G5PRISMIM

Formula for angle of minimum deviation

Calculation of the formula for the angle of minimum deviation depending on the refractive index and the top angle of the prism.

We do the following two differentiations where we represent the angles (variables for differentiations) just by one symbol.

$$\frac{d}{d\theta}(\sin(\theta) - n\cdot\sin(A-\theta)) \qquad \frac{d}{d\theta}(n\cdot\sin(\theta) - \sin(\delta + A - \theta))$$

the results are

$$\cos(\theta) + n\cdot\cos(-A+\theta) \qquad n\cdot\cos(\theta) + \cos(-\delta - A + \theta)$$

We introduce the different angles as differentials and have to set each to zero for optimum conditions

$$\cos(\theta_1)\cdot d\theta_1 + n\cdot\cos(-A+\theta_3)\cdot d\theta_3 = 0$$

$$n\cdot\cos(\theta_3)\cdot d\theta_3 + \cos(-\delta - A + \theta)\cdot d\theta_1 = 0$$

This is a system of two homogenious linear equtions and to have a solution the determinant of the matrix of the coefficients must be zero.

The determinant is

$$\begin{pmatrix} \cos(\theta_1) & n\cdot\cos(-A+\theta_3) \\ \frac{1}{n}\cdot\cos(-\delta - A + \theta_1) & \cos(\theta_3) \end{pmatrix}$$

and setting it to zero we get

$$\cos(\theta_1)\cdot\cos(\theta_3) - \cos(-A+\theta_3)\cdot\cos(-\delta - A + \theta_1) = 0$$

If we take

$$\theta_1 = \frac{\delta + A}{2} \qquad \theta_3 = \frac{A}{2}$$

the equation is fulfilled

$$\cos(\theta_1)\cdot\cos(\theta_3) - \cos(-A + \theta_3)\cdot\cos(-\delta - A + \theta_1) = 0$$

Using the law of refraction ,and by symmetry $\theta 2 = \theta 3$

We have

$$\sin\left(\frac{\delta + A}{2}\right) = n\cdot\sin\left(\frac{A}{2}\right)$$

For δm we get

$$\delta_m = 2\cdot\sin^{-1}\left(n\cdot\sin\left(\frac{A}{2}\right)\right) - A$$

And for N calcualted by the minimum angle δ and A

$$n = \frac{\sin\left(\frac{\delta + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$