

## ISSUES IN MODELING AND SIMULATIONS OF SOIL LIQUEFACTION

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### ABSTRACT

Fully coupled, porous solid-fluid formulation and related modeling and simulation issues are presented in this work. In particular, coupled dynamic field equations with  $u$ - $p$ - $U$  formulation are used to simulate pore fluid and soil skeleton responses. Present formulation allows, among other features, for water accelerations to be taken into account. This proves useful in modeling dynamic interaction of media of different stiffness (as in soil–foundation–structure interaction). Fluid compressibility is also explicitly taken into account, thus allowing excursions into modeling of limited cases of non-saturated porous media. In addition to that, no artificial damping is used (of Raleigh type), the formulation and implementation rather rely on (physical) interaction between solid particles and pore fluid, which is taken into account analytically in the  $u$ - $p$ - $U$  formulation.

One of the main challenges in modeling saturated soils is the appropriate modeling of elastic–plastic behavior of soil skeleton. This challenge is met with the use of an advanced material model (Dafalias and Manzari, 2004) and is discussed at some length.

Illustrative examples describing dynamical behavior of porous media (saturated soils) are presented. Of particular interest is the verification and validation (V&V) of current models and simulations, and examples will be used for that purpose.

### BACKGROUND

Various cases of soil failures induced by earthquakes, continue to challenge engineering research and practice. Failures are commonly occurring in loose to medium dense sands which are fully saturated and comprise the almost complete loss of strength (liquefaction), and partial loss of strength (cyclic mobility). To model these complex phenomena, a consistent and efficient coupled formulation must be utilized, including an accurate single-phase constitutive model for soil. Three general formulations (Zienkiewicz and Shiomi, 1984) are possible for modeling of coupled problems (soil skeleton – pore fluid) in geomechanics, namely the (a)  $u$ - $p$ , (b)  $u$ - $U$ , and (c)  $u$ - $p$ - $U$  formulations. Here, the unknowns are the soil skeleton displacements  $u$ ; the pore fluid (water) pressure  $p$ ; and the pore fluid (water) displacements  $U$ . The  $u$ - $p$  formulation captures the movements of the soil skeleton and the change of the pore pressure, and is the most simplistic one of the three mentioned above. This formulation neglects the accelerations of the pore fluid, and in one version neglects the compressibility of the fluid (assuming complete incompressibility of the pore fluid). In the case of incompressible pore fluid, the formulation requires special treatment of the approximation (shape) function for pore fluid to prevent the volumetric locking (Zienkiewicz and Taylor, 2000)). The majority of the currently available

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implementations are based on this formulation. For example Elgamal et al. (2002) and Elgamal et al. (2003) developed an implementation of the  $u$ - $p$  formulation with the multi-surface plasticity model (Prevost, 1985), while Chan (1988) and Zienkiewicz et al. (1998) used generalized theory of plasticity (Pastor et al., 1990). In addition to that Taiebat et al. (2007) used the model of Manzari and Dafalias (1997) in their  $u$ - $p$  formulation and showed good model performance for a boundary value problem from VELACS project (Arulanandan and Scott 1993). The  $u$ - $U$  formulations tracks the movements of both the soil skeleton and the pore fluid. This formulation is complete in the sense of basic variable, but might still experience numerical problems (volumetric locking) if the difference in volumetric compressibility of the pore fluid and the solid skeleton is large.

The  $u$ - $p$ - $U$  formulation resolves the issues of volumetric locking by including the displacements of both the solid skeleton and the pore fluid, and the pore fluid pressure as well. This formulation uses additional (dependent) unknown field of pore fluid pressures to stabilize the solution of the coupled system. The pore fluid pressures are connected to (dependent on) displacements of pore fluid, as, with known (given) volumetric compressibility of the pore fluid, pressure can be calculated. All three formulations were originally derived by Zienkiewicz and Shiomi (1984). Recently Cheng (2006) combined developments by Zienkiewicz and Shiomi (1984), and the nonlinear dynamics algorithms from Argyris and Mlejnek (1991), in order to use the consistent time marching scheme for nonlinear dynamic  $u$ - $p$ - $U$  discretization.

