

PILE-SOIL-PILE INTERACTION UNDER AXIAL HARMONIC VIBRATIONS: A SIMPLE APPROACH

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ABSTRACT

Piles are normally constructed in groups. Thus the response of closely spaced piles is different from isolated pile. This paper presents a closed form solution to determine the interaction between piles in groups using a simple method. The elasto-dynamic approach of Novak and his associates is used to characterize vertical isolated prismatic piles subjected to harmonic vibrations in the axial direction. An approximate approach based on the interference of cylindrical wave fields originating along each pile shaft and spreading radially outward is used to account for the influence of closely spaced piles in the group. The solution offers the stiffness and damping coefficients for the whole pile group. The results are comparable with those given by rigorous solutions, demonstrating the sound capability of the presented approach. The main advantage of the solution is its applicability to inhomogeneous soil profile along the pile shaft. The influence of pile-pile distance in the group and load excitation frequency will be presented.

Keywords: Pile group, harmonic vibration, vertical loading, pile-soil-pile interaction, stiffness, damping.

INTRODUCTION

Dynamically loaded piles under vertical vibrations have been investigated using various methods. These methods are lumped mass (Penzien, 1975; El Naggar and Novak, 1994) continuum approach (Novak, 1974; Novak and Aboul-Ella, 1978; Novak and Nogami, 1980), boundary element method (Kaynia and Kausel, 1982; Sen et al., 1985), and finite element solutions (Blaney et al., 1976; Wolf and von Arx, 1982; Chow, 1985). All these approaches are used for isolated piles.

The interaction between piles vibrating harmonically has also been investigated. For example, Kaynia and Kausel (1982) used elastic boundary element method to determine interaction factors.

In spite of significant progress achieved in pile dynamic analyses, there is still a serious need to develop simple procedures enabling users to calculate the dynamics interaction factor between piles. This paper presents a simple approach for determination of interaction effect between closely spaced piles subjected to harmonic vibrations.

ANALYSIS FOR SINGLE PILE

If a vertical pile is subjected to vertical harmonic vibration, by the use of continuum model the governing differential equation is expressed by (Novak, 1974):

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$$\mu \frac{\partial^2 w(z,t)}{\partial t^2} + c \frac{\partial w(z,t)}{\partial t} - E_p \cdot A_p \frac{\partial^2 w(z,t)}{\partial z^2} + K_z \cdot w(z,t) = 0 \quad (1)$$

where $w(z, t)$ is the pile settlement at depth z , t is time, μ is the mass of the pile per unit length, c is the coefficient of pile internal damping, E_p is Young's modulus of the pile, A_p is cross sectional area of the pile, and K_z is the soil complex stiffness per unit length of the pile, determined using (Novak and Beredugo, 1972):

$$K_z = G(S_{w1} + S_{w2}) \quad (2)$$

where G is the soil shear modulus and:

$$S_{w1} = 2 \cdot \pi \cdot a_0 \cdot \frac{J_1(a_0) \cdot J_0(a_0) + Y_1(a_0) \cdot Y_0(a_0)}{J_0^2(a_0) + Y_0^2(a_0)} \quad (3)$$

$$S_{w2} = \frac{4}{J_0^2(a_0) + Y_0^2(a_0)} \quad (4)$$

where $J_0(a_0)$ and $J_1(a_0)$ are Bessel functions of the first kind of order zero and one, $Y_0(a_0)$, $Y_1(a_0)$ are Bessel functions of the second kind of order zero and one, $a_0 = 2 \cdot r_0 \cdot \omega / V_s$ is dimensionless frequency, r_0 is the pile radius, ω is the circular frequency, $V_s = \sqrt{G/\rho_s}$ is shear wave velocity in the soil and ρ_s is mass density of the soil. Decomposing Eq. (1) results in:

$$w(z,t) = w(z) \cdot \exp(i \cdot \omega \cdot t) \quad (5)$$

where:

$$w(z) = A_{11} \cdot e^{\Lambda \cdot z} + B_{11} \cdot e^{-\Lambda \cdot z} \quad (6)$$

and A_{11} , B_{11} are integration constants calculated using appropriate boundary conditions and:

$$\Lambda = \sqrt{\frac{K_z + i \cdot \omega \cdot c - \omega^2 \cdot \mu}{E_p \cdot A_p}} \quad (7)$$

INTERACTION BETWEEN TWO PILES

The interaction factor between two identical piles is defined as the response atop a pile carrying no load at its head (hereafter called "Receiver Pile"), normalized by the corresponding response of a pile which is loaded with unit, vertical and harmonic load (hereafter called "Source Pile"). Fig. 1 shows two piles. The terms "Receiver" and "Source" were also used by others (Makris and Gazetas, 1992). It is obvious that the receiver pile is subjected to the ground vibrations produced by the vibrating source pile.

