N7PLPPS

Wave traveling with total internal reflection through a planar waveguide.

Resonance condition for p-polarization.

$\theta := 0, 1 .. 90$ global definition of n1,n2,n3, d and λ above the graph

$$y(\theta) := 2\cdot\pi\cdot n1\cdot\frac{d}{\lambda}\cdot\cos\left(2\cdot\pi\cdot\frac{\theta}{360}\right)$$

$$zp1(\theta) := \frac{\left(\sqrt{n1^2\cdot\sin\left(2\cdot\pi\cdot\frac{\theta}{360}\right)^2 - n2^2}\right)\cdot n1}{n2^2\cdot\cos\left(2\cdot\pi\cdot\frac{\theta}{360}\right)}$$

$$yp1(\theta) := -\operatorname{atan}(zp1(\theta))$$

$$zp3(\theta) := \frac{\left(\sqrt{n1^2\cdot\sin\left(2\cdot\pi\cdot\frac{\theta}{360}\right)^2 - n3^2}\right)\cdot n1}{n3^2\cdot\cos\left(2\cdot\pi\cdot\frac{\theta}{360}\right)}$$

$$yp3(\theta) := -\operatorname{atan}(zp3(\theta))$$

global definition

$n1 \equiv 1.5$
$n2 \equiv 1$
$n3 \equiv 1$
$d \equiv 6$
$\lambda \equiv 4.3$

$$\lambda\lambda := \frac{\lambda}{n1}$$

$\lambda\lambda = 2.867$

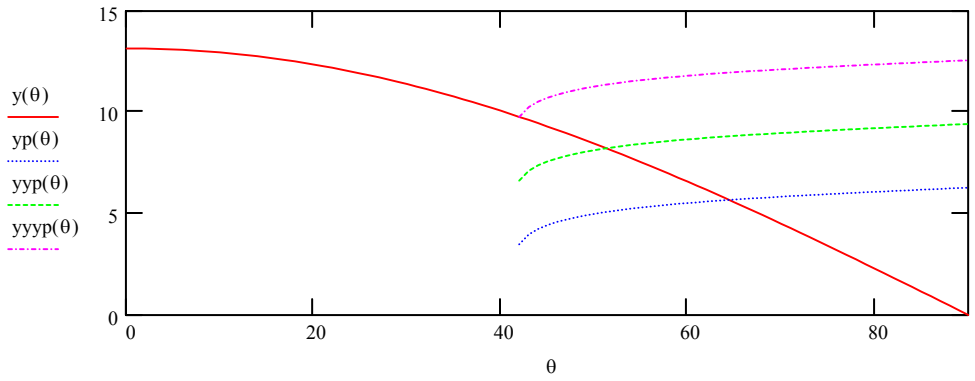
$$\theta c := \operatorname{asin}\left(\frac{n2}{n1}\right)$$

$$\theta\theta c := 360\cdot\frac{\theta c}{2\cdot\pi}$$

$\theta\theta c = 41.81$

For the graph we use yp for m=1, yyp for m=2, yyyp for m=3

$$yp(\theta) := -yp1(\theta) - yp3(\theta) + \pi$$
$$yyp(\theta) := -yp1(\theta) - yp3(\theta) + \pi\cdot 2$$
$$yyyp(\theta) := -yp1(\theta) - yp3(\theta) + \pi\cdot 3$$



At the cross-over point of y with yp, yyp, or yyyp respectively, the resonance condition is fulfilled.
The functions yp, yyp and yyyp are complex in the region from horizontal appearance to zero. This is shown in the next graph where the argument is plotted. The complex region has to be disregarded for the determination of the cross-over point.