G26HEM

Symbolic calculation of the product of 3 matrices corresponding to a hemisphere

Symbolic calculation of the product of 3 matrices
corresponding to a hemisphere with curved side to the left, refractive index n2 and width r1.

1. Symbolic calculation of the matrix for the hemisphere

$$\begin{pmatrix} 1 & 0 \\ 0 & \frac{n2}{n1} \end{pmatrix} \cdot \begin{pmatrix} 1 & r1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ P12 & \frac{n1}{n2} \end{pmatrix}$$

P12 = (-1/r1)((n2-n1)/n2)
P23 = (-1/r2)((n3-n2)/n3)

$$\begin{pmatrix} 1 + r1 \cdot P12 & r1 \cdot \frac{n1}{n2} \\ \frac{n2}{n1} \cdot P12 & 1 \end{pmatrix}$$

2. Determination of h and hh. For simpler calculation we define the matrix

$$\begin{pmatrix} M_{0,0} & M_{0,1} \\ M_{1,0} & M_{1,1} \end{pmatrix}$$

$M_{0,0} = 1 + r1 \cdot P12$ $\qquad$ $M_{0,1} = r1 \cdot \frac{n1}{n2}$

$M_{1,0} = \frac{n2}{n1} \cdot P12$ $\qquad$ $M_{1,1} = 1$

and determine h and hh

$$\begin{pmatrix} 1 & hh \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} M_{0,0} & M_{0,1} \\ M_{1,0} & M_{1,1} \end{pmatrix} \cdot \begin{pmatrix} 1 & -h \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} M_{0,0} + hh \cdot M_{1,0} & -h \cdot M_{0,0} - h \cdot hh \cdot M_{1,0} + M_{0,1} + hh \cdot M_{1,1} \\ M_{1,0} & -M_{1,0} \cdot h + M_{1,1} \end{pmatrix}$$

3. The results for h, hh, and f are

$hh = \frac{1 - M_{0,0}}{M_{1,0}}$ $\qquad$ $h = \frac{-(1 - M_{1,1})}{M_{1,0}}$ $\qquad$ $f = \frac{-1}{M_{1,0}}$

4. Numerical calculation

$P12 := -\frac{1}{r1} \cdot \frac{n2 - n1}{n2}$ $\quad P23 := 0$

$P12 = -0.033$ $\quad P23 = 0$

$M_{0,0} := 1 + r1 \cdot P12$ $\quad M_{0,1} := r1 \cdot \frac{n1}{n2}$

$M_{0,0} = 0.667$ $\quad M_{0,1} = 6.667$

$M_{1,0} := \frac{n2}{n1} \cdot P12$ $\quad M_{1,1} := 1$

$M_{1,0} = -0.05$ $\quad M_{1,1} = 1$

5. The result for h, hh, and f

$hh := \frac{1 - M_{0,0}}{M_{1,0}}$ $\quad h := \frac{-(1 - M_{1,1})}{M_{1,0}}$ $\quad f := \frac{-1}{M_{1,0}}$

$hh = -6.667$ $\quad h = 0$ $\quad f = 20$

6. The input values globally defined

$n1 \equiv 1$ $\quad n2 \equiv 1.5$ $\quad n3 \equiv 1$ $\quad r1 \equiv 10$