In order to calculate the interaction factor between two piles, the dynamic displacement around a vibrating is described by the following asymptotic cylindrical wave expression pile may be initially calculated using (Morse and Ingard, 1968 as reported by Dobry and Gazetas, 1988):

$$w(r) = A \sqrt{\frac{1}{r}} \cdot \exp\left(\frac{-\beta \cdot \omega \cdot r}{V_s}\right) \cdot \exp\left[i \cdot \omega \left(t - \frac{r}{V_s}\right)\right] \quad (8)$$

where r is the horizontal distance from the axis of pile, $A=A(z)$ is an amplitude constant, and β is damping ratio of the soil. The first exponential term represents the motion decay and a hysteretic damping. The second one describes the phase lag of the motion of the soil away from the vibrating pile.

The displacement field around the source pile may be initially calculated using:

$$w_{11}(z, r, t) = (A_{11} \cdot e^{\Lambda \cdot z} + B_{11} \cdot e^{-\Lambda \cdot z}) \cdot \sqrt{\frac{r_0}{r}} \cdot \exp\left(\frac{-\beta \cdot \omega \cdot r}{V_s}\right) \cdot \exp\left[i \cdot \omega \left(t - \frac{r}{V_s}\right)\right] \quad (9)$$

Where $w_{11}(z, r, t)$ is the soil displacement caused by the source pile being loaded at distance r , depth z and time t .

Due to the axial pile rigidity, the receiver pile does not follow exactly the free-field displacement generated by the source pile which is calculated from equation (9). For the interplay between receiver pile and the soil, the receiver pile is modeled as a beam supported with Winkler springs and dashpots.

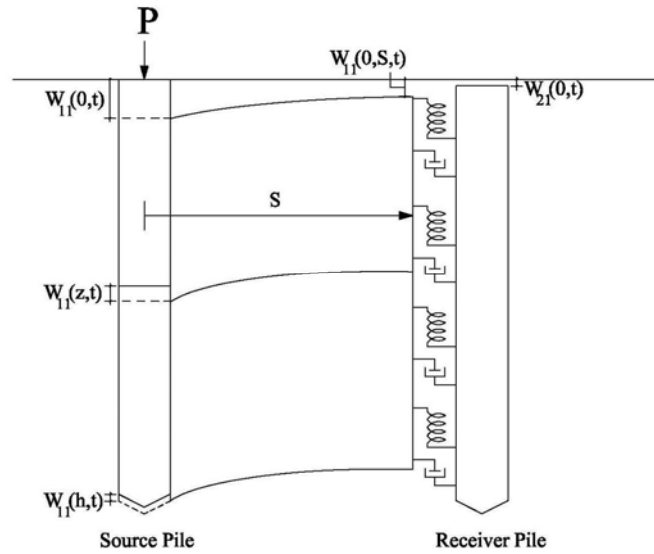


Figure 1. Concept of ‘source’ pile and ‘receiver’ pile

If the receiver pile does not obey exactly the displacement induced by the surrounding soil, the springs and dashpots reactions would be proportional to the relative displacement $w_{21}(z, t) - w_{11}(z, s, t)$ where s is horizontal distance between two vertical piles and $w_{21}(z, t)$ is the receiver pile displacement produced by loading the source pile. Therefore, the differential equation of the receiver pile will be concluded using:

$$\mu \frac{\partial^2 w_{21}(z, t)}{\partial t^2} + c \frac{\partial w_{21}(z, t)}{\partial t} - E_p A_p \frac{\partial^2 w_{21}(z, t)}{\partial z^2} + K_z [w_{21}(z, t) - w_{11}(z, s, t)] = 0 \quad (10)$$

Assuming $w_{21}(z, t) = w_{21}(z) \cdot \exp(i \cdot \omega \cdot t)$ and solving the above equation will result in:

$$w_{21}(z) = \frac{\Lambda}{2} \cdot \psi \cdot z \cdot (-A_{11} \cdot e^{\Lambda \cdot z} + B_{11} \cdot e^{-\Lambda \cdot z}) + A_{21} \cdot e^{\Lambda \cdot z} + B_{21} \cdot e^{-\Lambda \cdot z} \quad (11)$$

where B_{21} and A_{21} are integration constants calculated by applying appropriate boundary conditions for the receiver pile, for example zero force at the pile and zero force at the pile tip for floating piles.

$$\psi = \frac{K_z}{K_z + i \cdot \omega \cdot c - \omega^2 \cdot \mu} \cdot \sqrt{\frac{r_0}{s}} \cdot \exp\left(\frac{-\omega \cdot s}{V_s} \cdot (i + \beta)\right) \quad (12)$$

