G29SYST2LTC

Symbolic calculation of the principal planes for 2 thick lenses of refractive indices n and nn in air.

Distance between lenses is a and the thickness of the first is d1, of the second d2. Radii of curvature are r1 to r4. The matrix of the first lens is on the right.

$$\begin{pmatrix} 1 & 0 \\ P45 & \frac{nn}{1} \end{pmatrix} \cdot \begin{pmatrix} 1 & d2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ P34 & \frac{1}{nn} \end{pmatrix} \cdot \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \cdot \left[\begin{pmatrix} 1 & 0 \\ P23 & \frac{n}{1} \end{pmatrix} \cdot \begin{pmatrix} 1 & d1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ P12 & \frac{1}{n} \end{pmatrix} \right]$$

P12 = -(1/r1)(n-1)/n   P23 =-(1/r2)(1-n)   P34 = -(1/r3)(nn-1)/nn P45 =-(1/r4)(1-nn)

$$\begin{pmatrix} 1 & 0 \\ P45 & \frac{nn}{1} \end{pmatrix} \cdot \begin{pmatrix} 1 & d2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ P34 & \frac{1}{nn} \end{pmatrix} \qquad \left[\begin{pmatrix} 1 & 0 \\ P23 & \frac{n}{1} \end{pmatrix} \cdot \begin{pmatrix} 1 & d1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ P12 & \frac{1}{n} \end{pmatrix} \right]$$

Matrix for the second lens   Matrix for the first lens

$$\begin{bmatrix} 1 + d2 \cdot P34 & \frac{d2}{nn} \\ P45 + P34 \cdot P45 \cdot d2 + P34 \cdot nn & \frac{(P45 \cdot d2 + nn)}{nn} \end{bmatrix} \qquad \begin{bmatrix} 1 + d1 \cdot P12 & \frac{d1}{n} \\ P23 + P12 \cdot P23 \cdot d1 + P12 \cdot n & \frac{(P23 \cdot d1 + n)}{n} \end{bmatrix}$$

For the determination of h and hh

$$\begin{pmatrix} 1 & hh \\ 0 & 1 \end{pmatrix} \cdot \begin{bmatrix} 1 + d2 \cdot P34 & \frac{d2}{nn} \\ P45 + P34 \cdot P45 \cdot d2 + P34 \cdot nn & \frac{(P45 \cdot d2 + nn)}{nn} \end{bmatrix} \cdot \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \cdot \begin{bmatrix} 1 + d1 \cdot P12 & \frac{d1}{n} \\ P23 + P12 \cdot P23 \cdot d1 + P12 \cdot n & \frac{(P23 \cdot d1 + n)}{n} \end{bmatrix} \cdot \begin{pmatrix} 1 & -h \\ 0 & 1 \end{pmatrix}$$

Multiplication results in a very large expression, and we go right away to numerical calculations

See the input parameters below globally defined

We have for the power of refractions

$$P12 := -\frac{n-1}{r1\cdot n} \qquad P23 := -\frac{1-n}{r2} \qquad P34 := -\frac{nn-1}{r3\cdot nn} \qquad P45 := -\frac{1-nn}{r4}$$

The thick lens matrix is then

$$M := \begin{bmatrix} 1 + d2\cdot P34 & \frac{d2}{nn} \\ P45 + P34\cdot P45\cdot d2 + P34\cdot nn & \frac{(P45\cdot d2 + nn)}{nn} \end{bmatrix} \cdot \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \cdot \begin{bmatrix} 1 + d1\cdot P12 & \frac{d1}{n} \\ P23 + P12\cdot P23\cdot d1 + P12\cdot n & \frac{(P23\cdot d1 + n)}{n} \end{bmatrix}$$

The result is

$$M = \begin{pmatrix} 0.333 & 13.333 \\ -0.067 & 0.333 \end{pmatrix}$$ We define M as $$\begin{pmatrix} M_{0,0} & M_{0,1} \\ M_{1,0} & M_{1,1} \end{pmatrix}$$

For the determination of h and hh we multiply by the two translation matrices

$$\begin{pmatrix} 1 & hh \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} M_{0,0} & M_{0,1} \\ M_{1,0} & M_{1,1} \end{pmatrix} \cdot \begin{pmatrix} 1 & -h \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} M_{0,0} + hh\cdot M_{1,0} & -h\cdot M_{0,0} - h\cdot hh\cdot M_{1,0} + M_{0,1} + hh\cdot M_{1,1} \\ M_{1,0} & -M_{1,0}\cdot h + M_{1,1} \end{pmatrix}$$

$$hh := \frac{1-\left(M_{0,0}\right)}{M_{1,0}} \qquad h := \frac{1-\left(M_{1,1}\right)}{(-M)_{1,0}} \qquad f := -\frac{1}{M_{1,0}}$$

$$hh = -10 \qquad h = 10 \qquad f = 15$$

$$n \equiv 1.5 \qquad nn \equiv 1.5 \qquad d1 \equiv 10 \qquad d2 \equiv 10 \qquad a \equiv 0$$

$$r1 \equiv 10 \qquad r2 \equiv 10^{10} \qquad r3 \equiv 10^{10} \qquad r4 \equiv -10$$

Check the form of the final matrix product

$$MM := \begin{pmatrix} 1 & hh \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} M_{0,0} & M_{0,1} \\ M_{1,0} & M_{1,1} \end{pmatrix} \cdot \begin{pmatrix} 1 & -h \\ 0 & 1 \end{pmatrix}$$

$$MM = \begin{pmatrix} 1 & -1.776 \times 10^{-15} \\ -0.067 & 1 \end{pmatrix}$$