4th International Conference on
Earthquake Geotechnical Engineering
June 25-28, 2007
Paper No. 1132

CRACK PROPAGATION THRESHOLD IN FISSURED CLAYS

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ABSTRACT

Over-consolidated clays and shales forming part of the core section of earth dams, natural slopes, and clay deposits undergoing desiccation, can develop cracks that may propagate as a result of changes in the state of stresses produced by earthquakes. The dynamic loading produces very high stress concentrations around the crack tips which cause the crack to propagate. The pre-existing and the new cracks can produce severe weakness of the soil which could cause damaging or failure of structures and increased hydraulic conductivity of the clay. In the existing studies, the crack propagation in clays under dynamic loading has not been analyzed. In this study, the crack propagation threshold in fissured prismatic samples of clay subjected to uniaxial dynamic loading was determined in the laboratory. The stress propagation threshold is defined as the stress level below which no crack propagation occurs in the fissured clay independently of the number of applied cycles. Samples of unsaturated clay containing a crack inclined at 45° with respect to the horizontal axis were tested at different water contents. The threshold, as a function of the water content, was obtained from the laboratory tests.

Keywords: dynamic loading, crack propagation, stress propagation threshold, moisture content, clays.

INTRODUCTION

Over-consolidated clays and shales forming part of the core section of zoned earth dams and natural slopes have been found to exist in the fissured state (Bishop, 1967; Covarrubias, 1969; Duncan and Dunlop, 1969; Marsland, 1972; Morgenstern, 1977; Peterson et al. 1966; Rizkallah, 1977; Sherald, 1973; Skempton, 1964; Skempton and LaRochelle, 1965; Terzaghi, 1936; Vallejo, 1985; Williams and Jennings, 1977). According to Covarrubias (1969), fissures or cracks exist in the core section of earth dams as a result of deformation of the materials in the dam or in the foundation due to their weight; abrupt changes in the cross section of a valley; large deformations caused by saturation of the materials in the dam; excessively rapid filling of the reservoir that causes high rates of strain, especially of the materials undergoing substantial movement upon saturation; large transient stresses caused by earthquakes; large differences in stress-strain properties of materials in adjacent zones or layers. In the case of stiff clays forming natural slopes, Williams and Jennings (1977) found that fissures develop as a result of a variety of processes, the most important of which are: consolidation, swelling of the clay as a result of a decrease in overburden pressure, chemical reactions in the clay that induce volume distortions, tectonic stresses, desiccation of the clay, weathering process inherited from bedrock, and large lateral stresses.

The present research addresses the question of what happens to fissured soil deposits when they are subjected to dynamic loading such as that generated by earthquakes, wave action, traffic load, or machinery vibration.

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Sherard (1973) reports 15 cases of embankment dam cracking that have occurred around the world. He notes that 150 to 300 small dams (20 - 75 ft. high) are constructed in the United States each year and that most of the cracking in the dams in the U.S. results from embankment soils that are especially brittle and so susceptible to cracking.

Clay deposits subjected to desiccation often develop intense cracking that extends deep below the surface. Arizona, Mexico City, and Bogota are examples of areas particularly affected by such deep cracking. Vesga et al. (2003) found intensive deep cracking in the high plastic Bogota (Colombia) clay deposit that affects a flat area of 90000 ha located in a zone characterized by high seismic hazard: hundreds of kilometers of roadways and hundreds of small buildings in this area have been severely damaged or collapsed as a result of deep cracking caused by desiccation.

PREVIOUS RESEARCH

Previous Related Research on Crack Propagation in Clays

Vallejo (1986, 1988, 1989, 1994, 1993), and Vallejo and Shettima (1995) report several important findings related to the behavior of clays with pre-existing cracks. Vallejo and co-workers did several tests using rectangular kaolinite specimens with single or multiple cracks prepared in accordance with a special process that he developed; cracks of different orientations and specimens with different water contents were used. The specimens were subjected to monotonic uniaxial, biaxial, triaxial and shear stress fields. The researches used Linear Elastic Fracture Mechanics (LEFM) extensively to theoretically study the tension and compression stress concentrations around the cracks.