The interaction factor, α , may be defined as the dynamic displacement of the receiver pile divided by static displacement of the source pile (Kaynia and Kausel, 1982). For two piles embedded in a homogeneous soil layer α is given by:

$$\alpha = \frac{\psi}{2} \cdot \frac{\lambda}{\Lambda} \cdot \tanh(\lambda \cdot h) \cdot \left[\frac{2\Lambda h + \sinh(2\Lambda h)}{\cosh(2\Lambda h) - 1} \right] \quad (13)$$

Where h is the pile length and λ and r_m are calculated from (Randolph and Worth, 1978; Mylonakis and Gazetas, 1998):

$$\lambda = \sqrt{\frac{2 \cdot \pi \cdot G}{E_p \cdot A_p \cdot \ln(r_m/r_0)}} \quad (14)$$

$$r_m = 2.5h(1 - \nu) \quad (15)$$

where ν represents the soil Poisson's ratio.

VERIFICATION

Kaynia and Kausel (1982) presented a rigorous solution based on the boundary element method to determine interaction factors between two closely spaced piles. In this section, the results obtained from the present developed simple method are compared with those given by Kaynia and Kausel (1982). Figs. 2 to 4 show this comparison. The parameters used in this comparison are given by Kaynia and Kausel (1982). Three values are assumed for s/d where s and d denotes pile distance and the pile diameter, respectively. Other parameters have been extracted from the work of Kaynia and Kausel (1982) and shown at the bottom of figures. In the illustrated figures, E_s is Young's modulus of the soil and ρ_p is mass density of the pile. As seen, the simple method can predict the interaction factors reasonably.

The developed method can be easily used for the cases where the layered soil profile surrounds the piles (Ghadimi, 2006). This is very interesting and reveals the capability of the developed approach. The extension of the approach to layered soil profiles is beyond the scope of this paper. The interested readers are referred to Ghadimi (2006).

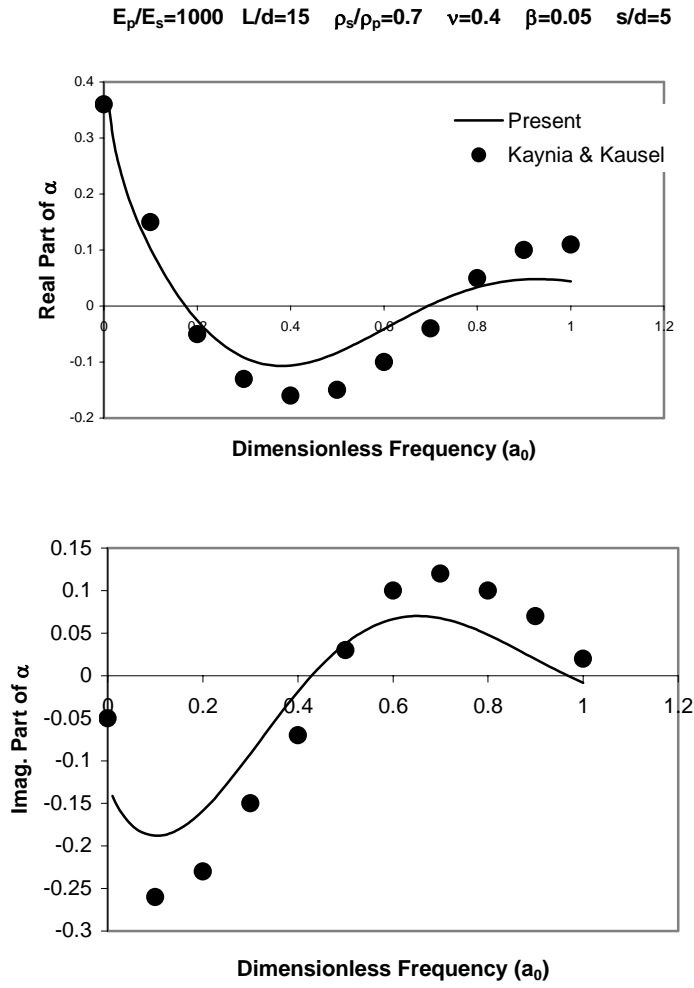
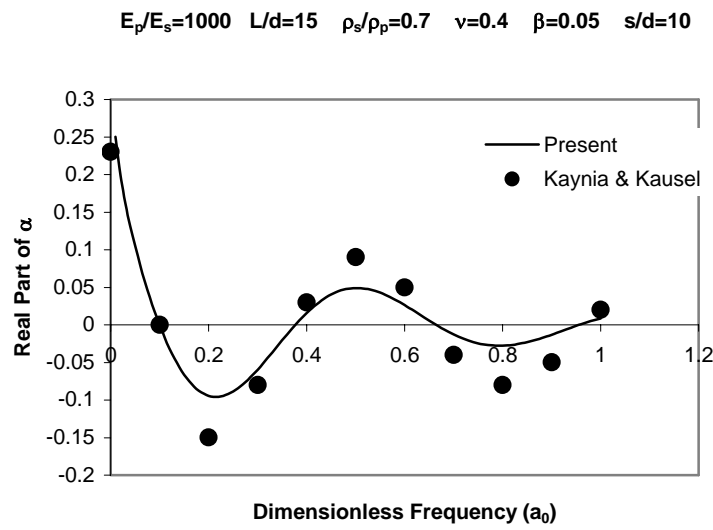


