

### N3SYMATPL Calculation of the transmitted intensity of a plane parallel plate

We set  $n_1 = n_3 = 1$ , call  $n_2 = n$ ,  $k' = k$ ,  $A_1' = AA_1$

$$\begin{bmatrix} A_3 \cdot e^{i \cdot k \cdot d} \\ -(A_3 \cdot e^{i \cdot k \cdot d}) \end{bmatrix} = \begin{pmatrix} \cos(k \cdot d) & -i \cdot \frac{\sin(k \cdot d)}{n_2} \\ -n_2 \cdot i \cdot \sin(k \cdot d) & \cos(k \cdot d) \end{pmatrix} \begin{pmatrix} A_1 + AA_1 \\ -A_1 + AA_1 \end{pmatrix}$$

Calculation without computer, we have set  $AA_1/A_1 = x$ ,  $A_3/A_1 = y$  and get the following system of linear equations, which have been written into the solution procedure "Given"....."Find"

Given

$$\left( \cos(k \cdot d) - i \cdot \frac{\sin(k \cdot d)}{n} \right) \cdot x - e^{i \cdot k \cdot d} \cdot y = -\cos(k \cdot d) - i \cdot \frac{\sin(k \cdot d)}{n}$$

$$[n \cdot (-i \cdot \sin(k \cdot d)) + \cos(k \cdot d)] \cdot x + e^{i \cdot k \cdot d} \cdot y = \cos(k \cdot d) + n \cdot i \cdot \sin(k \cdot d)$$

**Find(x,y) →**

$$i := \sqrt{-1}$$

Denominator without the phase factor and calculation of complex conjugate

$$-n^2 \cdot i \cdot \sin(k \cdot d) - 2 \cdot \cos(k \cdot d) \cdot n - i \cdot \sin(k \cdot d)$$

Multiply by complex conjugate

$$(-2 \cdot \cos(k \cdot d) \cdot n + n^2 \cdot i \cdot \sin(k \cdot d) + i \cdot \sin(k \cdot d))$$

$$[-2 \cdot \cos(k \cdot d) \cdot n + i \cdot (-n^2 \cdot \sin(k \cdot d) - \sin(k \cdot d))]$$

$$\text{result} \quad n^4 \cdot \sin(k \cdot d)^2 + 2 \cdot n^2 \cdot \sin(k \cdot d)^2 + 4 \cdot \cos(k \cdot d)^2 \cdot n^2 + \sin(k \cdot d)^2$$

without computer, adding and subtracting  $4n^2\sin(kd)^2$

$$\left(n^4 \cdot \sin(kd)^2 + 4 \cdot n^2 - 2 \cdot n^2 \cdot \sin(kd)^2\right) + \sin(kd)^2$$

$$\left(n^4 - 2 \cdot n^2 + 1\right) \cdot \sin(kd)^2 + 4 \cdot n^2$$

$$\left(n^4 - 2 \cdot n^2 + 1\right) \text{ factor } \rightarrow$$

$$(n - 1) \cdot (n + 1) \text{ expand } \rightarrow$$

result is for denominator

$$\left(n^2 - 1\right)^2 \cdot \sin(kd)^2 + 4n^2$$

numerator squared

$$4 \cdot n^2$$

We have for  $yy^*$

$$\frac{4 \cdot n^2}{\left(n^2 - 1\right)^2 \cdot \sin(kd)^2 + 4n^2}$$

Similar for  $xx^*$  one gets

$$\frac{\sin(kd)^2}{\sin(kd)^2 + \frac{4 \cdot n^2}{\left(n^2 - 1\right)^2}}$$

The sum of these two expression must be 1

We consider  $yy^*$  in a different form

$$\frac{1}{1 + \frac{(n^2 - 1)^2 \cdot \sin(kd)^2}{4 \cdot n^2}}$$

In the case we have treated we have set the refractive indices outside equal to 1 and in this case the transmitted intensity  $T = yy^*$

$T$  is the same expression one calculates using the "summation method" in interference and set the angle of incidence equal to zero, and again assuming that the index of refraction outside is one.