Despite its power, the  $u$ - $p$ - $U$  formulation has rarely been implemented into finite element code. In this paper, complete  $u$ - $p$ - $U$  formulation and discretization is presented. In addition to that, a recent critical state two-surface plasticity model accounting for the fabric dilation effects (Dafalias and Manzari, 2004) used for simulations is presented in some details as well. A number of examples, including cyclic behavior of a soil column are presented in order to illustrate previous developments. The soil column is drained at the top while sides are representing the symmetry boundary conditions (assuming saturated shear beam).

## GOVERNING EQUATIONS OF POROUS MEDIA

As the strength of the soil can be determined once the pore water pressures are known, it is possible to reduce the saturated soil mechanics problem to that of the behavior a single inelastic phase, while consistently coupling the fluid and porous solid. The concept of effective stress of the saturated mixture, that is, the relationship between effective stress, total stress and pore pressure (Zienkiewicz et al., 1998) are given by:  $\sigma'_{ij} = \sigma_{ij} + \alpha \delta_{ij} p$ , where  $\sigma'_{ij}$  is the effective stress tensor,  $\sigma_{ij}$  is total stress tensor,  $\delta_{ij}$  is Kronecker delta. For most of the soil mechanics problems,  $\alpha \approx 1$  can be assumed. The relation between total and effective stress becomes  $\sigma'_{ij} = \sigma_{ij} + \delta_{ij} p$ , which returns to the classical effective stress definition by Terzaghi (1943).

The overall equilibrium or momentum balance equation for the soil-fluid 'mixture' can be written as

$$\sigma_{ij,j} - \rho \ddot{u}_i - \rho_f \ddot{w}_i + \rho b_i = 0 \quad (1)$$

where  $\ddot{u}_i$  is the acceleration of the solid part,  $b_i$  is the body force per unit mass,  $\ddot{w}_i$  is the fluid acceleration relative to the solid part. For fully saturated porous media (no air inside),  $\rho = n\rho_f + (1-n)\rho_s$ , where  $n$  is the porosity,  $\rho_s$  and  $\rho_f$  are the soil particle and water density respectively.

For the pore fluid, the equation of momentum balance can be written as

$$-p_{,i} - \frac{R_i}{n} - \rho_f \ddot{w}_i - \rho_f \ddot{w}_i / n + \rho_f b_i = 0 \quad (2)$$

where it should be noted that the permeability  $k$  here is different from the permeability usually used in soil mechanics  $K$ . Their values are related by  $k=K/g\rho_f$ , where  $g$  is the gravitational acceleration and the permeability  $K$  is obtained from measurements. According to the Darcy's seepage law, the viscous drag forces ( $R$ ) between soil matrix and water can be written as  $R_i/n=k_{ij}^{-1}\dot{w}_j$ , where  $k_{ij}$  is the anisotropic Darcy permeability coefficient.

The final equation is the mass conservation of the fluid flow expressed by

$$\dot{w}_{i,i} + \alpha \dot{\varepsilon}_{ii} + \dot{p}/Q = 0 \quad (3)$$

where  $Q$  is expressed as  $1/Q = n/K_f + (\alpha - n)/K_s$  and  $K_s$  and  $K_f$  are the bulk moduli of the solid and fluid phases respectively.

In the above governing equations, convective and terms of lower order are neglected (Zienkiewicz et al., 1998), and a change of variables is performed. An new variable  $U_i$ , defined as  $U_i = u_i + U_i^R = u_i + w_i/n$ , is introduced in place of the relative pseudo-displacement  $w_i$ . The basic unknowns are now the soil skeleton displacements  $u_i$ , the water pore pressure  $p$ , and the water displacements  $U_i$ . These unknown variables can be approximated using shape functions and the corresponding nodal values, following standard Finite Element discretization procedure:

$$u_i = N_K^u \bar{u}_{Ki}, \quad p = N_K^p \bar{p}_K, \quad U_i = N_K^U \bar{U}_{Ki} \quad (4)$$

where  $N_K^u$ ,  $N_K^p$  and  $N_K^U$  are shape functions for solid displacement, pore pressure and fluid displacement respectively. The nodal degrees of freedom  $\bar{u}_{Ki}$ ,  $\bar{p}_K$ ,  $\bar{U}_{Ki}$  representing solid displacement, pore fluid pressure and fluid displacement respectively become the main unknowns of the discretized finite element system. It follows that each node of the  $u$ - $p$ - $U$  element has seven degrees of freedoms in 3D (3 solid displacement, 1 pore pressure and 3 fluid displacements).

