G24SYMBH

Symbolic calculation of the product of 3 matrices corresponding to a general thick lens.

1. Symbolic calculation of the matrix for the thick lens

$$\begin{pmatrix} 1 & 0 \\ P23 & \frac{n2}{n3} \end{pmatrix} \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ P12 & \frac{n1}{n2} \end{pmatrix}$$

P12 = (-1/r1)((n2-n1)/n2)
P23 = (-1/r2)((n3-n2)/n3)

$$\begin{bmatrix} 1 + d \cdot P12 & d \cdot \frac{n1}{n2} \\ \frac{(P23 \cdot n3 + P12 \cdot P23 \cdot d \cdot n3 + P12 \cdot n2)}{n3} & \frac{(P23 \cdot d \cdot n3 + n2)}{n3} \cdot \frac{n1}{n2} \end{bmatrix}$$

2. Determination of h and hh. For simpler calculation we define the matrix

$$\begin{pmatrix} M_{0,0} & M_{0,1} \\ M_{1,0} & M_{1,1} \end{pmatrix}$$

$$M_{0,0} = 1 + d \cdot P12 \qquad M_{0,1} = d \cdot \frac{n1}{n2}$$

$$M_{1,0} = \frac{(P23 \cdot n3 + P12 \cdot P23 \cdot d \cdot n3 + P12 \cdot n2)}{n3} \qquad M_{1,1} = \frac{(P23 \cdot d \cdot n3 + n2)}{n3} \cdot \frac{n1}{n2}$$

and determine h and hh

$$\begin{pmatrix} 1 & hh \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} M_{0,0} & M_{0,1} \\ M_{1,0} & M_{1,1} \end{pmatrix} \cdot \begin{pmatrix} 1 & -h \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} M_{0,0} + hh \cdot M_{1,0} & -h \cdot M_{0,0} - h \cdot hh \cdot M_{1,0} + M_{0,1} + hh \cdot M_{1,1} \\ M_{1,0} & -M_{1,0} \cdot h + M_{1,1} \end{pmatrix}$$

3. The results for h, hh, and f are

$$hh = \frac{1 - M_{0,0}}{M_{1,0}} \qquad h = \frac{-(1 - M_{1,1})}{M_{1,0}} \qquad f = \frac{-1}{M_{1,0}}$$

4. Numerical calculation

$$P12 := -\frac{1}{r1} \cdot \frac{n2 - n1}{n2} \qquad P23 := -\frac{1}{r2} \cdot \frac{n3 - n2}{n3}$$

$$P12 = -3.333 \times 10^{-11} \qquad P23 = -0.05$$

$$M_{0,0} := 1 + d \cdot P12 \qquad M_{0,1} := d \cdot \frac{n1}{n2}$$

$$M_{0,0} = 1 \qquad M_{0,1} = 6.667$$

$$M_{1,0} := \frac{(P23 \cdot n3 + P12 \cdot P23 \cdot d \cdot n3 + P12 \cdot n2)}{n3} \qquad M_{1,1} := \frac{(P23 \cdot d \cdot n3 + n2)}{n3} \cdot \frac{n1}{n2}$$

$$M_{1,0} = -0.05 \qquad M_{1,1} = 0.667$$

5. The result for h, hh, and f

$$hh := \frac{1 - M_{0,0}}{M_{1,0}} \qquad h := \frac{-(1 - M_{1,1})}{M_{1,0}} \qquad f := \frac{-1}{M_{1,0}}$$

$$hh = -6.667 \times 10^{-9} \qquad h = 6.667 \qquad f = 20$$

6. The input values globally defined (to play around)

$$n1 \equiv 1 \qquad n2 \equiv 1.5 \qquad n3 \equiv 1 \qquad r1 \equiv 10^{10} \qquad r2 \equiv -10 \qquad d \equiv 10$$