

AN INVESTIGATION INTO THE UNCERTAINTY OF THE ONE DIMENSIONAL SITE RESPONSE ANALYSIS

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ABSTRACT

The amplification of seismic motion due to the presence of overlying soil layers is a well-documented phenomenon. Various tools are used for its analysis, varying from 1D linear elastic or equivalent-linear closed-form solutions to highly sophisticated 2D and 3D finite element codes that model the phenomenon in a more appropriate way. However, even with sophisticated methods, the uncertainties involved are still too many (e.g. material properties, geometry of layers, etc.); therefore the use of simple procedures along with sophisticated ones is often the best way to validate the latter. This in turn means that such simple methods ought to be rendered as robust as possible. Bearing this in mind we focused our research on refining the widely used and well-regarded 'SHAKE' solution through a probabilistic procedure. The Monte Carlo simulation was employed for this purpose. Uncertainty in one or more input parameters (or any combination of them) may be taken into account, in order to obtain a probabilistic estimation of the ground response characteristics through an iterative process. The present work accounts for one variable at a time in a Monte Carlo analysis and yields some interesting results. The research shows that the influence of the uncertainty in the shear wave velocity may be significant. Although no definite 'rule' is derived, some general tendencies emerge.

Keywords: site response analysis; Shake; uncertainty; Monte Carlo

INTRODUCTION

The choice of the appropriate tools for the analysis of a complex phenomenon, such as the amplification of seismic motion due to the presence of softer soil layers overlying the rock, is not straightforward. According to Kramer and Paulsen (2004), practitioners in the USA are keener to use simplified methods such as 'SHAKE', while in Europe more sophisticated, non-linear methods are becoming more and more popular. But in order to achieve a reliable solution with a sophisticated program, more detailed and therefore expensive data is required. Besides, for researchers, who usually tend to reinforce the complexity of their analyses and for whose work time is a less restricting parameter, non-linear finite element codes become the only option. At the same time, while the use of 'SHAKE' can save time and money, because a large number of analyses can be preformed with easily obtainable data and by making use of the large experience accumulated over the 30 years that the program exists, the uncertainties involved are too many. Moreover in a conventional 'SHAKE' analysis there is no capability to quantify uncertainty. However, the aforementioned advantages establish enough reasons for practitioners to continue using 'SHAKE' as their main tool for seismic response analysis. The writers of the present article support the idea that a balanced use of both simple and more sophisticated methods is the best way to achieve reliable estimations, under the appropriate time and cost constraints imposed by engineering practice, which is after all the overall criterion for research. The refinement of simple methods such as the equivalent-linear steady-state solution, known as 'SHAKE' was therefore a major aim for the present work. Monte Carlo (MC) method was employed for the quantification of the uncertainty involved with the input parameters of the analysis.

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Monte Carlo method can be defined as a numerical modeling procedure that makes use of random numbers to simulate processes that involve an element of chance; a particular experiment is repeated many times with different randomly determined data that follow a pre-assumed distribution to allow statistical conclusions to be drawn. In the case of ‘SHAKE’, a specific level of uncertainty is assumed for some of the input parameters (e.g. the shear wave velocity of each soil layer) and it is ‘transferred’ through the ‘SHAKE’ algorithm to the output. The results of the response analysis can then be analyzed statistically and thus achieve a quantification of the uncertainty involved with the acceleration, velocity or displacement spectra (or any other output) of the free surface motion.

SETTING UP THE INVESTIGATION

The program used for the investigation

For the scope of our investigation a modern version of the well known ‘SHAKE’ program was used. The computer program ‘EERA’ (Equivalent-linear Earthquake Response Analysis, by Bardet et al, 2000) is a program based on the original ‘SHAKE’ (Schnabel et al, 1972) and its modified version ‘SHAKE91’ (Idriss et al, 1992). EERA is written in FORTRAN 90 and it is fully integrated in MS Excel. This proved useful for the implementation of the Monte Carlo iterations through the use of a simple Visual Basic macro and for the presentation of the results in an easily visualized manner.

