consisting of component weights α as well as regression coefficients θ and noise variances r of particular factors in components. The parameters's values are given in the following table:

component, c	factor, i	regression coefficient, θ_{ic}	noise variance, r_{ic}
1	1	[1.81 -0.8189 0 0 0.00438 0.00468 0]	.1
1	2	[0 0 0 0];	.1
2	1	[1.81 -0.8189 0.00438 0.00468 0 0 0];	1
2	2	[-.33 -94 81 0]	.1
3	1	[2.1040 -1.1067 0 0.00468 0.00438 0 0];	.1
3	2	[0 0 0 0]	.1

The specified components, with weights $\alpha = [1/3 \quad 1/3 \quad 1/3]$, have special meaning. The first component represents open-loop dynamic system with a transport delay. The second component describes closed loop system with well designed controller and the third component represents unstable open loop system.

Data simulated by the assumed mixture model are plotted in Fig.1.

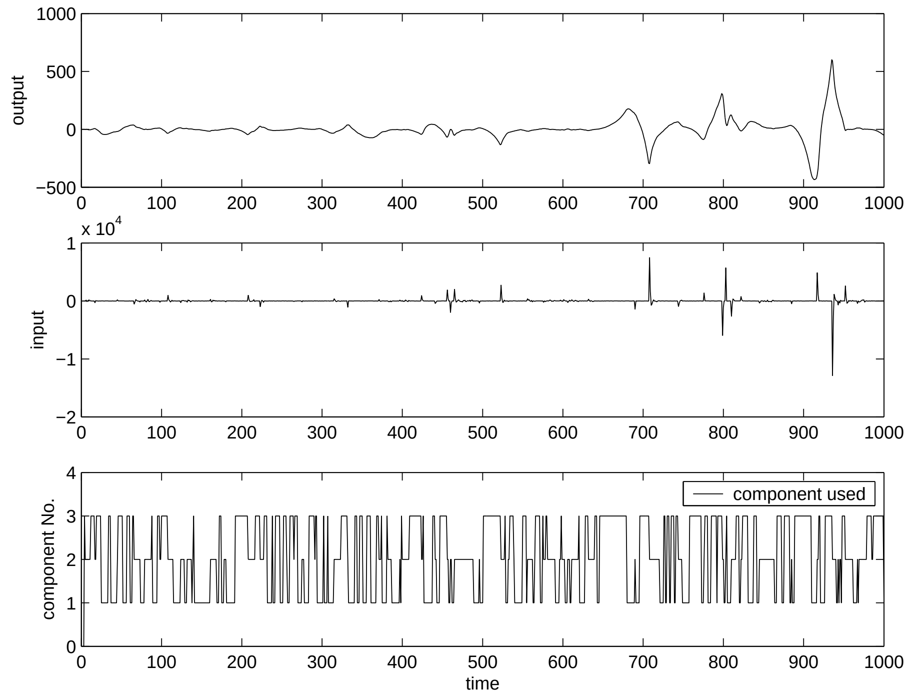

Figure 1: Original system data and active components

The first two plots show time dependencies of the data record while the third one depicts the component, active at particular time. Notice, that even if the unstable component is active 1/3 of time, the whole process remains stable.

The *original system data* possess the following values of covariance and mean

- covariance of the output = 7.7317e+003
- mean of the output = 2.5059
- covariance of the input = 3.7870e+005

Assuming fully cooperating user, the advised recognizable actions represent an optimal feedback controller. Thus it is possible to change the dynamic of individual components. The instability of particular component is not dangerous here, as the component can be stabilized by the strategy designed.

Compare to industrial design, the optimized recommended pointers determine the best controllable mixture which possesses the pdf on data to be even more closer to the true user's ideal pdf. As the result of optimization is mainly influenced by its choice, user have to choose true user's ideal more carefully. The user's wishes on the system's behavior are represented by the true user's ideal. In particular, parameters of true user's ideal pdf represent the desired values of corresponding data records while noise covariances define the degree of tolerance of these values. When restrictions on particular data records are competitive, the user can express his preferences by changing parameters of user's ideal pdf to reach acceptable results.

Similarly to academic design, additional requirements on the advising strategy, specified at design stage, can influence user's ideal on recommended pointers to components ${}^{\lfloor U}f(c)$, which cannot be *directly* influenced by the user. However, use of this optional design parameter is redundant for setting component stability requirements, as a component can be stabilized by the strategy designed. To demonstration purposes, this example allows direct change of ${}^{\lfloor U}f(c)$. Reader is advised to set different values ${}^{\lfloor U}f(c)$ to imitate an exclusion some of the components, or to make it less desirable. For example, to avoid the second component completely, one can specify: ${}^{\lfloor U}f(c) = [1/2 \quad 0 \quad 1/2]$.