Four important conclusions can be derived from their findings as follows. (1) The critical pre-existing crack inclination, which corresponds to the condition of the lowest compression strength for crack propagation, varies between 45º and 60º with respect to the direction of the applied principal stress. (2) The maximum tangential stress criterion for a sharp crack of the type earlier proposed by Erdogan and Sih (1963) was used to predict the angle between the pre-existing crack plane and the crack propagation direction; this criterion was selected by Vallejo et al. (1995) as the closest to their findings between of all the criteria that were applied; (3) Fissures propagate as a result of constant compressive stresses (creep), which are much less than the crack-propagation compression strength of monotonically loaded clays (Vallejo and Shettima, 1997). (4) Multiple cracks will make the clay weaker, especially if superposition of tensile-stress concentration zones develops (Vallejo, 1993).

Previous Related Research on Stability Threshold

Lefebvre et al. (1988) studied the cyclic undrained resistance of non-fissured, intact saturated Hudson Bay clay and described the threshold as the stress level below which the soil suffers no failure regardless of the number of applied cycles. The researchers used the term *cyclic stress ratio* to describe this stress level which relates to both the applied triaxial cyclic stress and the triaxial compression strength of the intact clay. They found that for the saturated Hudson Bay clay, the stability threshold is defined by a cyclic stress ratio of between 0.60 and 0.65. In similar way, the stability threshold concept is applied in this research to the study of crack propagation in clays under dynamic loads as explained later on.

STABILITY THRESHOLD APPLIED TO FISSURED CLAYS

The stability threshold research will now be extended to fissured clays. The laboratory tests for this research focused on specimens of fissured clays that were subjected to cyclic loads; the loads applied were just a fraction of the static loads that caused the failure of the clay (Vesga, 2005). The research investigated the influence of the resulting fatigue on the propagation of cracks in unsaturated kaolinite clay and on the threshold stress with respect to cyclic loads levels below which the cracks do not propagate. Samples of fissured clays were subjected to uniaxial cyclic stress conditions. The cyclic stress ratio was defined as the ratio between the applied deviator dynamic vertical stress (σ_d) on a fissured specimen and the monotonic compression strength (σ_u) of a similar specimen having the same water content and crack geometry. The cyclic stress ratio is given as:

$$r_d = \frac{\sigma_d}{\sigma_u} \qquad (1)$$

The purpose of the dynamic-load testing was to find the threshold load (fraction of the static load), expressed as the cyclic stress ratio r_d, at which, regardless of the number of cycles applied, there is no crack propagation in an unsaturated kaolinite clay subjected to dynamic loading conditions.

LEFM THEORY AS APPLIED TO THE ANALYSIS OF CRACK PROPAGATION

Linear Elastic Fracture Mechanics Theory (LEFM)
According to LEFM theory, a crack or fissure in clay can be stressed in three different modes illustrated in Figure 1 (Vallejo, 1994). With Mode I, the stress normal to the crack walls produces a different type of cracking in which the disarrangements of the crack surfaces are perpendicular to the plane of the crack. With Mode II, a type of cracking is produced by shear stresses along the crack plane which causes the walls of the crack to slide with the crack plane. With Mode III, a tearing and cracking is caused by out of-plane shear stresses. Cracks can propagate in materials as a result of one or more of these modes (Vallejo, 1994).

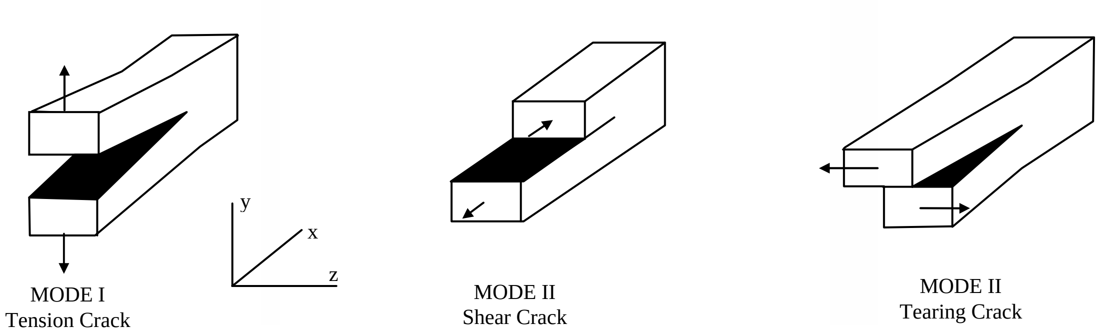

Figure 1. The three modes of cracking (Adapted from Vallejo, 1994).

