

# Industrial design with dynamic mixtures

Industrial design optimizes recommended [recognizable actions](#) and offers them as recommended operator's actions while keeping component weights unchanged. The industrial design provides such [strategy](#) that the resulting [mixture](#) has minimal [KL divergence](#) from the [user's ideal pdf](#). The example supposes recognizable actions coincide with system [inputs](#).

The second order dynamic system with two-dimensional [data record](#), consisting of system input and system [output](#), is simulated by the [dynamic mixture](#) of three normal components with constant probabilistic [component weights](#)  $\alpha_c$ :

$$d_t \sim \sum_{c=1}^3 \alpha_c \mathcal{N}_{d_{1,t}}(\theta_{c1}\psi_{1,t}, r_{1c}) \mathcal{N}_{d_{2,t}}(\theta_{c2}\psi_{2,t}, r_{2c}),$$

where  $\psi_{1,t}$ ,  $\psi_{2,t}$  are [regression vectors](#) of respective [factors](#) and  $\mathcal{N}_d(\mu, R)$  stands for normal distribution with the expectation  $\mu$  and covariance matrix  $R$ . The regression vectors have the form  $\psi_{1,t} = [d_{1,t-1}, d_{1,t-2}, d_{2,t}, d_{2,t-1}, d_{2,t-2}, d_{2,t-3}, 1]$  for factors describing  $d_1$  and  $\psi_{2,t} = [d_{2,t-1}, d_{1,t-1}, d_{1,t-2}, 1]$  for those describing  $d_2$ . In dynamic models the mean value  $\mu$  depends on the past data.

The introduced mixture model is determined by multivariate [parameter](#) consisting of component weights  $\alpha$  as well as [regression coefficients](#)  $\theta$  and noise variances  $r$  of particular factors in components. The parameters's values are given in the following table:

component, $c$	factor, $i$	regression coefficient, $\theta_{ic}$	noise variance, $r_{ic}$
1	1	[1.81 -0.8189 0 0 0.00438 0.00468 0]	.1
1	2	[0 0 0 0];	.1
2	1	[1.81 -0.8189 0.00438 0.00468 0 0 0];	1
2	2	[-.33 -94 81 0]	.1
3	1	[2.1040 -1.1067 0 0.00468 0.00438 0 0];	.1
3	2	[0 0 0 0]	.1

The specified components, with weights  $\alpha = [1/3 \ 1/3 \ 1/3]$ , have special meaning. The first component represents open-loop dynamic system with a transport delay. The second component describes closed loop system with well designed controller and the third component represents unstable open loop system.

Data simulated by the assumed mixture model are plotted in Fig.1. The first two plots show time dependencies of the data record while the third one depicts the component, active at particular time. Notice, that even if the unstable component is active 1/3 of time, the whole process remains stable.

The *original system data* possess the following values of covariance and mean

- covariance of the output = 7.4835e+003
- mean of the output = 0.6978
- covariance of the input = 3.2890e+005

Assuming fully cooperating user, the advisory strategy, generated by industrial design, is equivalent to an optimal feedback controller. Thus it is possible to change the dynamic of individual components. The instability of particular component is not dangerous here, as the component can be stabilized by the strategy designed.

Compare to [academic design](#), the optimized pdf on data can be much closer to the [true user's ideal pdf](#). As the result of optimization is mainly influenced by its choice, user have to choose true user's ideal more carefully. The user's wishes on the system's [behavior](#) are represented by the true user's ideal. In particular, parameters of true user's ideal pdf represent the desired values of corresponding data records while noise covariances define the degree of tolerance of these values. When restrictions on particular data records are competitive, the user can express his preferences by changing parameters of user's ideal pdf to reach acceptable results. Notice, that user's ideal on pointers to components  ${}^U f(c)$  does not have any influence here as the component weights are considered to be fixed.

Let, for the example considered, the user's wish is to have quantity  $d_{1,t}$  as close as possible to the prescribed value  $y_0$ . Regarding the second quantity, input,  $d_{2,t}$ , two possibilities will be inspected: i) to keep the variance of input small; and, ii) to let input almost arbitrary. Thus, the true user's ideal pdf  ${}^U f(d(t))$  can be one static component:

$${}^U d_t \sim \mathcal{N}_{d_{1,t}}({}^U \theta_1 \psi_{1,t}, {}^U r_1) \mathcal{N}_{d_{2,t}}({}^U \theta_2 \psi_{2,t}, {}^U r_2) \quad (1)$$

