

## THE EFFECT OF INHOMOGENEITIES ON THE DYNAMIC RESPONSE OF LAYERED SOIL WITH VARIABLE DAMPING

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### ABSTRACT

This work investigates the effect of inhomogeneities in layered soil with frequency-dependent damping on the kinematic fields that develop at the free-surface. More specifically, inhomogeneities are understood in the form of (a) discontinuities such as tunnels and solid bodies and (b) the presence of a variable shear modulus. The standard multiple-degree-of-freedom (MDOF) model of structural dynamics is used herein to represent the scalar problem of horizontally polarized shear (SH) waves moving vertically through a soil column. In order to handle frequency-dependent damping, the complex frequency response matrix of the MDOF system is first defined in the Fourier domain and then numerically transformed in the time domain. The resulting matrix-equivalent of the impulse response function is finally convoluted as a Duhamel integral with synthetic accelerograms that filter the white noise, power spectrum density function through a triple filter representing bedrock. It thus becomes possible to recover both displacement and acceleration time histories at the free surface of a multi-layered deposit that may or may not contain discontinuities. Finally, a series of parametric studies clearly shows the differences between the kinematic fields that develop in media that are inhomogeneous, as compared to the reference homogeneous background.

Keywords: Free-field motion, discontinuities, layered media, specific barrier model, shear waves

### INTRODUCTION

Wave motion in geological media presents certain peculiarities that have to do with the inherent difficulty of providing an accurate description of the underlying soil and rock formations. If the wave motion is earthquake-induced, additional complications arise in the description of the fault area and the transmission of seismic signals up to bedrock [Kausel and Manolis, 2000]. As a result, many specialized methods for analyzing seismically-induced ground motions have been devised, classified as analytical, numerical and hybrid [Helbig, 1994].

Seismic motions carry substantial amounts of energy that are absorbed by both buried infrastructure and conventional above-ground structures. In the former case, the presence underground construction such as tunnels, pipelines, etc., modifies the seismic signal that would register the free surface of the ground in the absence of such structures. This is one of the parameters of the phenomenon of soil-structure-interaction (SSI), which comprises (a) kinematic interaction, (b) inertial interaction and (c) structural response [Johnson, 1981]. Broadly speaking, SSI is understood within the context of above-ground structures, whose foundation system modifies the kinematic fields that develop in the immediate soil region. Furthermore, SSI is not considered to affect buried structures in any significant way, provided they can conform to the localized motions induced by the earthquake. In this work, we

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investigate the modification of the seismic signal at the ground's free surface due to the presence of infrastructure with shallow burial. Two basic parameters are introduced, in addition to the depth of burial and type of tunnel, namely inhomogeneous distribution of mass and stiffness in the deposit plus frequency-dependent material damping. Additional effects that will be added in future work are the presence of buried geological cracks, either alone or interacting, and of surface topography [Dineva et al., 2004; 2006]

The long-term aim of this work is to produce software for the construction of artificial seismic motions (displacements, velocities, accelerations) based on spectra that go beyond the usual 'seismic waves on bedrock model' and encompass correction factors accounting for (a) soil layering, (b) surfacetopography, (c) cavities, (d) tunnels, (e) geological cracks, (f) material inhomogeneity and finally (g) material anisotropy, for the basic cases of incoming body (i.e., pressure and shear) waves. All these effects will be summarized in frequency dependent, dimensionless functions constructed through comparisons with the equivalent homogeneous case. The resulting free-surface motions can then be used as input in routine structural analysis. At present, some of this information can be recovered by accessing the internet site <http://infoseismo.civil.auth.gr/>.

## PROBLEM STATEMENT

In here, we present the well-known shear-column model for layered soil with variable (i.e., frequency-dependent) damping under seismically-induced, horizontally polarized shear (SH) waves for the purpose of computing ground acceleration time histories (see Fig. 1). Furthermore, the presence of a shallow buried structure in the form of either an unlined tunnel (modeled as a hole that weakens the layer) or a lined one (modeled as a disc that reinforces the layer) is also accounted for, albeit in an approximate way. The input is a synthetic acceleration / displacement / velocity at bedrock, as produced by the specific barrier model [Papageorgiou and Aki, 1983]. The next step is to introduce overburden in the form of layered soil, which is modelled as an MDOF system, whereby each individual oscillator represents a single layer. Given that all DOF correspond to horizontal motions, this model describes SH wave propagation through layers of constant thickness, parallel to the free surface of the ground (see [Hadjian, 2002] for a practical method of estimating the equivalence of thin soil layers to a single deposit for SH wave motion). Also, soil properties for near-surface layers can be inferred from comparing vertical to horizontal spectra resulting from micro-tremor recordings at the surface of a soil deposit, known as the quasi-transfer spectra technique [Carniel et al., 2006].