Figure 2 Comparison of interaction factors determined using present simple approach with those given by Kaynia and Kausel (1982)



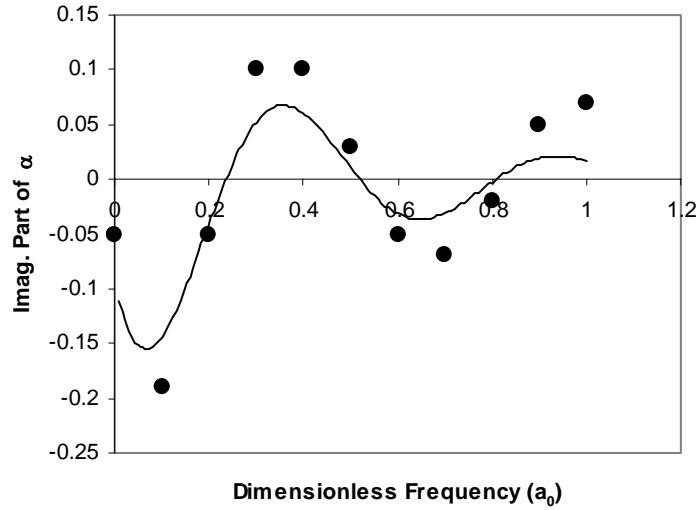
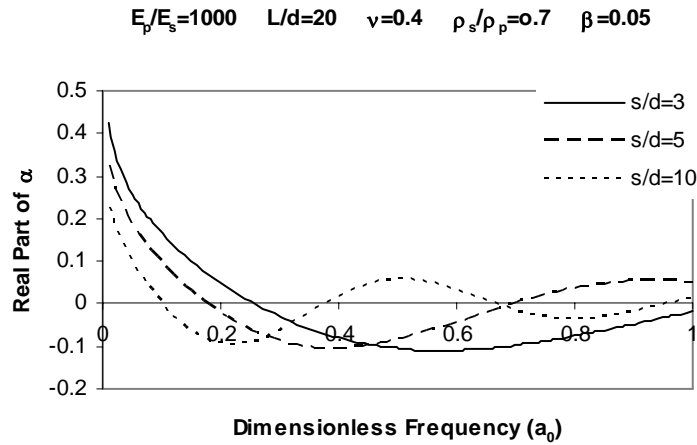


Figure 3 Comparison of interaction factors determined using present simple approach with those given by Kaynia and Kausel (1982)

PARAMETRIC STUDY

In this section, parametric studies are performed to show the capability of the simple method described above. The contributing parameters are shown in Fig. 4. These figures show the variation of interaction factors versus the dimensionless frequency. All parameters were defined in previous sections.



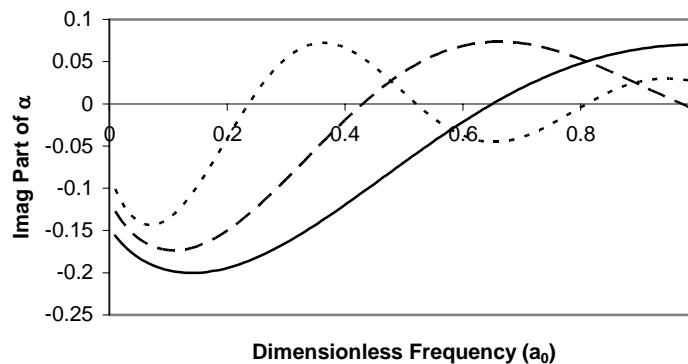


Figure 4 Variation of interaction factor

CONCLUSION

A simple method has been presented for calculating dynamic interaction factor in pile groups, which leads to a closed-form expression. This method assumes that the soil is elastic and a perfect bounding exists between the pile and the soil. An approximate concept based on the interference of cylindrical wave fields originating along each pile shaft and spreading radially outward is used to account for the influence of closely spaced piles in the group. A comparison of results obtained from the developed simple approach with those given by a rigorous solution shows a very good agreement. It should be noted that further analyses are required to generalize the findings in this paper. It is also noted that the method may not be applicable to cases involving strong nonlinearities such as strong motions.

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