Covarrubias (1969) used the three modes of fracture to investigate the propagation characteristics of cracks in cohesive materials used in earth barriers. The propagation of the cracks was found by Covarrubias to be the result from the tensile stresses normal to the plane of the cracks (mode I type of loading). More recently, Lee et al. (1988), Fang et al. (1989), and Morris et al. (1992) have used LEFM theory to analyze the crack propagation in solids that result from tensile stresses (mode I type loading).

Vallejo (1985, 1986, 1987, 1988a, 1989) used the principle of LEFM theory to explain the failure mechanisms of rigid fissured kaolinite clay samples that had been subjected to compression and direct shear stress conditions. Failure resulted when stress concentrations induced by the tips of the fissures caused the propagation and interaction of the fissures in the samples. The fissures in the samples were subjected to a combination of Mode I and Mode II types of loading. Saada et al. (1985, 1994) applied LEFM theory to normally and over-consolidated clays with cracks tested under a combination of normal and shear loads (Modes I and II type of loading). Saada et al (1994) determined that the degree of consolidation as well as the degree of anisotropy of the clay samples effect the way the cracks propagated.

Stresses around a Crack Tip

According to LEFM, the tangential stress (σ_θ), the radial stress *(*σ_r) and the shear stress *(*$\tau_{r\theta}$*)* in the material located in the vicinity of a crack subjected to a mixed mode type of loading (Mode I plus Mode II) can be obtained from the following formulas (Vallejo, 1994)

$$\sigma_\theta = \frac{1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[\mathbf{K}_I \cos^2\frac{\theta}{2} - \frac{3}{2}\mathbf{K}_{II}\sin\theta\right] \tag{2}$$

$$\sigma_r = \frac{1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[\mathbf{K}_I\left(1+\sin^2\frac{\theta}{2}\right) + \frac{3}{2}\mathbf{K}_{II}\sin\theta - 2\mathbf{K}_{II}\tan\frac{\theta}{2}\right] \tag{3}$$

$$\tau_{r\theta} = \frac{1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[\mathbf{K}_I \sin\theta + \mathbf{K}_{II}(3\cos\theta - 1)\right] \tag{4}$$

In the equations above, $\boldsymbol{r}$ is the radius between the tip of the crack and a point in the clay surrounding the crack where the stresses are being measured, θ is the angle that the radius r makes with the axis of the crack and K_I and K_{II} are the stress intensity factors for an open crack under Mode I or Mode II type of loading (Figure 1). The stress intensity factors (Vallejo, 1994) are represented as follows

$$K_I = 1.1215\sigma_n(\pi c)^{1/2} \qquad K_{II} = 1.1215\tau(\pi c)^{1/2} \tag{5}$$

where σ_n is the normal stress that acts perpendicular to the plane of the open crack, $\tau_{r\theta}$ is the shear stress that acts parallel to the crack, and $\boldsymbol{c}$ is the semi-length of the crack.

The above mentioned equations, formulated with LEFM were implemented through an electronic sheet using Mathcad (MathSoft Engineering & Education, Inc, 2001). Figure 3 shows the isobars for the tangential stress σ_θ normalized with respect to an arbitrary stress. Positive isobars indicate compressive stresses and negative isobars indicate tensile stresses. Compressive stress concentrations can be seen on the right side of the crack tip and tensile stress concentrations on the left side.

Principal Stresses

The principal stresses can be deducted from the tangential, radial, and shear stresses presented above. The maximum and minimum principal stresses are calculated as follows

$$\sigma_1 = \frac{(\sigma_\theta + \sigma_r)}{2} + \sqrt{\left(\frac{\sigma_\theta - \sigma_r}{2}\right)^2 + (\tau_{r\theta})^2} \qquad \sigma_3 = \frac{(\sigma_\theta + \sigma_r)}{2} - \sqrt{\left(\frac{\sigma_\theta - \sigma_r}{2}\right)^2 + (\tau_{r\theta})^2} \quad \text{(6 and 7)}$$

The center (p) and the radius (q) of the Mohr's circle that represents the stresses at a point are calculated as

$$p = \frac{(\sigma_1 + \sigma_3)}{2} \qquad q = \frac{(\sigma_1 - \sigma_3)}{2} \qquad \text{(8 and 9)}$$