Let, for the example considered, the user's wish is to have output $d_{1,t}$ as close as possible to the prescribed value $y0$. Regarding the second quantity, input, $d_{2,t}$, two possibilities will be inspected: i) to keep the variance of input small; and, ii) to let input almost arbitrary. Thus, the true user's ideal pdf

${}^{\lfloor U}f(d(\mathring{t}))$ can be described by one static component:

$$
{}^{\lfloor U}d_t \sim \mathcal{N}_{d_{1,t}}({}^{\lfloor U}\theta_1 \psi_{1,t},\ {}^{\lfloor U}r_1)\mathcal{N}_{d_{2,t}}({}^{\lfloor U}\theta_2 \psi_{2,t},\ {}^{\lfloor U}r_2) \tag{1}
$$

with parameters

A: ${}^{\lfloor U}\theta_1 = 5,\ \ {}^{\lfloor U}\theta_2 = 5,\ \ {}^{\lfloor U}r_1 = 1,\ \ {}^{\lfloor U}r_2 = 1$, in the case when variance of $d_{2,t}$ should be small

B: ${}^{\lfloor U}\theta_1 = 5,\ \ {}^{\lfloor U}\theta_2 = 0,\ \ {}^{\lfloor U}r_1 = 1,\ \ {}^{\lfloor U}r_2 = 10000$, in the case when $d_{2,t}$ can be large

C: ${}^{\lfloor U}\theta_1 = 0,\ \ {}^{\lfloor U}\theta_2 = 0,\ \ {}^{\lfloor U}r_1 = 1,\ \ {}^{\lfloor U}r_2 = 10000$, in the case when $d_{2,t}$ can be large

If the user wishes to have output $d_{1,t}$ around some fixed point $y0$, he must either set the mean of the true user's ideal or the respective input to some value $u0 \equiv y0$ as was done in case **A**, or must allow wide range of input, as in the cases **B** and **C**.

The optimization is performed according to Algorithm 9.2.19{368}. The result is substantially influenced by the chosen optimization horizon $1 \leq$*hor*$< \infty$. Significant influence has the grouping part of the horizon hor.

Data generated by the advised optimized system for the variant **A** of the true user's ideal (1) and *hor*$= 500$, are shown in Fig.2

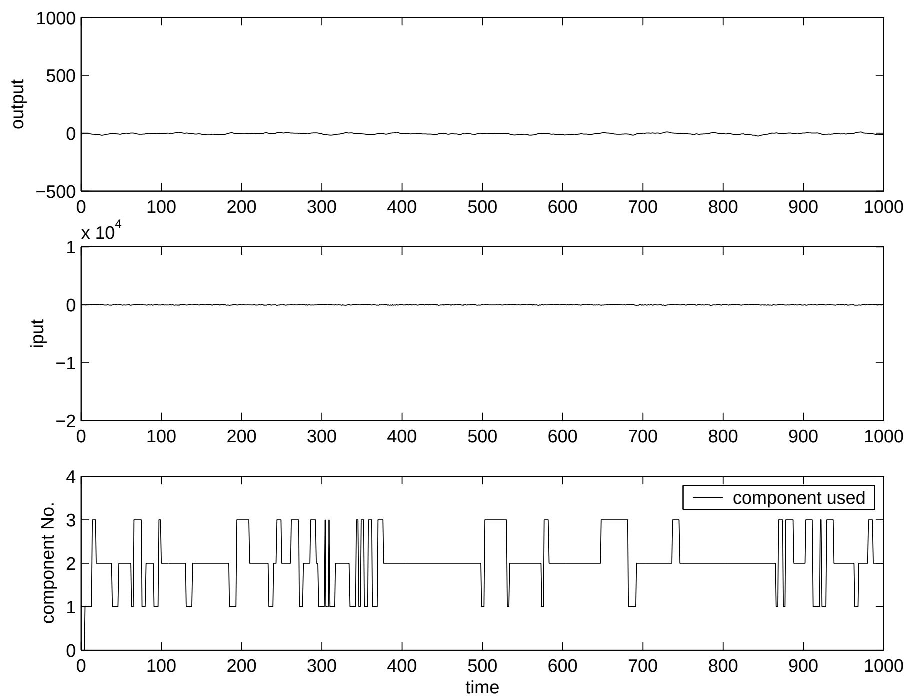

Figure 2: Optimized data and advised components

The *optimized system* (variant **A** of the true user's ideal) possess the following values of covariance and mean. For comparison a corresponding value for the data, generated by original system, is given.