with parameters

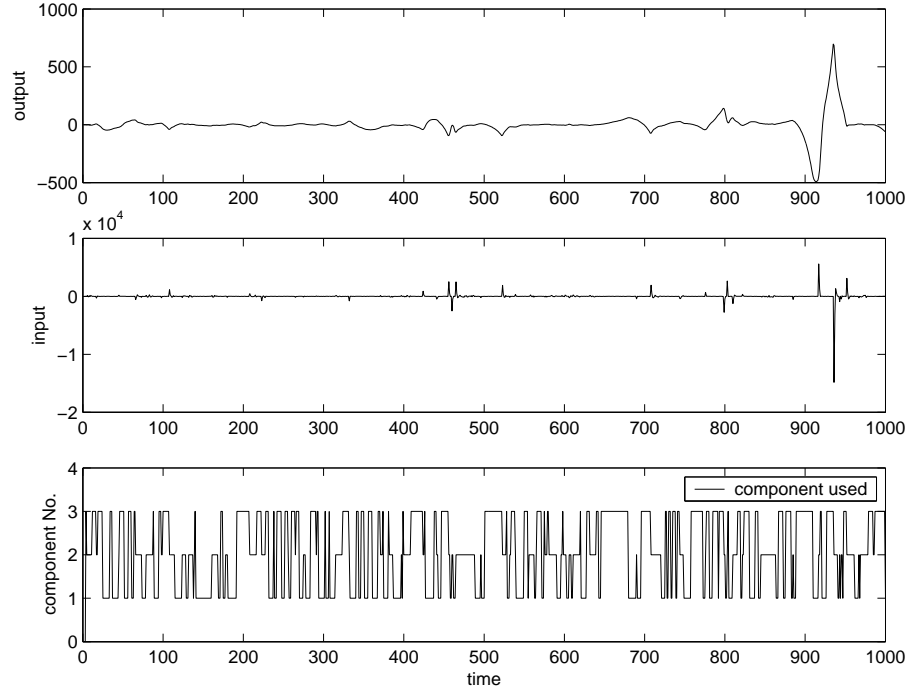


Figure 1: Original system data and active components

**A:**  $\mathbb{I}^{U\theta_1} = -5$ ,  $\mathbb{I}^{U\theta_2} = -5$ ,  $\mathbb{I}^{Ur_1} = 1$ ,  $\mathbb{I}^{Ur_2} = 10$  in the case when variance of  $d_{2,t}$  should be small, and

**B:**  $\mathbb{I}^{U\theta_1} = 5$ ,  $\mathbb{I}^{U\theta_2} = 0$ ,  $\mathbb{I}^{Ur_1} = 1$ ,  $\mathbb{I}^{Ur_{12}} = 0$ ,  $\mathbb{I}^{Ur_2} = 10000$  in the case when  $d_{2,t}$  can be large

If the user wishes to have output  $d_{1,t}$  around some fixed point  $y_0$ , he must either set the mean of the true user's ideal or the respective input to some value  $u_0 \equiv y_0$  as was done in case **A**, or must allow wide range of input, as in the case **B**.

The optimization is performed according to Algorithm 9.2.10{358}. The result is substantially influenced by the chosen optimization horizon  $1 \leq \text{hor} < \infty$ . Data generated by the advised optimized system for the variant **A** of the true user's ideal (1) and  $\text{hor} = 100$ , are shown in Fig.2

