

G25SYMBGTH

Symbolic calculation of the product of 3 matrices
corresponding to a thin lens between two different media.

1. Symbolic calculation of the matrix for the thick lens, with $d = 0$

$$\begin{pmatrix} 1 & 0 \\ P23 & \frac{n2}{n3} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ P12 & \frac{n1}{n2} \end{pmatrix} \quad \begin{aligned} P12 &= (-1/r1)((n2-n1)/n2) \\ P23 &= (-1/r2)((n3-n2)/n2) \end{aligned}$$

$$\begin{bmatrix} 1 & 0 \\ \frac{(P23 \cdot n3 + n2 \cdot P12)}{n3} & \frac{1}{n3} \cdot n1 \end{bmatrix}$$

2. Determination of h and hh . For simpler calculation we define the matrix

$$\begin{pmatrix} M_{0,0} & M_{0,1} \\ M_{1,0} & M_{1,1} \end{pmatrix}$$

$$M_{0,0} = 1$$

$$M_{0,1} = 0$$

$$M_{1,0} = \frac{(P23 \cdot n3 + P12 \cdot n2)}{n3}$$

$$M_{1,1} = \frac{n1}{n3}$$

and determine h and hh

$$\begin{pmatrix} 1 & hh \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} M_{0,0} & M_{0,1} \\ M_{1,0} & M_{1,1} \end{pmatrix} \cdot \begin{pmatrix} 1 & -h \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} M_{0,0} + hh \cdot M_{1,0} & -h \cdot M_{0,0} - h \cdot hh \cdot M_{1,0} + M_{0,1} + hh \cdot M_{1,1} \\ M_{1,0} & -M_{1,0} \cdot h + M_{1,1} \end{pmatrix}$$

3. The results for h, hh, and f are

$$hh = \frac{1 - M_{0,0}}{M_{1,0}} \quad h = \frac{-(1 - M_{1,1})}{M_{1,0}} \quad f = \frac{-1}{M_{1,0}}$$

4. Numerical calculation

$$P12 := -\frac{1}{r1} \cdot \frac{n2 - n1}{n2} \quad P23 := -\frac{1}{r2} \cdot \frac{n3 - n2}{n3}$$

$$P12 = -0.033 \quad P23 = -0.015$$

$$M_{0,0} := 1 \quad M_{0,1} := 0$$

$$M_{0,0} = 1 \quad M_{0,1} = 0$$

$$M_{1,0} := \frac{(P23 \cdot n3 + P12 \cdot n2)}{n3} \quad M_{1,1} := \frac{n1}{n3}$$

$$M_{1,0} = -0.054 \quad M_{1,1} = 0.769$$

5. The result for h, hh, and f

$$hh := \frac{1 - M_{0,0}}{M_{1,0}} \quad h := \frac{-(1 - M_{1,1})}{M_{1,0}} \quad f := \frac{-1}{M_{1,0}}$$

$$hh = 0 \quad h = 4.286 \quad f = 18.571$$

6. The input values globally defined

$$n1 \equiv 1 \quad n2 \equiv 1.5 \quad n3 \equiv 1.3 \quad r1 \equiv 10 \quad r2 \equiv -10$$