- covariance of the output = 31.7611 (original data: 7.7317e+003)
- mean of the output = -4.9727 (original data: 2.5059)
- covariance of the input = 1.4417e+003 (original data: 3.7870e+005)

Data generated by the advised optimized system for the variant **B** of the true user's ideal (1) and *hor*$= 500$, are shown in Fig.3.

The *optimized system* (variant **B** of the true user's ideal) possess the following values of covariance and mean. For comparison a corresponding value for the data, generated by original system, is given.

- covariance of the output = 3.1655 (original data: 7.7317e+003)
- mean of the output = -5.0272 (original data: 2.5059)

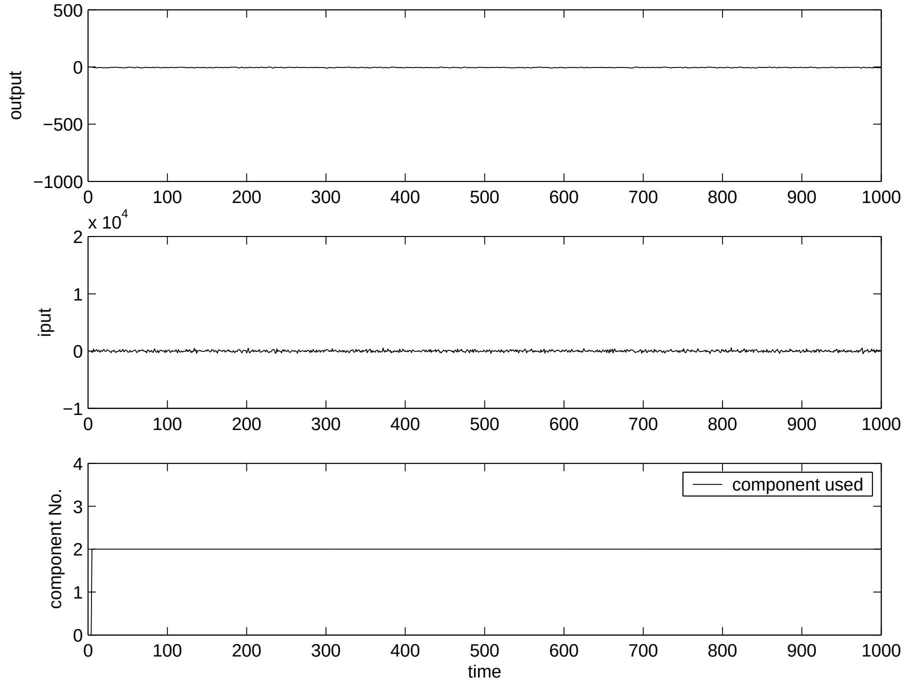

Figure 3: Optimized data and advised components

- covariance of the input = 2.5500e+004 (original data: 3.7870e+005)

Data generated by the advised optimized system for the variant **C** of the true user's ideal (1) and *hor*= [10 50] , are shown in Fig.4.

The *optimized system* (variant **C** of the true user's ideal) possess the following values of covariance and mean. For comparison a corresponding value for the data, generated by original system, is given.

- covariance of the output = 2.5556 (original data: 7.7317e+003)
- mean of the output = -0.0807 (original data: 2.5059)
- covariance of the input = 3.3063e+003 (original data: 3.7870e+005)

Data generated by the advised optimized system for the variant **C** of the true user's ideal (1) and *hor*= 500 , are shown in Fig.5.