The incorporation of Monte Carlo simulation in the EERA program

The VB macro works as follows: Through a random number generation procedure those parameters of the program’s input that are assumed to be random variables assume a value; the macro does not change the values of the parameters that have been set to be fixed. Hence, after the generation of one set of random input parameters, EERA (which is a deterministic transfer algorithm) is run and an output is produced. The same procedure is repeated as many times as is required by the user. After each iteration the data for the peak ground acceleration (PGA), the relative velocity (PGV) and the relative displacement (PGD) along with their respective response spectra and Fourier spectra are accumulated into new spreadsheets and are ready for statistical interpretation.

The random number generation

In the present work the uncertainty of the shear wave velocity is of interest. In general, it is accepted with confidence that this parameter follows a Normal (Gaussian) distribution. The most basic method for generating values that have a normal distribution, among others (e.g. Box-Muller transform, ziggurat algorithm), is to invert the standard normal cumulative function (cdf). MS Excel gives the invert of any normal distribution with the function:

NORMINV(probability,mean,standard_deviation)

with ‘probability’ being any number in the space (0,1). For random generation of the ‘probability’, MS Excel uses the function RAND(), which is a reliable algorithm for the generation of pseudo-random numbers.

The management of the output

The output parameters produced with the MC iterations are not deterministic values anymore, but they should follow a normal distribution; the closeness of the output distribution to the normal is actually the criterion for the efficiency of the number of iterations of the Monte Carlo method. The produced normal distribution has a mean and a standard deviation $N(\mu, \sigma^2)$. A number of other parameters can be assessed as well, the most important of which being the coefficient of variation (COV):

$$COV(\%) = \frac{\sigma}{\mu} \cdot 100\%$$

which is particularly useful as it provides a normalised measure for the variation of the parameter.

The investigation

The random variables involved in the site response problem are primarily the shear wave velocity and soil layer’s depth and probably secondarily the soil’s unit weight and the normalized shear modulus

reduction and material damping curves (e.g. Stokoe et al, 2004, propose new curves with a quantified uncertainty). For a more extensive reference on the estimation of uncertainties in Geotechnical properties see Jones et al (2002). **In the present work only shear wave velocity (S) is taken as the random variable.**

The soil profiles and the earthquake records used

Two published borehole logs (taken from the website of Boore) were used as soil profiles for our investigation. The estimated mean and the one standard deviation lines of the distribution are drawn. In Figure 1 the COV of the shear wave velocity of the bedrock is much larger for the bedrock (reaching 15%) than for the rest of the layers. The average COV does not exceed 7%. However, for practical purposes the average COV might be assumed larger (15-20% may be realistic). It is necessary to notice that shear wave velocity can never be negative and therefore the normal distribution can never be true for this parameter. In Figure 1 the uncertainty distribution of the bedrock velocity is shown skewed, which may be due to this reason. However, one could assume that the mean values are sufficiently large positive numbers to show much effect of this skewness. Therefore, in our analyses the input uncertainty was assumed to be symmetric (not truncated normal distributions).