After some tensor algebra manipulations, the final  $u$ - $p$ - $U$  form can be written as

$$\begin{aligned} & \begin{bmatrix} (M_s)_{KijL} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (M_f)_{KijL} \end{bmatrix} \begin{bmatrix} \ddot{\bar{u}}_{Lj} \\ \ddot{\bar{p}}_N \\ \ddot{\bar{U}}_{Lj} \end{bmatrix} + \begin{bmatrix} (C_1)_{KijL} & 0 & -(C_2)_{KijL} \\ 0 & 0 & 0 \\ -(C_2)_{LjiK} & 0 & (C_3)_{KijL} \end{bmatrix} \begin{bmatrix} \dot{\bar{u}}_{Lj} \\ \dot{\bar{p}}_N \\ \dot{\bar{U}}_{Lj} \end{bmatrix} + \\ & \begin{bmatrix} 0 & -(G_1)_{KiM} & 0 \\ -(G_1)_{LjM} & -P_{MN} & -(G_2)_{LjM} \\ 0 & -(G_2)_{KiL} & 0 \end{bmatrix} \begin{bmatrix} \bar{u}_{Lj} \\ \bar{p}_M \\ \bar{U}_{Lj} \end{bmatrix} + \begin{bmatrix} \int_{\Omega} N_{K,j}^u \sigma_{ij}'' d\Omega \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \bar{f}_{Ki}^u \\ 0 \\ \bar{f}_{Ki}^U \end{bmatrix} = 0 \end{aligned} \quad (5)$$

It is interesting to note the coupling of solid and fluid through the damping tensor (matrix). This damping is dynamically consistent as it represents the real interaction between solid and fluid, which results in energy losses proportional to velocity. The other energy losses (dissipation) are resulting from the so called displacement proportional damping, better known as elasto-plasticity.

The dynamic system of equations was integrated using the Hilber-Hughes-Taylor (HHT)  $\alpha$ -method (Hilber et al., 1977) which provides excellent accuracy and stability for such coupled systems. The finite model developed using this formulation uses eight node brick elements were used. Since the pore fluid is compressible, there are no problems with locking, particularly if good equation solvers are used, therefore there is no need for lower order of interpolation for pore fluid pressures. On the constitutive level an initial explicit integration algorithm applied to Dafalias and Manzari (2004) was

used. It should also be noted that no additional physical (Raleigh) damping was used. Small amount of numerical damping was introduced through constants of the HHT algorithm in order to stabilize the dynamic time stepping.

## MATERIAL MODEL

Soil constitutive model plays the key role in simulating the liquefaction phenomenon. The constitutive platform here is based on the recent work of Dafalias and Manzari (2004). A brief description of the basic equations of the model is given below. Isotropic hypoelasticity is defined by

$$\dot{e}_{ij}^e = \frac{\dot{s}_{ij}}{2G}; \quad \dot{\epsilon}_v^e = \frac{\dot{p}}{K} \quad (6)$$

with  $s_{ij}$  is the stress deviator;  $p$  mean effective pressure;  $e_{ij}^e$  strain deviator; and  $[\epsilon]_v$  volumetric strain. Superscript  $e$  denoting elastic; superposed dot denoting the rate.  $G$  and  $K$  are the hypoelastic shear and bulk moduli, respectively given by

$$G = G_0 p_{at} \frac{(2.97 - e)^2}{1 + e} \left( \frac{p}{p_{at}} \right)^{0.5}; \quad K = \frac{2(1 + \nu)}{3(1 - 2\nu)} G \quad (7)$$

where  $G_0$  is a dimensionless constant;  $\nu$  is the elastic Poisson's ratio; and  $p_{at}$  the atmospheric pressure used for normalization.