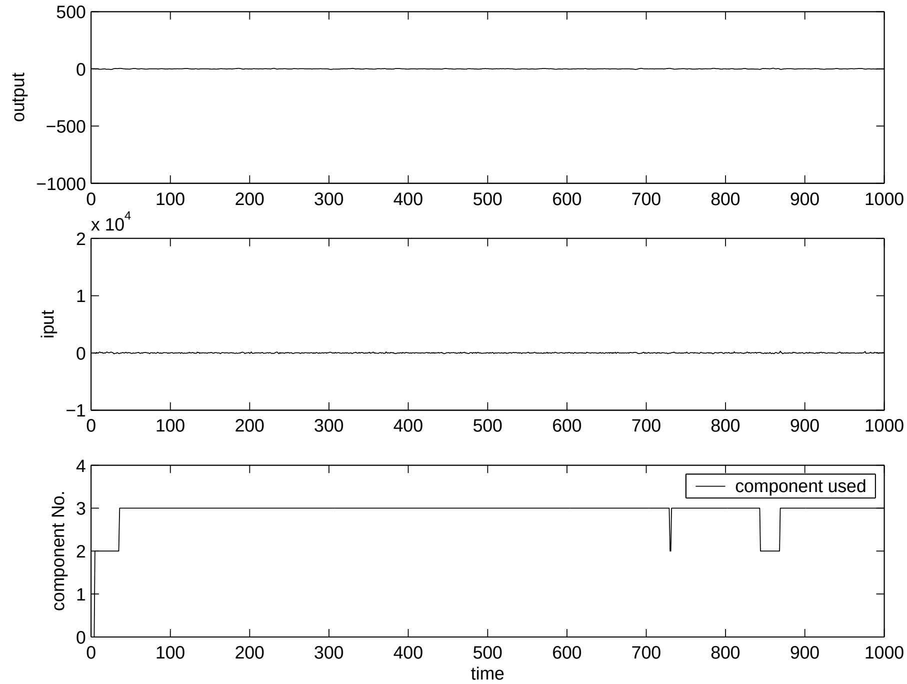

Figure 4: Optimized data and advised components

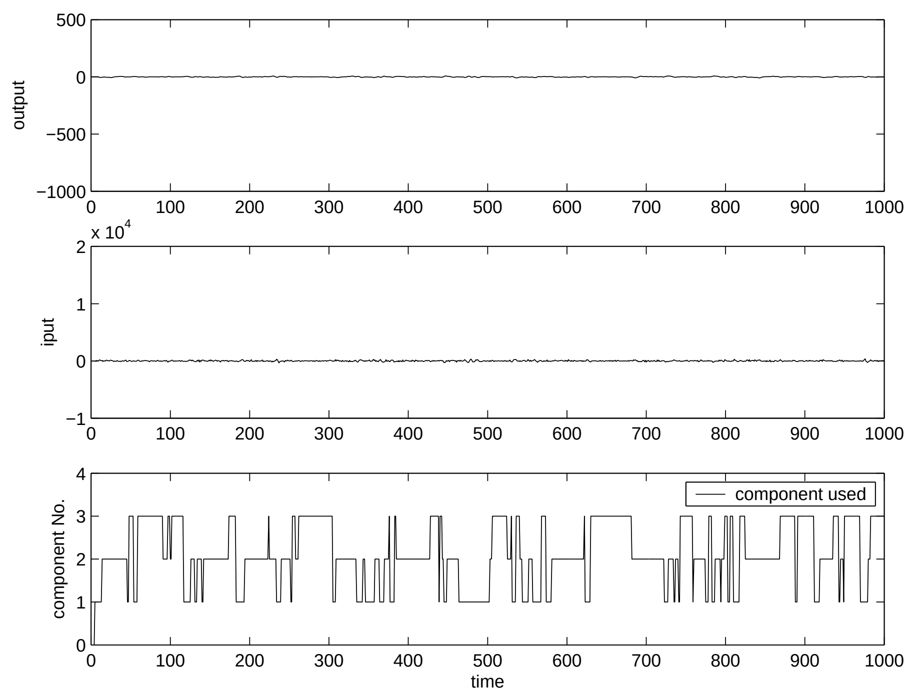

Figure 5: Optimized data and advised components

The *optimized system* (variant **C** of the true user's ideal) possess the following values of covariance and mean. For comparison a corresponding value for the data, generated by original system, is given.

- covariance of the output = 7.0237 (original data: 7.7317e+003)
- mean of the output = -0.2339 (original data: 2.5059)
- covariance of the input = 8.0884e+003 (original data: 3.7870e+005)

Remark:
Industrial design needs to be specified which quantities entering data record can be manipulated in order to change the behavior of the rest data. Simultaneous design provides the feedback action to the system described by a mixture. If original system contained some feedbacks, they are substituted by an optimized one. The optimization depends on the mixture model parameters, true user ideal pdf and the optimization horizon *hor*. For *hor*$\rightarrow \infty$ the solution approaches a steady state values. The original component weights does not influence the optimization.

Contents Back to example