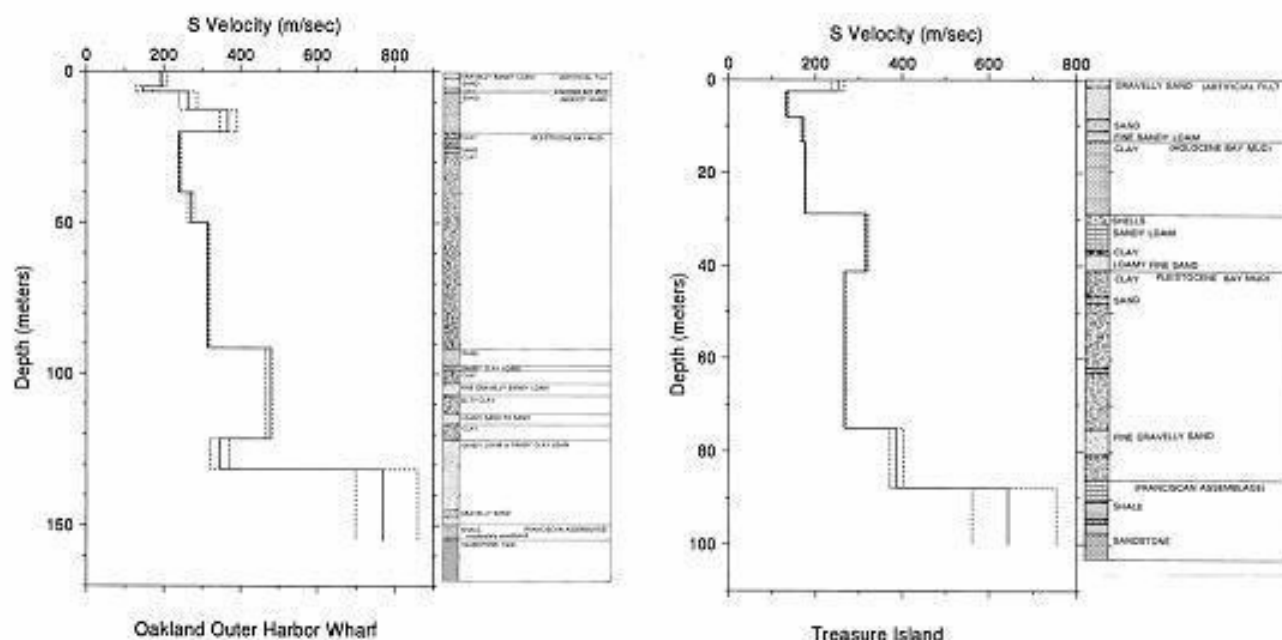


Figure 1. The two boreholes published by Boore, which were used for the analyses. The asymmetry shown in the velocity band of the bedrock was not taken into account in the analysis.

Recorded accelerograms (not necessarily of rock records) were taken from reliable databases (ISESD, PEER). These were applied through EERA to the soil profiles. In the analyses published in this paper the N-S Diamond Heights record from the Loma Prieta earthquake (1989) was used. In Figure 2 this record is shown scaled to 0.10g maximum acceleration.

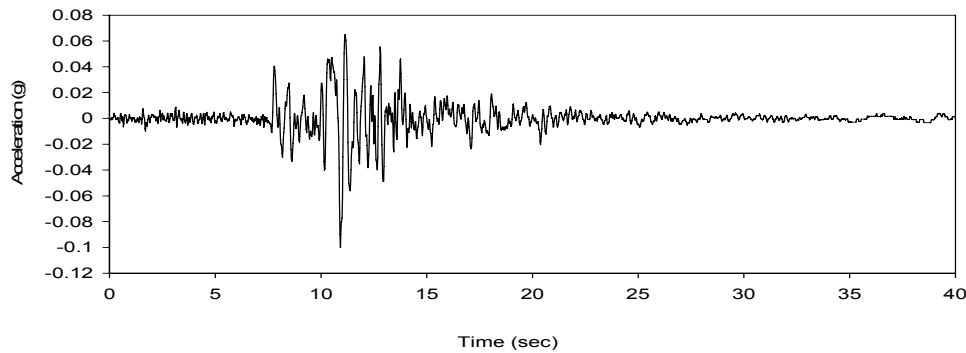


Figure 2. The N-S Diamond Heights record from the Loma Prieta earthquake scaled to 0,1g.

Questions arising during the research and some answers given

The scope of the present work was preliminary. The aim was to give some clear, if possible, answers to basic questions raised in a stochastic treatment of the SHAKE method. The most fundamental of these questions, along with the answers we were able to provide, are the following:

□ What number of iterations is sufficient for a reliable one-variable Monte Carlo analysis?

Analyses of 100, 1,000 and 10,000 iterations showed that even 100 iterations were sufficient. Figure 3 shows that the COV of the acceleration spectra of the three analyses practically coincide at all periods. It can be shown also that there is a good match between the distributions for each period and the respective normal distribution, even for the 100 iterations.