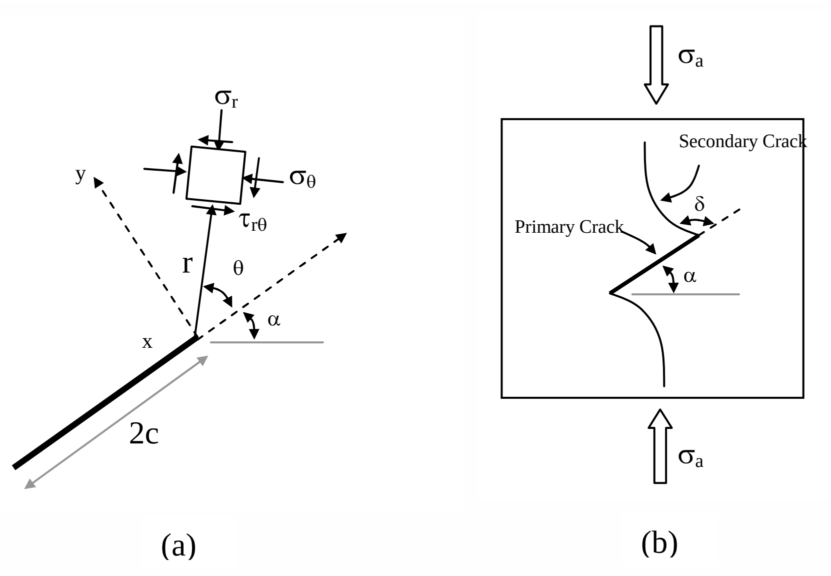

Figure 2. a) Tangential, radial, and shear stresses around a crack tip. b) Crack propagation angle δ.

Direction of Crack Propagation

Erdogan and Sih (1963) have proposed the hypothesis that crack extension in fragile materials takes place in a direction in which σ_θ, given by Eq.2, reaches its maximum value – i.e., that crack will take place in a radial direction from the crack tip and that the direction of crack growth is normal to the direction of the maximum tangential stress σ_θ. (Vallejo, 1994). This theory is known as the maximum tangential stress criterion (MTS).

Hence the direction of crack propagation taking place when θ reaches a value equal to δ is given by

$$\frac{d\sigma_\theta}{d\theta} = 0 \quad \text{and} \quad \frac{d^2\sigma_\theta}{d^2\theta} < 0 \quad \text{at} \quad \theta = \delta \qquad \text{(10 and 11)}$$

where δ is the value reached by θ when crack propagation takes place (Figure 2). After manipulating the above equations, the direction of crack propagation (δ) can be obtained from the following equation

$$K_I \sin\delta + K_{II}(3\cos\delta - 1) = 0 \qquad \cot\alpha \times \sin\delta + (3\cos\delta - 1) = 0 \qquad \text{(12 and 13)}$$

After replacing the values of K_I and K_{II} in the equation above, the relationship between δ and α is obtained. The direction of crack propagation follows the tensile concentration stresses shown in Figure 3. Eq 12 applies to an open crack for which the stress intensity factors K_I and K_{II} are involved. If the crack closes, the stress intensity factor, K_I becomes equal to zero (Broek, 1984), and Eq 13 can be written as

$$3\cos\delta - 1 = 0 \qquad (14)$$

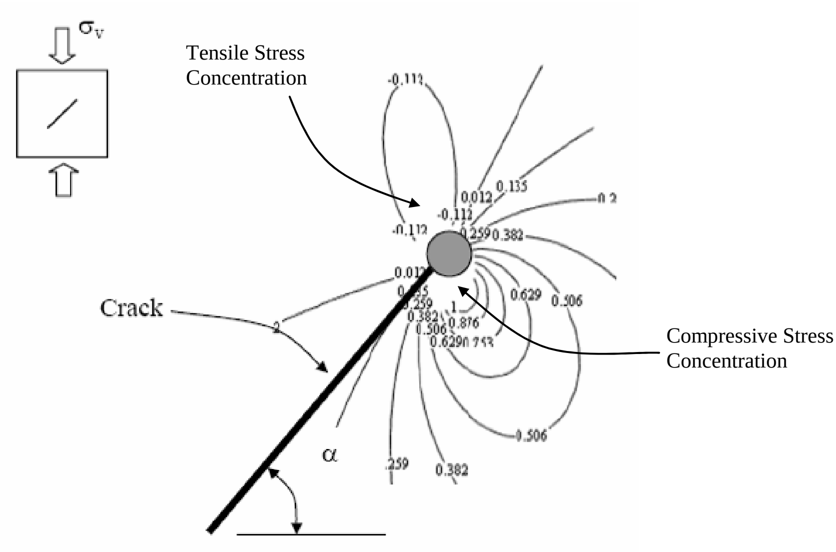

Figure 3. Normalized tangential stress σ_θ around a crack tip derived using LEFM. The stresses were normalized with respect to an arbitrary compression stress. To increase clarity of the image, the stresses inside the gray circle are not shown.

