**A4SPHLSIPIS Spherical Aberration and the pi-sigma equation**

**We assume n=1.5 and compare the cases of real and virtual images.**

**1. Image for f = 10, and xo to the left of focal point, LSA may not be eliminated**

$$f := \frac{1}{(n-1)\cdot\left(\frac{1}{r1} - \frac{1}{r2}\right)}$$

$r1 \equiv 10 \qquad r2 \equiv -10 \qquad n \equiv 1.5$

$ro \equiv 4 \qquad xo \equiv 4$

$f = 10$

$$xi := \frac{1}{\left(\frac{1}{f} + \frac{1}{xo}\right)} \qquad xi = 2.857$$

**2. Definitions**

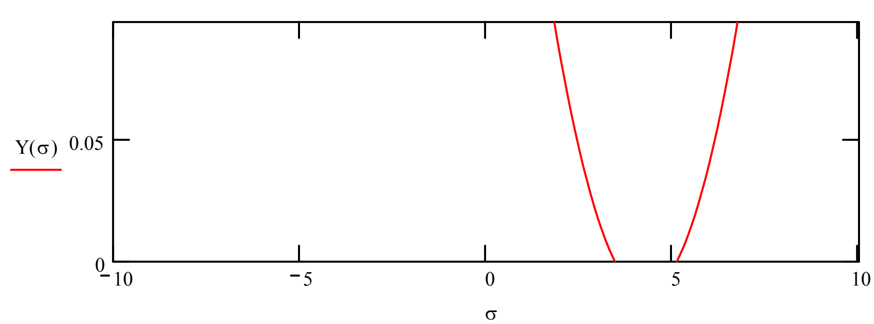
Definition:σ = (r2-r1)/(r2+r1) $\qquad \sigma := -10, -9.9 .. 10$

$$\pi := \frac{xi + xo}{(xi - xo)} \qquad \pi = -6$$

**3. π-σ Equation**

$$A(n) := \frac{n+2}{8\cdot n\cdot(n-1)^2} \qquad B(n) := \frac{n+1}{2\cdot n\cdot(n-1)} \qquad C(n) := \frac{3\cdot n+2}{8\cdot n} \qquad D(n) := \frac{n^2}{8\cdot(n-1)^2}$$

$$Y(\sigma) := \frac{(ro)^2}{f^3}\cdot\left(A(n)\cdot\sigma^2 + B(n)\cdot\sigma\cdot\pi + C(n)\cdot\pi^2 + D(n)\right)$$



**4. Minimum value of $Y(\sigma)$**

The value of $Y(\sigma)$ at the minimum is obtained by differentiation and equating equal to 0. The result is

$$\sigma min := -B(n)\cdot\frac{\pi}{2\cdot A(n)}$$

$$\sigma min = 4.286$$

Calculation of the corresponding value of $Y(\sigma\ min)$

$$Y(\sigma min) = -0.013$$

for our choice of parameters, $Y(\sigma\ min)$ is negative and LSA may be eliminated.