Note that for the aims of this presentation the graphs between COV and the period were chosen as more suitable for the illustration of the effects of uncertainty in the whole spectral range. These plots were preferred to the $S_a - T$ plots, as the lines required for the description of the influence of uncertainty are reduced from three (the mean and the standard deviation) to one.

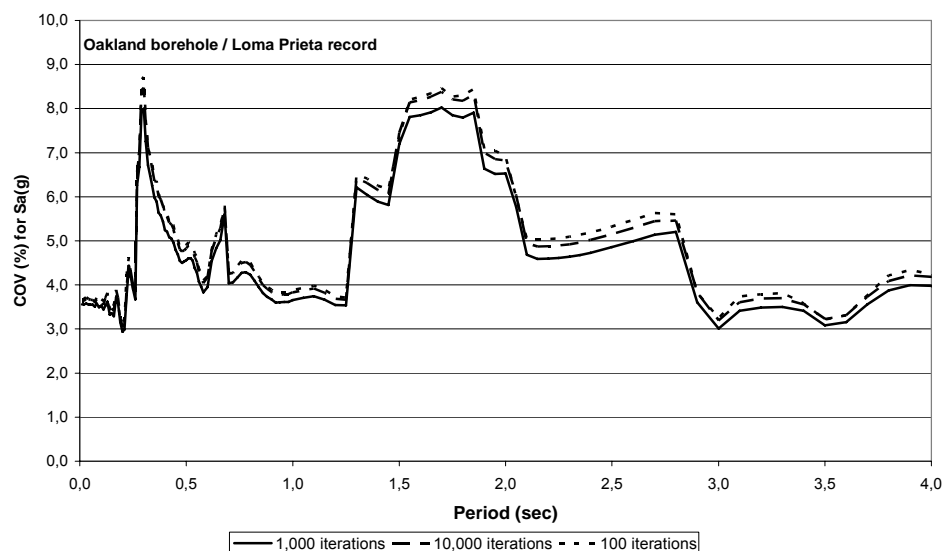


Figure 3. The COV of the acceleration spectra practically does not differ for 100, 1000 and 10000 iterations.

□ Does the mean (μ) of the normal distribution of any output parameter coincide with the respective output of a conventional deterministic solution that uses as input parameters mean estimate values?

Only one out of three analyses showed that the deterministic solution lied at least within the limits of $\mu \pm \sigma$. In the other two analyses this was not the case, as seen in Figure 4. This means that although the input uncertainty was symmetric, as stated earlier, the output showed an asymmetry between the

deterministic and the mean in the whole spectral range. This is an interesting finding whose explanation may not be straightforward.

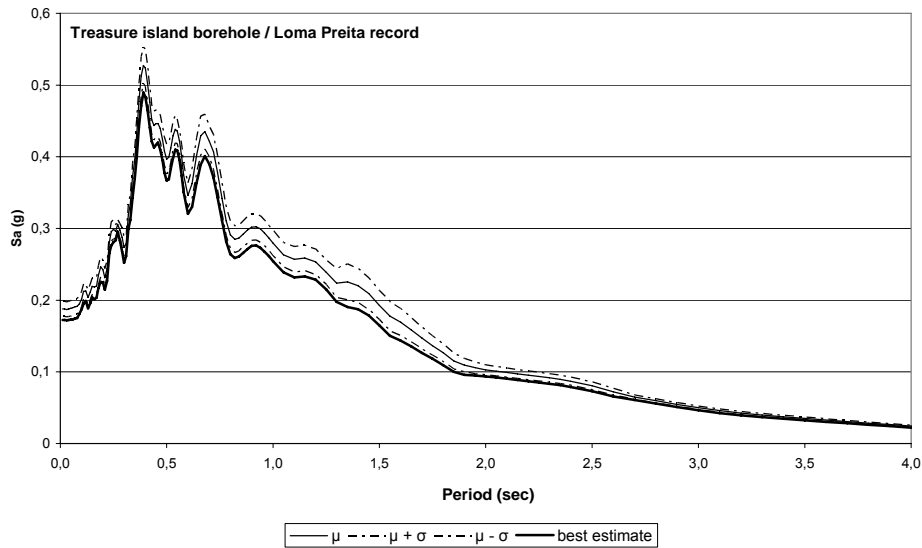


Figure 4. The deterministic solution lies well out of the $\mu \pm \sigma$ range of the respective MC solution.