The yield surface is defined by

$$f = [(s_{ij} - p\alpha_{ij})(s_{ij} - p\alpha_{ij})]^{1/2} - \sqrt{2/3}mp = 0 \quad (8)$$

which describes geometrically a "cone" in terms of deviatoric back-stress ratio  $\alpha_{ij}$  which defines the position of the axis of the cone, and the stress ratio scalar variable  $m$  the "size" of cone. The intersection of the yield surface with the  $\pi$ -plane of deviatoric stress ratio  $r_{ij} = s_{ij}/p$  space is shown as a circle with center  $\alpha_{ij}$  and radius  $\sqrt{2/3}m$ . The gradient to the yield surface at  $r_{ij}$  is obtained as

$$\frac{\partial f}{\partial \sigma_{ij}} = n_{ij} + \frac{1}{3}(n_{mn}r_{mn})\delta_{ij}; \quad n_{ij} = \frac{r_{ij} - \alpha_{ij}}{\sqrt{2/3}m} \quad (9)$$

and  $n_{ij}$  is a deviatoric unit tensor. Using  $n_{ij}$ , the Lode angle  $\theta$  has been defined by  $\cos 3\theta = \sqrt{6}n_{ij}n_{jk}n_{ki}$ . Three concentric and homologous surfaces, the bounding, dilatancy, and critical surface, associated with the tensors  $(\alpha_\theta^b)_{ij}$ ,  $(\alpha_\theta^d)_{ij}$ , and  $(\alpha_\theta^c)_{ij}$ , along the direction  $n_{ij}$  emanating from the origin at the Lode angle  $\theta$  have been defined as

$$(\alpha_\theta^a)_{ij} = \sqrt{\frac{2}{3}}[g(\theta, c)M \exp(\mp n^a \psi) - m]n_{ij} \quad (10)$$

here the function  $g(\theta, c) = (2c)/[(1+c) - (1-c)\cos 3\theta]$  has been used for interpolation of related quantities for a given  $\theta$  between their triaxial compression and extension values.  $a=b, c, d$ ; and sign of  $\mp$  corresponds to  $a=b$  and  $d$ , respectively, and  $n^c = 0$ .  $\psi = e - e_c$  is the state parameter where

$e_c = e_0 - \lambda_c (p/p_{at})^\xi$  defines the corresponding void ratio at confining pressure  $p$  on the critical state line with the constants  $e_0$ ,  $\lambda_c$ , and  $\xi$ .

The plastic strain tensor is given by

$$\dot{\epsilon}_{ij}^p = \langle L \rangle R_{ij} \quad (11)$$

$$R_{ij} = [B n_{ij} + C(n_{ik} n_{kj} - \frac{1}{3} \delta_{ij})] + \frac{1}{3} D \delta_{ij} \quad (12)$$

where  $L$  is the loading index and  $\langle \rangle$  the MacCauley brackets.  $B$  and  $C$  are the functions of the Lode angle  $\theta$  and compression-tension ratio  $c$  as follows

$$B = 1 + \frac{3}{2} \frac{1-c}{c} g(\theta, c) \cos 3\theta; \quad C = 3 \sqrt{\frac{3}{2}} \frac{1-c}{c} g(\theta, c) \quad (13)$$

The dilatancy coefficient  $D$  is defined by

$$D = -A_d [(\alpha_\theta^d)_{ij} - \alpha_{ij}] n_{ij} \quad (14)$$

with parameter  $A_d$  a function of the state variables. The rate of  $\alpha_{ij}$  is defined by

$$\dot{\alpha}_{ij} = \langle L \rangle \frac{2}{3} h [(\alpha_\theta^b)_{ij} - \alpha_{ij}] \quad (15)$$

i.e.,  $\dot{\alpha}_{ij}$  is directed along  $(\alpha_\theta^b)_{ij} - \alpha_{ij}$ , and  $h$  is a hardening coefficient. The consistency condition applied to the yield surface equation together with the presented hardening law yields the value of the loading index. While constant values of  $h$  yield reasonable simulations, better results are obtained with the following  $h$  function

$$h = G_0 h_0 \frac{(1 - c_h e)(p/p_{at})^{-0.5}}{(\alpha_{ij} - \alpha_{ij}^{in}) n_{ij}} \quad (16)$$

with two model constants  $c_h$  and  $h_0$ . The  $(\alpha_{ij}^{in})$  is the initial value of  $\alpha_{ij}$  updated at any new loading initiation.