DESCRIPTION OF LABORATORY TESTS PERFORMED

Preparation of Samples

Prismatic kaolinite specimens were prepared for the tests. The liquid, plastic and shrinkage limits of the kaolinite are 44, 26 and 24 respectively and the specific gravity is 2.59. The specimens for the direct and indirect tensile tests had moisture contents between 3% and 32%, with intervals of around 3%. The sample preparation was done with the following process:

a) Mixing the dry powdered kaolinite with distilled water to moisture content of 40% and until a smooth and uniform paste was obtained.
b) Casting the paste into molds (prismatic of 7.5 cm x 7.5 cm x 2.5 cm or cylindrical 6.35 cm in diameter and 2 cm thick).
c) Applying a vertical pressure of 30 kPa for consolidation during 24 hours in a closed environment with 75% of relative humidity (*RH*).
d) Extracting the specimens and subjecting them to a drying process into an environment with a *RH* ~30%.
e) Cracks with inclination angle 45º with respect to the horizontal were made in the prismatic specimens; samples were still saturated during this stage. Each crack was made using a blade which produced a crack having a length of 25 mm and a thickness of 1 mm.
f) After each specimen reached the desired moisture content (between 2% and 34%) it was stored into a plastic membrane during a minimum 24 hour period in order to obtain equilibrium of the moisture content through the sample.

Uniaxial Compression Tests

Uniaxial constant water content tests were done using both intact and pre-cracked specimens. Specimens with moisture contents of between 3% and 32% with intervals around 3% were used. Each sample tested was maintained inside a plastic membrane in order to prevent water content changes during the tests.

Dynamic Loading Tests

A MTS hydraulic compression machine was used for the dynamic loading tests. The purpose of these tests was to find the threshold cyclic stress ratio (fraction of the ultimate static strength) at which there is no crack propagation under dynamic loading conditions regardless of the number of applied cycles. The cyclic stress ratio r_d was defined as the ratio between the applied dynamic vertical stress and the ultimate monotonic compression strength of a similar specimen having the same water content and the same crack inclination. Thus, the cyclic axial load was a fraction of the static load at which the specimen failed.

Dynamic uniaxial compression tests were performed on fissured specimens having moisture contents of between 3% and 32% with intervals around 3%. A sine wave loading type with a frequency of 1 Hz was applied to the specimens. The data acquisition system DATAQ ID-194 (Dataq Instruments, 2003) recorded the load and deformation of the samples occurred during the tests. The level of the cyclic stress ratio r_d was changed in order to determine the crack propagation threshold.

RESULTS FROM LABORATORY TESTS

Uniaxial Ultimate Compression Strength σ_u

The ultimate uniaxial compressive strength σ_u of the intact and pre-fissured specimens is presented as a function of the water content in Figures 4 and 6 respectively. The ultimate compression strength for intact specimens and for specimens with a crack is very close

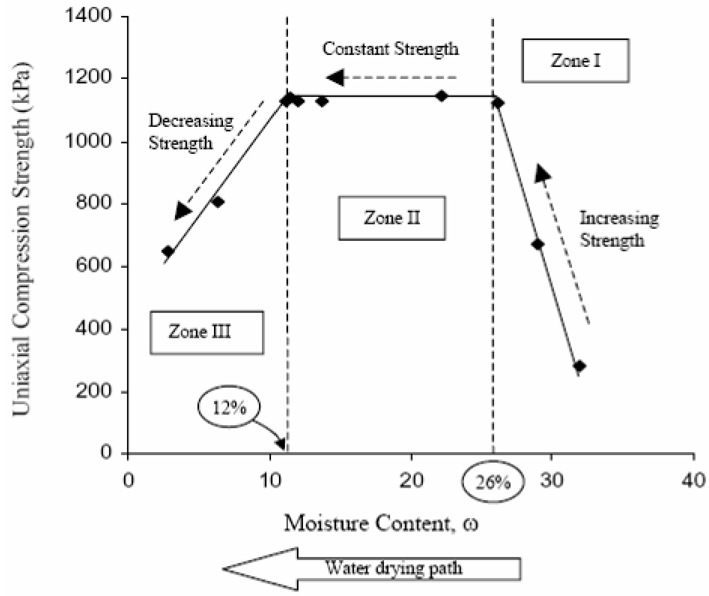

Figure 4. Uniaxial compression tests on intact specimens subjected to a drying path. Zone I: the strength increases as water content is reduced. Zone II: constant strength. Zone III: the strength

diminishes as the water content is reduced (Note: Zones are numbered from right to left here and elsewere to reflect the drying path).