- ☐ Could we simply assume as input a uniform COV for the whole soil column and attain similar uncertainty patterns for the output as if the input COV of each layer were taken separately?

According to Figure 5, only for some periods of the acceleration spectrum is there some similarity in the patterns. This means that especially for cases where uncertainty is concentrated to a certain layer (e.g. the bedrock) this simplification could lead to wrong conclusions.

However, despite the large variation shown (already for a small range of input COV's), one cannot yet eliminate the possibility that for practical purposes this simplification may be valid. Further analyses (with larger COV values – 10% to 20%) are required in order to lead to 'final' and useful conclusions.

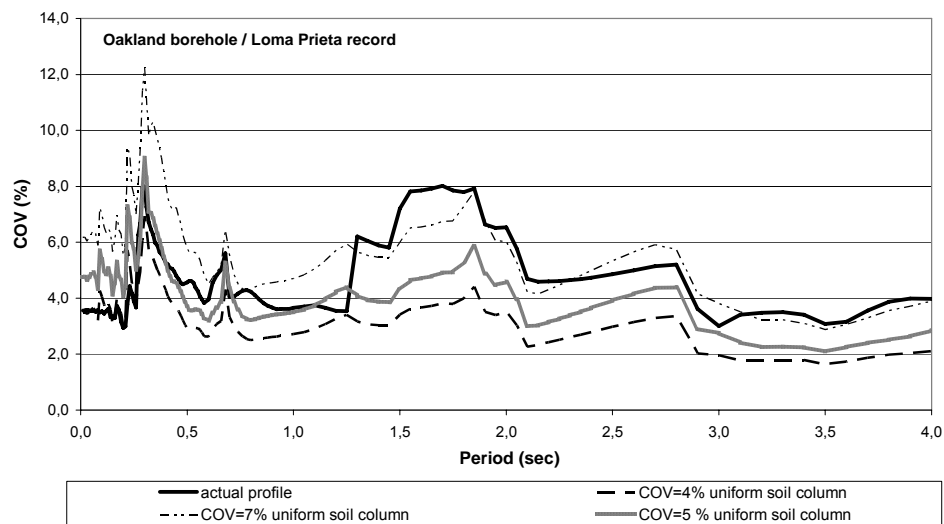


Figure 5. Comparison of the COV of the acceleration spectra when the shear wave velocities of the whole soil column is assumed uniform with COV=4, 5, 7% and when each layer has its actual properties.

- ☐ If there is a specific soil layer in a profile (e.g. the bedrock) that has a dominant uncertainty compared to the other layers (COV of bedrock \gg COV of the rest of the layers), then could the

shear wave velocity of that single layer be assumed to be the only variable of the problem, while the velocities of the rest of the layers are taken as fixed?

Figure 6 shows the COV of the acceleration spectra produced from an analysis with all the layers to have uncertain shear wave velocities and an analysis with the velocity of the bedrock as the only variable. For periods of 0.6sec and up, the hypothesis in this case is true. For periods shorter than 0.6sec there are large differences (for $T=0,25\text{sec}$ $\text{COV}_1 \approx 4 \cdot \text{COV}_2$). This means that this hypothesis cannot always be used with confidence as a simplification.