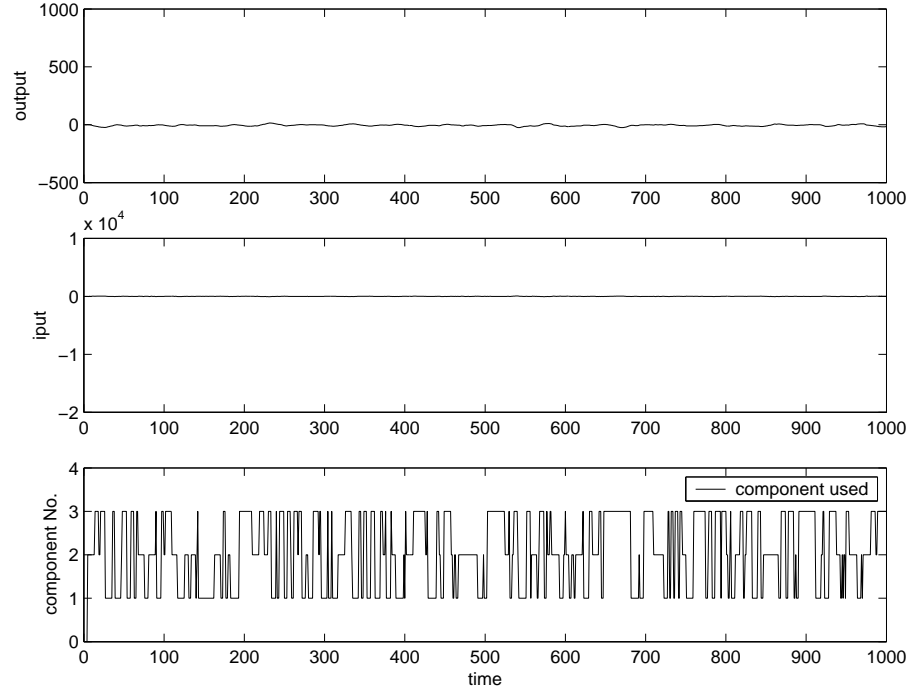


Figure 2: Optimized data and advised components

The *optimized system* (variant **A** of the true user's ideal) possess the following values of covariance and mean. For comparison a corresponding value for the data, generated by original system, is given.

- covariance of the output = 43.8888 (original data: 7.4835e+003)
- mean of the output = -5.7084 (original data: 0.6978)
- covariance of the input = 369.1894 (original data: 3.2890e+005)

Data generated by the advised optimized system for the variant **B** of the true user's ideal (1) and  $hor=100$ , are shown in Fig.3

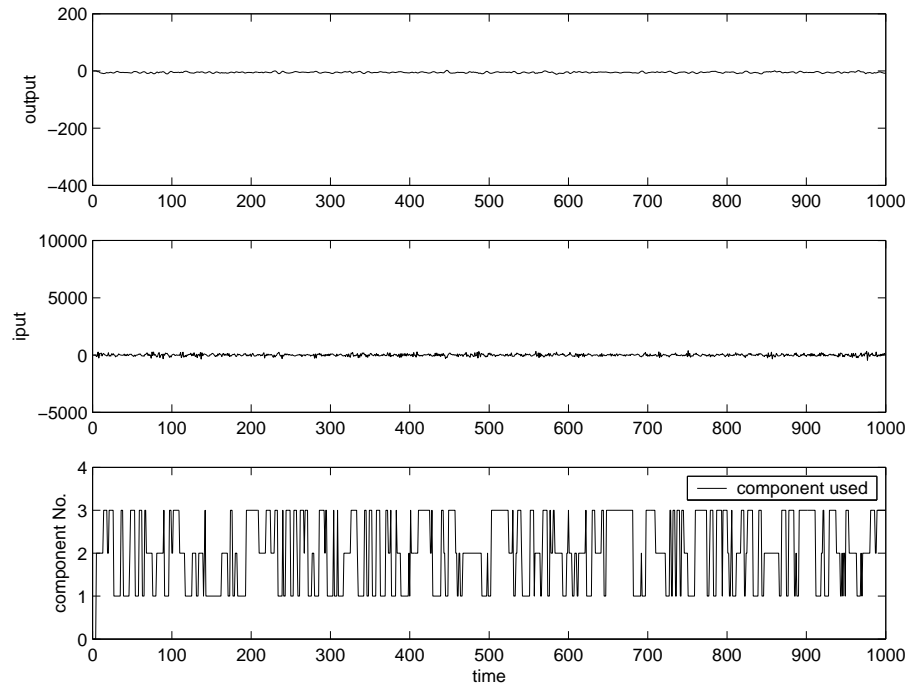


Figure 3: Optimized data and advised components

The *optimized system* (variant **B** of the true user's ideal) possess the following values of covariance and mean. For comparison a corresponding value for the data, generated by original system, is given.

- covariance of the output = 4.1999 (original data: 7.4835e+003)
- mean of the output = -5.0837 (original data: 0.6978)
- covariance of the input = 1.0309e+004 (original data: 3.2890e+005)

Remark:

Industrial design needs to be specified which quantities entering data record can be manipulated in order to change the [behavior](#) of the rest data. Industrial design provides the feedback action to the system described by a mixture. If original system contained some feedbacks, they are substituted by an optimized one. The optimization depends on the mixture model parameters, true user's ideal and optimization horizon [hor](#). For  $hor \rightarrow \infty$  the solution approaches a steady state values. Comparing the third plot in Fig.1 with corresponding plots in Fig.2 and Fig.3 it can be verified that the component weights have not been changed during optimization.

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