and indicates that the presence of the primary crack has no effect on the ultimate uniaxial compressive strength of the tested specimens. As Figure 5 shows, a shear failure plane was observed in the lateral face of all of the specimens. This confirms that the two types of failure occur independently one of each other.

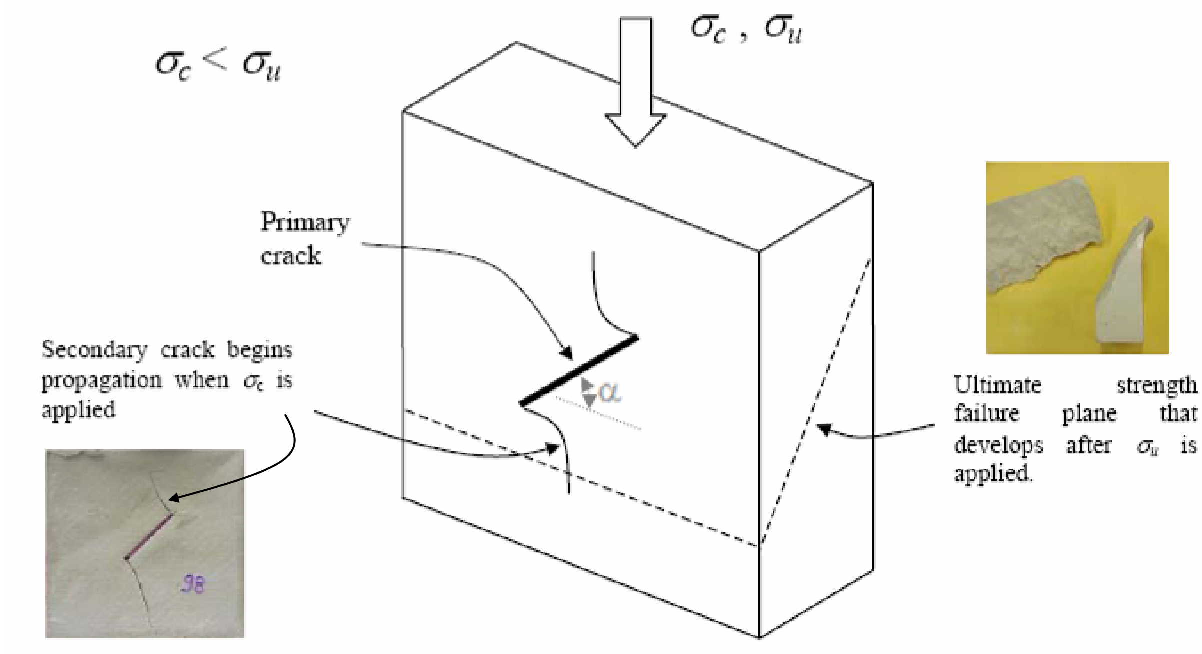

Figure 5. Crack propagation and failure plane observed from the uniaxial compression tests.

Vesga and Vallejo (2006) explain the observed variation of the uniaxial strength with the moisture content using the concept of equivalent effective stress (EES) proposed by Cho and Santamarina (2001). Vesga and Vallejo (2006) propose that the EES is dependent on the water distribution in the pores of the clay. Zone I corresponds to saturated-funicular state in which the soil increases strength as the water content is diminished. Zone II corresponds to complete pendular state in which the soil strength is almost constant, and Zone III corresponds to partial pendular state in which strength diminishes as the water content is reduced. Saturated, funicular, and pendular states are defined by German (1989). All the pores within the soil are full of water in the saturated state, air penetrates in the soil but water phase is still continuous in the funicular state, and capillary meniscus between particles (discontinuous water phase) exist in the pendular state (German, 1989).