Finally taking into account the effect of fabric change during dilatancy, the so-called fabric-dilatancy internal variable  $z_{ij}$  has been introduced in the model which evolves according to

$$\dot{z}_{ij} = -c_z \langle LD \rangle (z_{\max} n_{ij} + z_{ij}) \quad (17)$$

while  $A_d$  is given by

$$A_d = A_0 (1 + z_{ij} n_{ij}) \quad (18)$$

with three model parameters  $c_z$  and  $z_{\max}$ , for the fabric-dilatancy tensor and  $A_0$  for the dilatancy.

This model was verified and validated with the laboratory testing data for Toyoura sand.

## NUMERICAL EXAMPLES

The examples consist of a multiple-element soil column subjected to an earthquake shaking. The soil is assumed to be Toyoura sand and the calibrated parameters are from Dafalias and Manzari (2004). The column is horizontally excited after self-weight loading. It should be noted that the self weight loading is performed on an initially zero stress (unloaded) soil column and that the material model is powerful enough to follow through this early loading with proper parameter evolution. The boundary conditions are such that the soil and water displacement degrees at the bottom surface are fixed, the pore pressure degrees are free; the soil and water displacement degrees at the upper surface are free, however the pore pressure degrees are fixed to simulate the upward drainage. In order to simulate the one dimensional shaking, all DOFs at the same level are connected in a master-slave fashion. The

permeability is assumed to be isotropic  $k=5.0 \times 10^{-4}$  m/s. The input acceleration time history is taken from the recorded horizontal acceleration of Model No.1 of VELACS project (Arulanandan and Scott 1993) by Rensselaer Polytechnic Institute (<http://geoinfo.usc.edu/gees/velacs/>). The magnitude of the motion is close to 0.2 g. Simulated results for the initial void ratio of 0.85 (before applying self-weight) are shown in Figure (1). It should be emphasized that the soil parameters are related to Toyoura sand, not Nevada sand which is used in VELACS project. The purpose of this simulation is to show the overall performance of the model. Input files for examples shown in this paper are available at the following web site: <http://sokocalo.engr.ucdavis.edu/~jeremic>.

The calculated maximum horizontal shear strain can reach up to 5%. It is apparent that the shear modulus is decreasing as the shaking progresses and the pore fluid pressure increases. All soil layers are experiencing liquefaction ( $R_u \approx 1$ ). The excess pore pressure dissipation at the lower layers are

quicker than those at the upper layers. The upper layers have to dissipate their own excess pore pressure, but they also receive a relatively large additional volume of water from lower levels (dissipation is upward), so one can note increase in pore fluid pressure at top layers even after the shaking has stopped. The soil settlement and water drainage continue even after the initial shaking is over mainly due to the continuous pore fluid movement upward.

In a similar numerical experiment, the effect of initial void ratio on cyclic behavior of soil column is investigated. Results for a denser soil example with an initial void ratio of 0.75 are shown in Figure (2). All other input parameters remain identical. For this denser example, the calculated maximum horizontal shear strain is reduced to less than 1%. The upper layers are liquefied (possibly by receiving large water volumes from layer beneath), while the lower layers are not even close to liquefaction ( $R_u \approx 0.8$ ). The strength in all layers is evidently reduced. The excess pore pressure dissipation happens

faster than in the looser soil example mainly because there is less compression of soil skeleton, which results in smaller amount of pore fluid that is dissipated.

The comparison illustrates that initial conditions (initial void ratio) are significant to the behavior of fully saturated soil-water systems. There are a number of other interesting (and possibly significant) findings steaming from our simulations, but space limitations for this paper make it impossible to present them here. They will be presented in a full length paper that is in the process of being submitted for a journal publication.