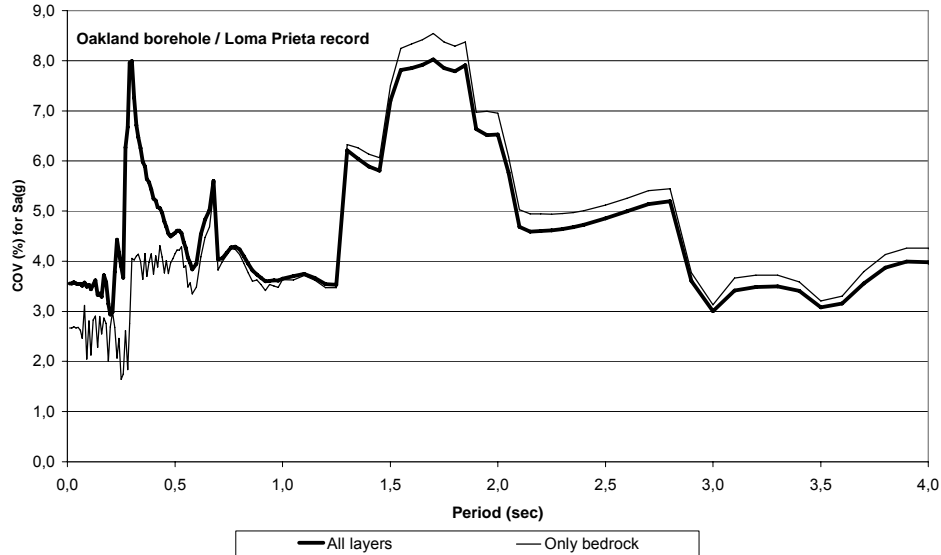


Figure 6. Comparison of the COV of the acceleration spectra for two cases: In the first the uncertainty for the whole soil column is taken into account; in the second only the uncertainty of the bedrock (which is dominant in the profile) is taken as variable.

- ☐ If the same accelerogram is scaled to various maximum accelerations and is applied through EERA to the same site (with a given input uncertainty), would the uncertainty of the input parameters be scaled in a predictable way?

The analyses did not support such a hypothesis. According to Figure 7, the COV e.g. for PGA varies largely between 0.10g, 0.25g and 0.50g maximum accelerations ($\text{COV}_{\text{PGA}} \approx 3,5, 7,5$ and 10% respectively). For the acceleration spectrum the shape of COV does not follow a standard rule and there is no standard relationship between the spectral COV of the three cases. Thus, although for increasing maximum input acceleration the $S_a(g)$ is scaled accordingly (although not linearly), the COV does not show such a certain tendency.

Therefore, we may assume that the influence of the input uncertainty of the soil profile plays a more significant role on the output uncertainty than the level of input motion. The validity and the limits of such a hypothesis and the combined effect of the input excitation and the soil profile have to be investigated in future work.

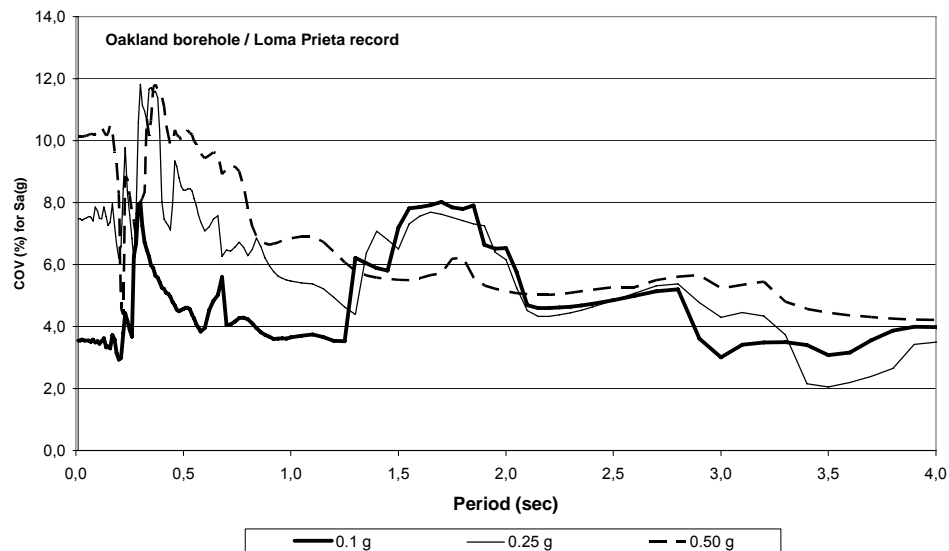


Figure 7. Comparison of the COV of the acceleration spectrums when the same record is scaled to different maxima is applied to the same profile.