### Earthquake Source Mechanism

Barrier-type models are used to simulate ground motions emanating from a fault system and propagating through the lithosphere to the surface. These models incorporate the simulation of fault geometry and the relation between seismic moment and stress drop to estimate the number of cracks appearing in the fault. The simulated peak ground accelerations (PGA) are then calibrated against strong motion data. Since the variability in the simulated motions is generally less than that manifested in recorded motions, randomness is added to key parameters of these models. Regarding the specific barrier model, [Papageorgiou and Aki, 1983] constructed acceleration-source spectra that show good agreement with recorded data, except at high frequencies where spectral amplitudes may be over-predicted.

Regarding the present implementation, three filters are used in sequence to model the passage of fault-generated seismic motion through the upper lithosphere. In sum, white noise emanating from the earthquake fault source is filtered through a Kanai-Tajimi (KT) filter and a high-pass (HP) filter across the lithosphere, and then brought up to rock through a low-pass (LP) filter within a fixed frequency band of  $0.2 - 20.0 \text{ Hz}$ . The mechanical interpretation of these filters is three single-degree-of freedom (SDOF) oscillators in series, all of them highly damped, whose natural frequencies are strategically placed at the low, middle and high end of the aforementioned frequency spectrum. No

interaction phenomena between the incoming wave and the structure of the upper soil deposits (e.g., layering, cavities, geological cracks, the presence of the free surface and of surface topography) are taken into account, and neither is the possible inhomogeneous structure of the said deposits considered in any way. A typical range of parameters entering the model is taken from [Papageorgiou and Aki, 1983] and listed in Table 1, where PSDF is the power spectral density function.

**Table 1. Input parameters for synthetic ground motion generation by the specific barrier model**

Magnitude $M = 6.0$ Event						
<i>PSDF</i>	<i>KT-filter</i>	<i>KT-filter</i>	<i>HP-filter</i>	<i>HP-filter</i>	<i>LP-filter</i>	<i>LP-filter</i>
$S_0 (cm^2 / s^3)$	$\omega_g (Hz)$	$\xi_g$	$\omega_f (Hz)$	$\xi_f$	$\omega_m (Hz)$	$\xi_m$
150.0	5.0	0.7	0.8	1.0	10.0	1.4

### The Layered Half-plane

If the half-plane is homogeneous, there are some simple relations for computing its dynamic characteristics. Starting with a deposit height of  $h (m)$ , a unit weight  $\gamma (t / m^3)$ , a shear wave speed  $c_s (m / s)$  and a damping ratio  $\xi (\%)$ , the fundamental period and frequency, as well as the mass, stiffness and damping coefficients are respectively given below as:

$$T = 4h / c_s (s), \quad \omega = 2\pi / T (rad / s), \quad (1)$$

$$M = \gamma h l d (t), \quad K = m\omega^2 (kN / m), \quad C = 2m\omega\xi (kNs / m)$$

where  $l, d$  are (unit) lateral dimensions. The presence of discrete layers requires modeling of each layer by a one horizontal DOF, and the entire deposit is built by superimposing layers, thus giving rise to a MDOF dynamic model. This in turn is solved by standard methods of structural dynamics [Clough and Penzien, 1993]. In particular, we follow an eigenvalue analysis, and solve for each modal DOF separately using Duhamel's integral that employs the unit impulse response function  $h(t)$ . Next, Duhamel's integral is convoluted with the input signal  $f(t)$  and numerically integrated over time  $t$  to yield the response  $x(t)$ , as shown below