Vesga (2005) subdivides the pendular state in complete pendular and partial pendular states. All the contacts between particles in the soil have capillary meniscus in the complete pendular state, and some capillary contacts break in the partial pendular state as the soil is dried. A capillary contact can break for two principal reasons: cavitation pressure in the water is reached as the soil is dried, or the particles forming the contact are separated a distance and the capillary neck cannot survive if the water volume is reduced and some distance between particle remains. Capillary neck breakings produce the soil to be weaker. The soil has a minimum resistance after it is completely dried; its resistance depends on the van der Waals inter-particle attractions (Vesga, 2005).

Stress level at Crack Propagation σ_c

Two stress levels were registered for each uniaxial compression loading test performed on fissured specimens. With the stress level of σ_c, crack propagation occurred in the front and rear faces of the specimen (see Figure 5). Secondary crack initiation was established visually with continuous inspection of the specimen faces. With the stress level of σ_u – which is the maximum stress specimens' were able to withstand – a shear failure plane appeared in the lateral faces of the specimen.

The stress level at crack propagation σ_c as a function of the water content that resulted from the uniaxial compression tests is shown in Figure 6. The three zones that were first observed with the uniaxial compression tests of the intact specimens are again present in fissured specimens.

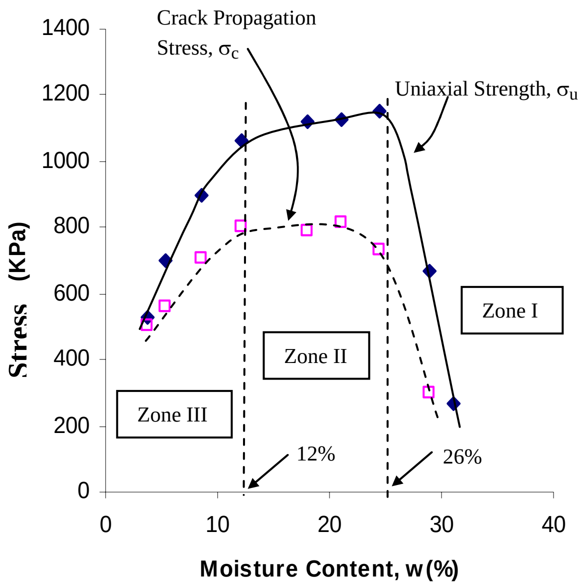

Figure 6. Uniaxial strength and crack propagation stress on fissured specimens subjected to a drying path. Zone I: the strength increases as water content is reduced. Zone II: constant strength. Zone III: the strength diminishes as the water content is reduced (Note: Zones are numbered from right to left here and elsewhere to reflect the drying path).

Zone I (saturated-funicular state) shows increments of the clay strength σ_c as the water content is reduced from 32% to 26%. In Zone II (complete pendular state), the strength σ_c reaches a maximum value and remains almost constant for water contents between 26% and 12%. Zone III (partial pendular state) shows the reduction in clay strength σ_c as the water content falls below 12%.

Crack Propagation angle and Crack Propagation Path

Figure 7 shows the condition of two samples after crack propagation occurred and one sample which primary crack closed and didn't propagate; one specimen had a water content of 9.5%, and the other two had water content of 30.6% and 32.4%. The specimen with the lower water content had a typical brittle behavior characterized by a primary crack that remains open after the secondary crack propagates. The specimen having moisture content of 30.6% had a typical ductile type I behavior: the primary crack closed after the propagation of the secondary crack took place. In this specimen, it was observed that the secondary crack finished its propagation after the primary crack closed.

The specimen having a water content of 32.4% demonstrated a ductile, Type II behavior -i.e., the crack closed and didn't propagate. This is consistent with the more general finding that in some ductile specimens no propagation occurs. The crack in the brittle specimen propagated at an angle of 90º with respect to the plane of the primary crack while the crack in the ductile type I specimen propagated at an angle of 92º. Comparison of results from different tests does not show important variation of the crack propagation angle with water content. After the secondary crack started the propagation, the crack curved towards the vertical direction as Figure 7 shows. The MTS criterion proposed by Erdogan and Sih (1963) for the prediction of the crack propagation angle predicts very well the angle δ for primary crack inclination angle α of 45º. Vallejo et al (1995) present results and discussion for the crack propagation angle using specimens with different pre-crack inclination angles.