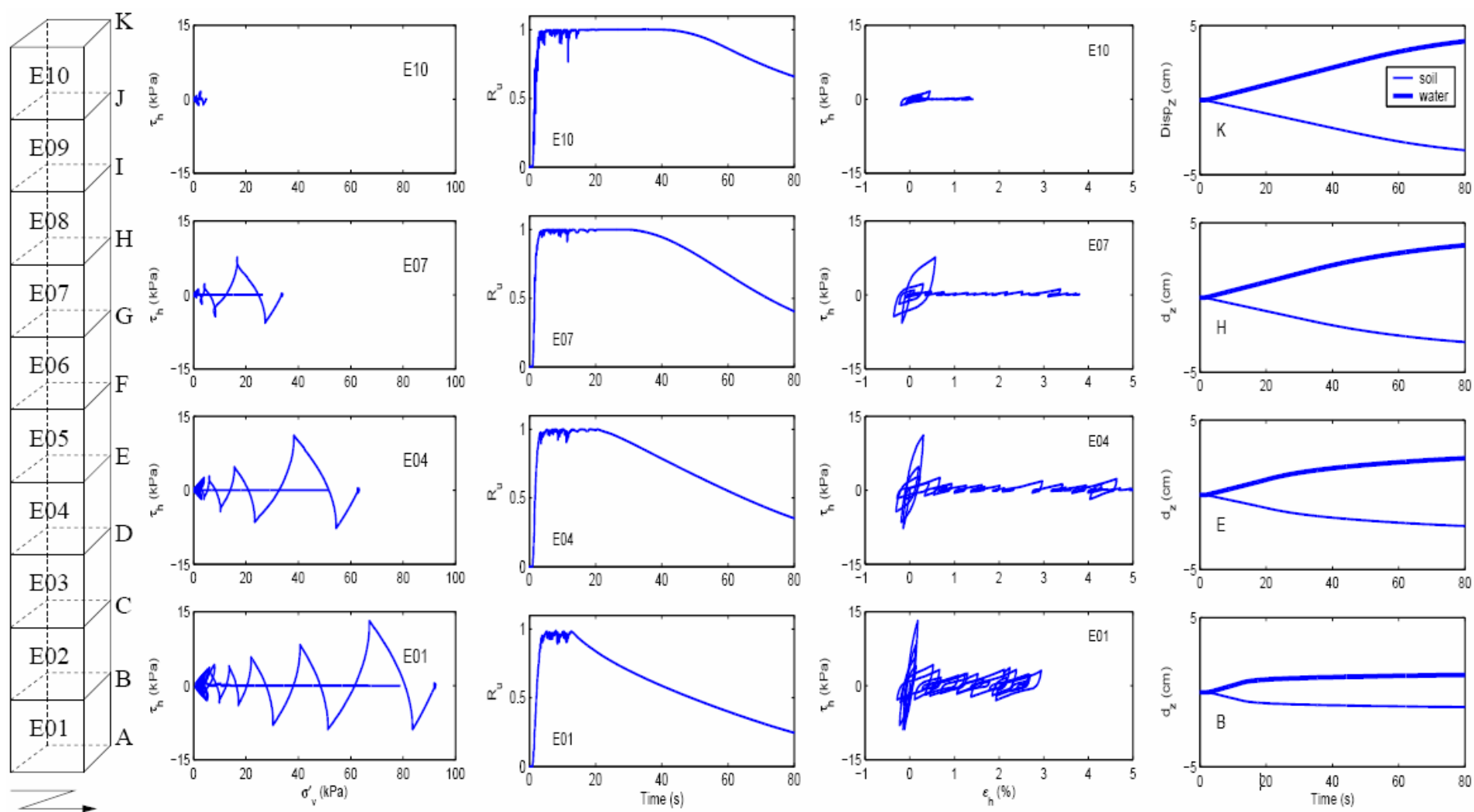


Figure 1. Simulated results of one soil column subjected to earthquake ( $e_0=0.85$ ).

