

G27SPH

Symbolic calculation of the product of 3 matrices corresponding to a sphere.

Refractive index n_2 and width $2r_1$

1. Symbolic calculation of the matrix for the hemisphere

$$\begin{pmatrix} 1 & 0 \\ P_{23} & \frac{n_2}{n_1} \end{pmatrix} \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ P_{12} & \frac{n_1}{n_2} \end{pmatrix}$$
$$P_{12} = (-1/r_1)((n_2 - n_1)/n_2)$$
$$P_{23} = (-1/r_2)((n_3 - n_2)/n_3)$$

$$\begin{bmatrix} 1 + d \cdot P_{12} & d \cdot \frac{n_1}{n_2} \\ \frac{(P_{23} \cdot n_1 + P_{12} \cdot P_{23} \cdot d \cdot n_1 + P_{12} \cdot n_2)}{n_1} & \frac{(P_{23} \cdot d \cdot n_1 + n_2)}{n_2} \end{bmatrix}$$

2. Determination of h and hh . For simpler calculation we define the matrix

$$\begin{pmatrix} M_{0,0} & M_{0,1} \\ M_{1,0} & M_{1,1} \end{pmatrix}$$

$$M_{0,0} = 1 + d \cdot P_{12}$$

$$M_{0,1} = d \cdot \frac{n_1}{n_2}$$

$$M_{1,0} = \frac{(P_{23} \cdot n_1 + P_{12} \cdot P_{23} \cdot d \cdot n_1 + P_{12} \cdot n_2)}{n_1}$$

$$M_{1,1} = \frac{(P_{23} \cdot d \cdot n_1 + n_2)}{n_2}$$

and determine h and hh

$$\begin{pmatrix} 1 & hh \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} M_{0,0} & M_{0,1} \\ M_{1,0} & M_{1,1} \end{pmatrix} \cdot \begin{pmatrix} 1 & -h \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} M_{0,0} + hh \cdot M_{1,0} & -h \cdot M_{0,0} - h \cdot hh \cdot M_{1,0} + M_{0,1} + hh \cdot M_{1,1} \\ M_{1,0} & -M_{1,0} \cdot h + M_{1,1} \end{pmatrix}$$

3. The results for h, hh, and f are

$$hh = \frac{1 - M_{0,0}}{M_{1,0}}$$

$$h = \frac{-(1 - M_{1,1})}{M_{1,0}}$$

$$f = \frac{-1}{M_{1,0}}$$

4. Numerical calculation

$$P12 := -\frac{1}{r1} \cdot \frac{n2 - n1}{n2}$$

$$P12 = -0.033$$

$$P23 := \frac{-1}{r2} \cdot \frac{n3 - n2}{n3}$$

$$P23 = -0.05$$

$$M_{0,0} := 1 + d \cdot P12$$

$$M_{0,0} = 0.333$$

$$M_{0,1} := d \cdot \frac{n1}{n2}$$

$$M_{0,1} = 13.333$$

$$M_{1,0} := \frac{(P23 \cdot n1 + P12 \cdot P23 \cdot d \cdot n1 + P12 \cdot n2)}{n1}$$

$$M_{1,0} = -0.067$$

$$M_{1,1} := \frac{(P23 \cdot d \cdot n1 + n2)}{n2}$$

$$M_{1,1} = 0.333$$

5. The result for h, hh, and f

$$hh := \frac{1 - M_{0,0}}{M_{1,0}}$$

$$hh = -10$$

$$h := \frac{-(1 - M_{1,1})}{M_{1,0}}$$

$$h = 10$$

$$f := \frac{-1}{M_{1,0}}$$

$$f = 15$$

6. The input values globally defined

$$n1 \equiv 1$$

$$n2 \equiv 1.5$$

$$n3 \equiv 1$$

$$r1 \equiv 10$$

$$r2 \equiv -10$$

$$d \equiv 20$$