Crack Propagation Threshold derived from Dynamic Loading Tests

The cyclic stress ratio was earlier defined as the ratio between the applied dynamic vertical stress and the ultimate monotonic compression strength of a similar specimen (with the same water content and the same primary crack inclination). The purpose of the dynamic loading test was to find the threshold load (fraction of the static load) at which no crack propagation occurred under dynamic loading conditions regardless of the number of applied cycles.

(a) (b) (c)

Figure 7. (a) Secondary tensile crack in a brittle sample with a moisture content of 9.5%. (b) Secondary tensile crack in a ductile sample with a moisture content of 30.6%; the primary crack closed after propagation occurred. c) The primary crack closed and didn´t propagate in a ductile sample of 32.4%. The failure that is shown occurred by shear.

Figure 8 shows the obtained results; in this figure, the cyclic stress ratio is expressed as a function of the water content in the specimens. The dark points in this figure represent the cyclic stress ratios (r_d) for which crack propagation occurred; the clear points represent the cases for which no crack propagation was observed after 7200 cycles of load applications.

The general trend observed in these tests was that the crack stability threshold is almost constant regardless of the moisture content for water contents of less than 26% (complete and partial pendular states). For water content above 26% in funicular-saturated state, the crack stability threshold diminishes noticeably; at a moisture content of 30%, the threshold was found to be as low as 0.15. As noted, the reduction in the crack stability threshold for high moisture contents is very significant and is very possible to be related to locally developed pore pressures in the compression zones (local liquefaction) around the crack. These pore pressures can be developed when the material is close to saturation. Pore pressures in unsaturated clays subjected to dynamic loading are not easy to be measured and even more difficult would be measuring local pore pressures around the crack tips in the clay.

CONCLUSIONS

The crack stability threshold is defined as the cyclic stress ratio r_d, at which, regardless of the number of cycles applied, there is no crack propagation in a clay specimen subjected to uniaxial dynamic loading conditions. Uniaxial static and dynamic laboratory tests were performed in order to measure the crack stability threshold on kaolinite clay specimens having a crack inclined 45° with the horizontal and different moisture contents. The threshold is almost constant under moisture contents of 26% in the complete and partial pendular states of water distributions in the pores of the clay, and diminishes remarkably for higher moisture contents when the water distribution is in the saturated-funicular state.

More research is necessary in order to confirm that the low threshold values observed for high moisture contents close to saturation are due to the pore pressures developed in the clay during dynamic loading and if there occurs local liquefaction around the crack tips due to the high compressive stress concentrations which produce the pore pressure to rise and the crack to propagate.

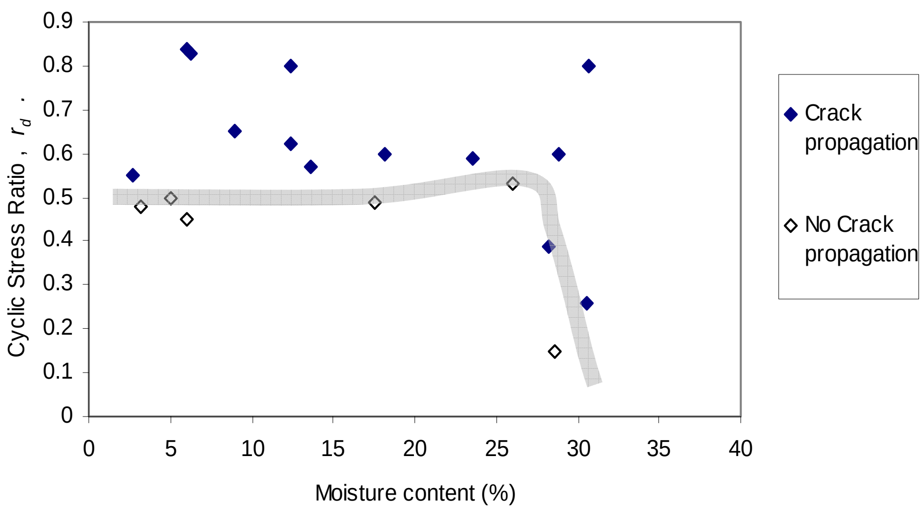

Figure 8. Crack stability threshold for specimens with a crack having an inclination angle α=45°

AKNOWLEDGEMENTS

This research was developed by the author as Graduate Student in the School of Engineering in the University of Pittsburgh. The support received from the University is gratefully acknowledged. The author would also like to show his gratitude to his advisor in the University of Pittsburgh, Professor Luis E. Vallejo, and to Edifica Colombia Ltda for the financial support provided through this investigation.

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