G23SYMB3M

Thin lens matrix

The special case of the thin lens matrix.
We start with the symbolic calculation of two surfaces at distance d

P12 =(-1/r1)(n2 -n1)/n2 P23 =(-1/r2)(n3 -n2)/n3

$$\begin{pmatrix} 1 & 0 \\ P23 & \dfrac{n2}{n3} \end{pmatrix} \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ P12 & \dfrac{n1}{n2} \end{pmatrix}$$

$$\begin{bmatrix} 1 + d \cdot P12 & d \cdot \dfrac{n1}{n2} \\ \dfrac{(P23 \cdot n3 + P12 \cdot P23 \cdot d \cdot n3 + P12 \cdot n2)}{n3} & \dfrac{(P23 \cdot d \cdot n3 + n2)}{n3} \cdot \dfrac{n1}{n2} \end{bmatrix}$$

P = P23 +d P12P23 + (n2/n3)P12

We go to the thin lens and set d = 0

$$\begin{pmatrix} 1 & 0 \\ P23 & \dfrac{n2}{n3} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ P12 & \dfrac{n1}{n2} \end{pmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ \dfrac{(P23 \cdot n3 + P12 \cdot n2)}{n3} & \dfrac{1}{n3} \cdot n1 \end{bmatrix}$$

Since n3 and n1 are both set to 1 we have

$$\begin{bmatrix} 1 & 0 \\ (P23 + P12 \cdot n2) & 1 \end{bmatrix}$$

We set

$$P = (P23 + P12 \cdot n2)$$

and

$$P = \frac{-1}{f}$$ f is the focal length of the lens

With P12 =(-1/r1)(n2 -n1)/n2 P23 =(-1/r2)(n3 -n2)/n3

we obtain for 1/f =-((-1/r2)(1 -n2) + (-1/r1)(n2 -1))

and have finally fo the thin lens matix

$$\begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix}$$