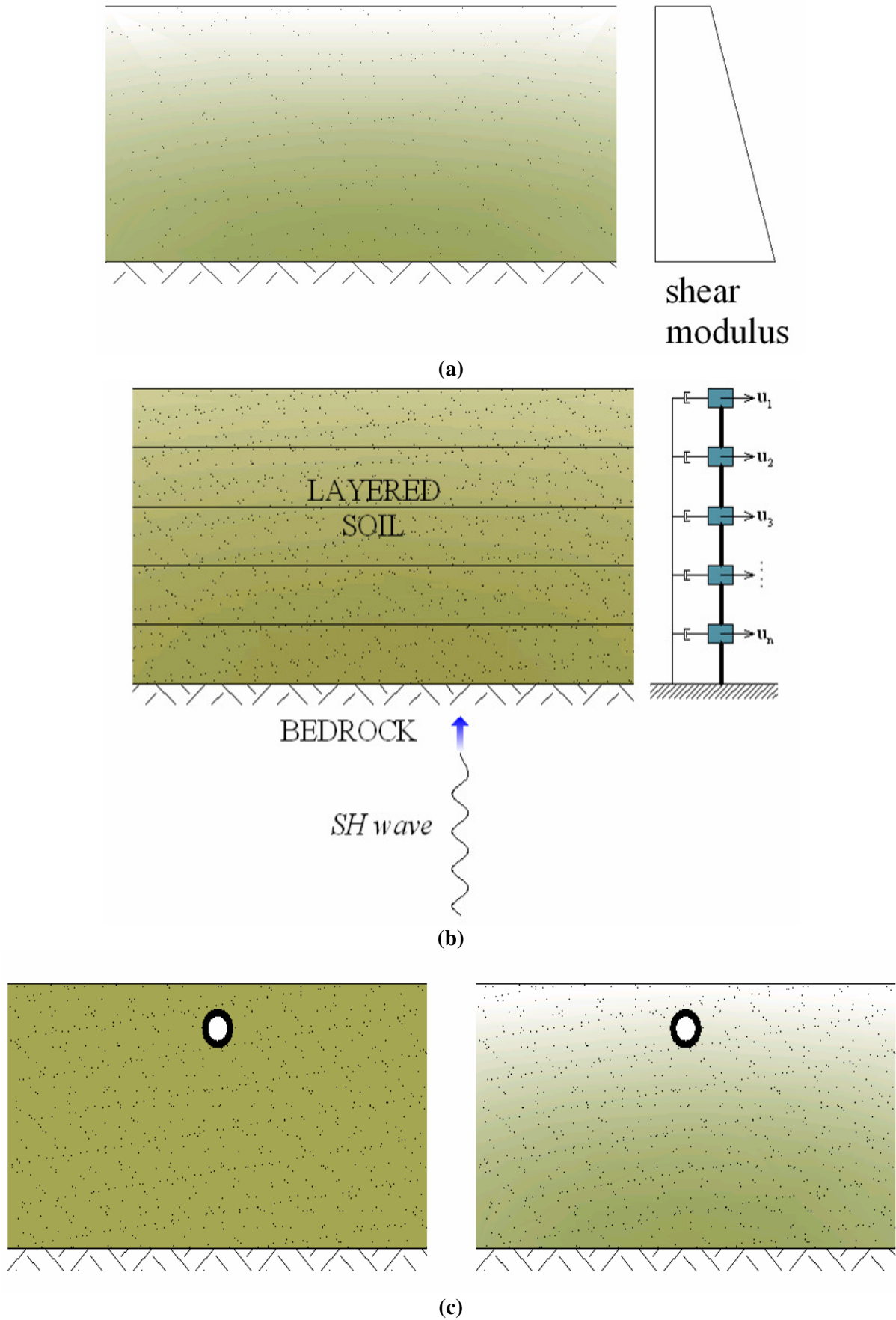
$$x(t) = \int_0^t h(t-\tau) f(\tau) d\tau = h(t) \otimes f(t) \quad (2)$$

The response in the original physical coordinates is then reconstituted from the modal coordinate response through use of the modal matrix containing the system eigenvectors.

