M10POELIPSES

The equation of the ellipse may be written in matrix form

$$(E1\ \ E2)\cdot\begin{pmatrix} 1 & -\cos(\phi) \\ -\cos(\phi) & 1 \end{pmatrix}\cdot\begin{pmatrix} E1 \\ E2 \end{pmatrix} = \sin(\phi)^2$$

$$(E1 - E2\cdot\cos(\phi))\cdot E1 + (-E1\cdot\cos(\phi) + E2)\cdot E2 = \sin(\phi)^2$$

$$E1^2 - 2\cdot E1\cdot E2\cdot\cos(\phi) + E2^2 = 1 - \cos(\phi)^2$$

equivalent to

$$E1^2 - 2\cdot E1\cdot E2\cdot\cos(\phi) + E2^2 = \sin(\phi)^2$$

1. Linear polarized light $\phi = 0$

$$(E1\ \ E2)\cdot\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\cdot\begin{pmatrix} E1 \\ E2 \end{pmatrix} \quad 0 = \sin(\phi)^2$$

$$(E1 - E2)\cdot E1 + (-E1 + E2)\cdot E2 = 0$$

$0 = E1^2 - 2\cdot E1\cdot E2 + E2^2$ or E1-E2 is 0, or E1 = E2 Linear polarized light

2. Elliptically polarized light $\phi = \frac{\pi}{4}$ $\cos\left(\frac{\pi}{4}\right) = 0.707$ that is $\frac{1}{\sqrt{2}}$

$$(E1\ \ E2)\cdot\begin{pmatrix} 1 & -\cos(\phi) \\ -\cos(\phi) & 1 \end{pmatrix}\cdot\begin{pmatrix} E1 \\ E2 \end{pmatrix} = \sin(\phi)^2$$ that is $\frac{1}{2}$

$$(E1\ \ E2)\cdot\begin{pmatrix} 1 & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & 1 \end{pmatrix}\cdot\begin{pmatrix} E1 \\ E2 \end{pmatrix}$$

$$\left(E1 - \frac{1}{2}\cdot E2\cdot\sqrt{2}\right)\cdot E1 + \left(\frac{-1}{2}\cdot E1\cdot\sqrt{2} + E2\right)\cdot E2$$

$$E1^2 - E1\cdot E2\cdot\sqrt{2} + E2^2 = \frac{1}{2}$$

3. Transformation to principal axes in the coordinate system of EE1 and EE2 by determination of the eigenvalues of the matrix.

$$\text{eigenvals}\left(\begin{pmatrix} 1 & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & 1 \end{pmatrix}\right)$$

$$\begin{pmatrix} 1+\frac{1}{2}\cdot\sqrt{2} \\ 1-\frac{1}{2}\cdot\sqrt{2} \end{pmatrix}$$

The transformed matrix equation is now

$$(EE1\ \ EE2)\cdot\begin{pmatrix} 1+\frac{1}{\sqrt{2}} & 0 \\ 0 & 1-\frac{1}{\sqrt{2}} \end{pmatrix}\cdot\begin{pmatrix} EE1 \\ EE2 \end{pmatrix}$$

$$EE1^2\cdot\left(1+\frac{1}{2}\cdot\sqrt{2}\right)+EE2^2\cdot\left(1-\frac{1}{2}\cdot\sqrt{2}\right)$$

This can be written with the right side = 1/2

$$EE1^2\cdot\left(1+\frac{1}{2}\cdot\sqrt{2}\right)+EE2^2\cdot\left(1-\frac{1}{2}\cdot\sqrt{2}\right)=\frac{1}{2}$$

or the right side equal 1

$$\frac{EE1^2}{\left(1-\frac{1}{\sqrt{2}}\right)}+\frac{EE2^2}{\left(1+\frac{1}{\sqrt{2}}\right)}=1$$

This is the equation of an ellipse in normalized form

4. Circular polarized light $\phi = \frac{\pi}{2}$

$$(E1\ \ E2)\cdot\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\cdot\begin{pmatrix}E1\\E2\end{pmatrix} = 1$$

$$E1^2 + E2^2 = 1$$

This is the equation of a circle