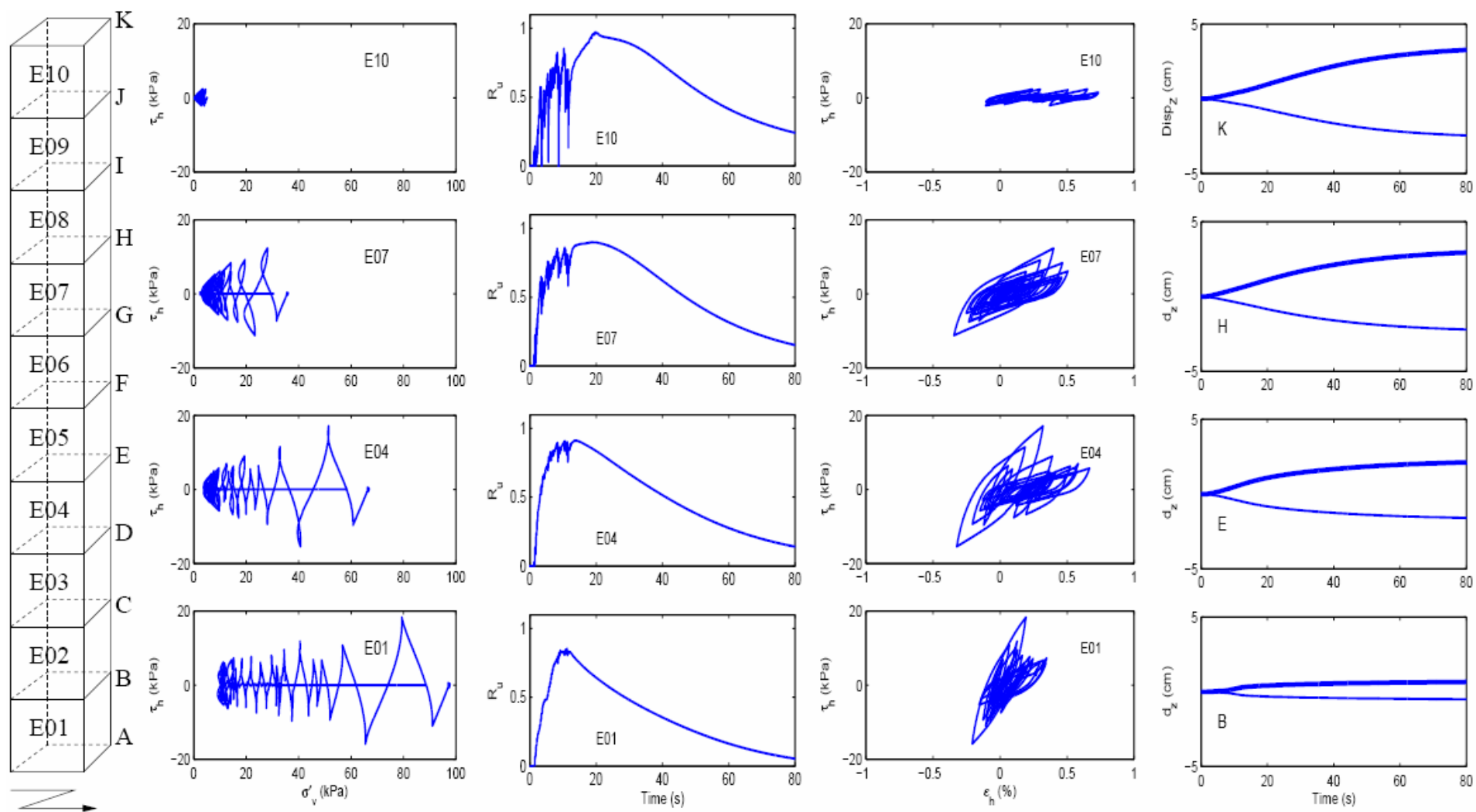


Figure 2. Simulated results of one soil column subjected to earthquake ( $c_0=0.75$ ).



## SUMMARY

A numerical modeling approach is presented accounting for fully coupled (solid-fluid) behavior of soils ( $u$ - $p$ - $U$  formulation), and using a critical state two-surface elasto-plastic model accounting for the fabric dilatancy effects. The formulation and resulting implementation are applied to the problem of sand liquefaction and cyclic mobility phenomena. Results for the seismic behavior of soil columns with liquefaction and cyclic mobility are presented and briefly commented upon.

Presented formulation, discretization and implementation in conjunction with a powerful material model show versatility which we hope will make our approach applicable for practical problems involving seismic behavior of soils and soil-foundation-structure systems.

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