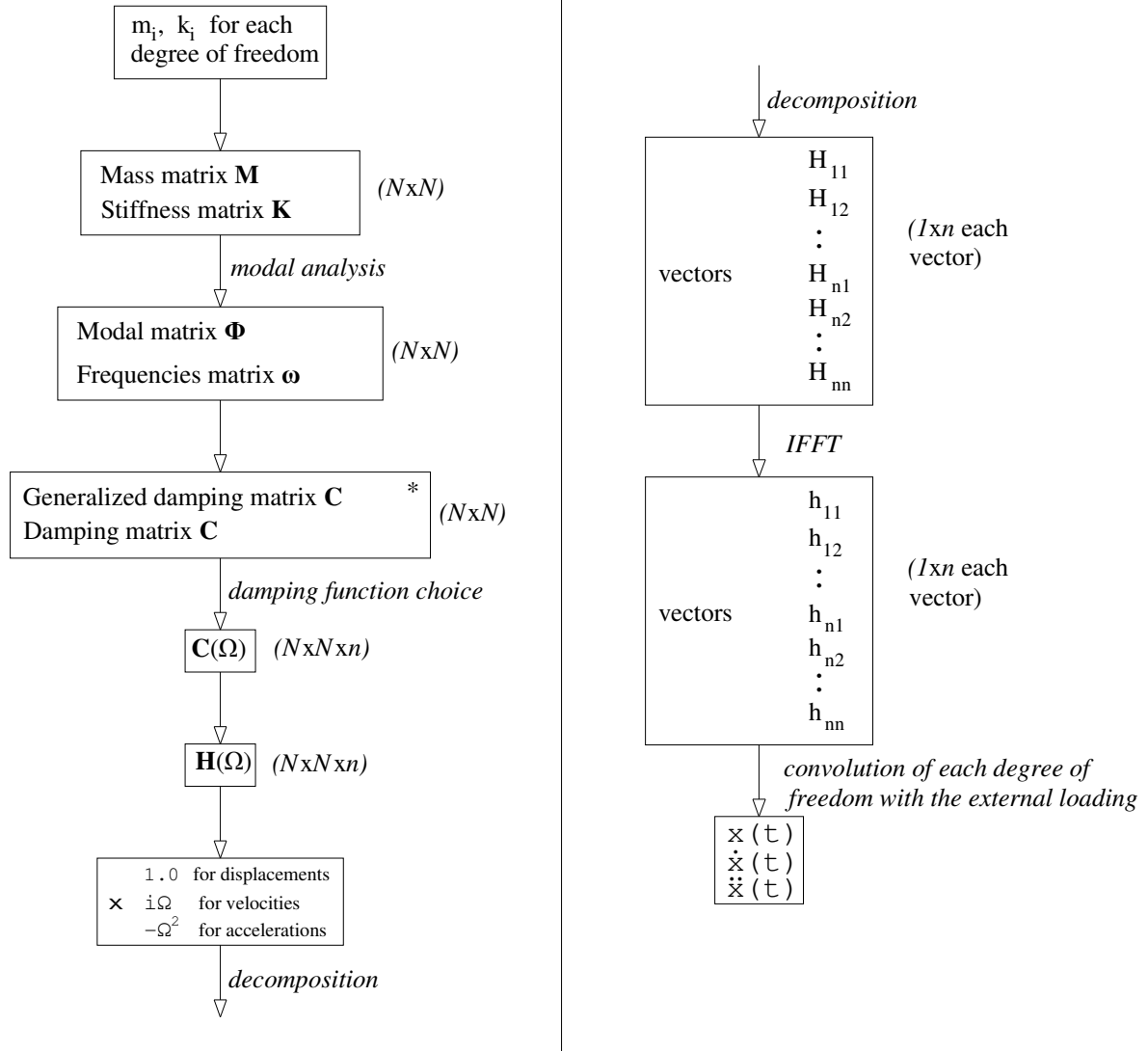
In the presence of frequency-dependent damping, which is perhaps the best way to capture viscoelastic response in the layers, solution for the MDOF system takes place in the Fourier transformed (FT) domain so as to take advantage of the correspondence principle [Flugge, 1975]. The unit impulse response function  $h(t)$  for each DOF is then recovered from the corresponding complex frequency response function

$$H(\Omega) = 1 / \left[ (K - M\Omega^2) + iC(\Omega)\Omega \right] \quad (3)$$

through use of the inverse Fourier transformation (or more precisely, through the fast Fourier transform (FFT)), where  $\Omega$  is the frequency spectrum. The entire approach is summarized in the flowchart of Fig. 2. We also note here that all computations were performed using the Matlab [2004] software package.