- Finally, a most important question is this: even if we manage to determine a distribution of probability for the spectral ordinates of a specific problem through the incorporation of MC analysis into the EERA program, how close to reality would our solution be?

For this reason a number of analyses were performed in order to make comparisons with published data of Idriss (1990), based on SHAKE analyses for the soft soil site of Treasure Island after the Loma Prieta (1989) earthquake. Figure 8 shows that the Monte Carlo $\mu \pm \sigma$ range is very well correlated to the Idriss range, which came from a number of deterministic analyses (parametric study). It can be noticed that both predictions are much lower (10-30%) than the recorded spectrum, shown in figure 10 as well. For the larger periods ($T > 1.5$ sec) the bad matching of the plots can be attributed to the strain softening due to liquefaction, which cannot be accommodated in a SHAKE type analysis.

Another paper by Hryciw et al (1990) based on respective data, shows that 'SHAKE' can reach the recorded values if the uncertainty of the soil layer is also taken into account.

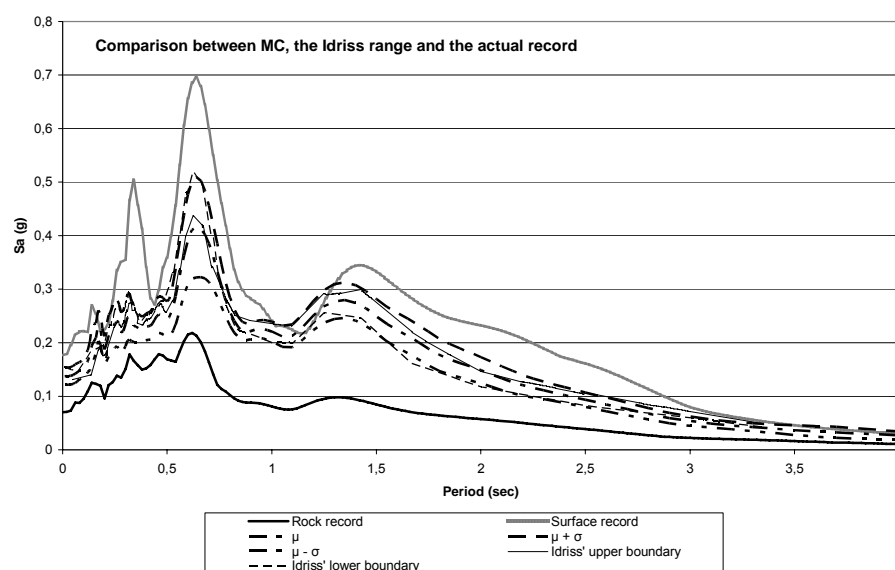


Figure 8. Comparison between the MC $\mu \pm \sigma$ range, the Idriss range and the actual recordings.

CONCLUSIONS

The paper presents the procedures and the most characteristic results of some preliminary work done within the scope of an MSc thesis. Nevertheless, although limited to one variable analyses, the Monte Carlo method was proved useful in refining the 'SHAKE' approach for the problem of the seismic response of soil layers. A number of simple questions were raised, namely about the required number of iterations for a reliable Monte Carlo simulation and the sensitivity of the uncertainty patterns of the output due to the variation of the uncertainty patterns on the input. Some of the findings may be interesting for future research and probably provide useful information for practical purposes. In general, the lack of reference literature rendered the scope of this work preliminary and the conclusions limited. However, the method and the investigation may be enriched; two or more variable analyses are necessary in order to extend this approach. Nevertheless, keeping always in mind the restrictions of the equivalent linear method, the results should in any case be checked with other, more sophisticated solutions.

ACKNOWLEDGEMENTS

The writers are grateful to Professor J.P. Bardet of USC who politely gave the necessary codes of EERA, with which we were able to create the VB Macro for our analyses.

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Strong Motion Databases:

European Strong Motion Database: <http://www.isesd.cv.ic.ac.uk>

PEER Strong Motion Database: <http://peer.berkeley.edu/smdat/search.html>