**Figure 1. (a) Inhomogeneous 2D soil deposit modeled as a (b) layered medium and (c) including a near-surface discontinuity under seismically-induced SH waves**



**Figure 2. Flow chart depicting the formulation and numerical implementation process**

### The Presence of Discontinuities

As previously mentioned, two types of discontinuities in the top layers close to the surface are taken into account, as shown in Fig. 1(c). First, we have the unlined circular cylindrical tunnel that weakens the layer, a process that results in a reduction of the layer mass to 10% of its original value and gives a terminal stiffness of  $K = 1000 \text{ kN/m}^2$ . Next, the presence of a lined tunnel also leads to the same mass reduction, but the layer stiffness is recomputed using the concrete liner's shear wave speed of  $c_s = 1730 \text{ m/s}$ . These approximate considerations were formulated following extensive parametric studies with 2D finite element simulations of layers containing discontinuities using the commercial program Ansys [2004].

## NUMERICAL RESULTS

In the tables that follow, we summarize the results of an extensive parametric study of a soil deposit of unit thickness (see Fig. 1 again) with the following basic properties:

$$h = 210\text{ m}, \gamma = 2.0\text{ t/m}^3, \nu = 0.3, \xi = 7\% \quad (4)$$

Regarding the variable viscous damping function, we have the basic form

$$C(\Omega) = C_0(a\Omega + b)^n \quad (5)$$

in the frequency range  $0 \leq \Omega \leq 40\pi$ , plus a constant value past  $\Omega > 40\pi$ , where  $C_0$  is the dashpot coefficient corresponding to a fixed viscous damping ratio  $\xi$ . For a viscoelastic model whose damping values increase with increasing frequency,  $a = 26.54, b = 1, n = 0.5$ , while one that shows a decrease,  $a = 0.024, b = 1, n = -0.5$ . As previously mentioned, input to the system is the synthetic acceleration time history of Fig. 3 produced by the specific barrier model.

### The Effect of Layering

The SDOF model cannot in theory be used for inhomogeneous deposits, since a number of horizontal layers are required to capture variable stiffness. On the other hand, should the soil layer turn out to be homogeneous, the use of multiple layers of equal thickness skews the final results. This is so because the equivalent layer stiffness  $K$  is inversely proportional to the third power of the layer thickness, which implies a ‘stiff’ model compared to the correct one for many layers. In particular, Table 2 depicts the percent error (in absolute value terms) when a homogeneous layer with wave speed  $c_s = 300\text{ m/s}$  is modeled with three and subsequently with ten equal thickness layers (roughly 10% and 50%), for both free-surface accelerations and displacements. The use of ten variable thickness layers (where layer thickness increases gradually as we move downwards) improves the situation (to about 35%). Next, Table 3 is for the same deposit as before, only this time the wave speed is variable and increases linearly with depth from 300–900 m/s. Using the three DOF model as reference, we again observe the effect of using more layers than necessary in the modeling effort.

**Table 2. Maximum displacement and acceleration values for a deposit with fixed  $c_s = 300\text{ m/sec}$  and constant damping**

Displacement (m)			Acceleration ( $\text{m/s}^2$ )		
Model	Value	Deviation	Model	Value	Deviation
SDOF	0.032	-	SDOF	1.809	-
3DOF	0.035	9.38%	3DOF	2.155	19.13%
10DOF (equal h)	0.047	46.88%	10DOF (equal h)	2.559	41.46%
10DOF (variable h)	0.043	34.06%	10DOF (variable h)	2.418	33.68%

**Table 3. Maximum displacement and acceleration values for a deposit with variable  $c_s = 300 - 900\text{ m/sec}$  and constant damping**

Displacement (m)			Acceleration ( $\text{m/s}^2$ )		
Model	Value	Deviation	Model	Value	Deviation
3DOF	0.049	-	3DOF	2.616	-
10DOF (equal h)	0.031	36.33%	10DOF (equal h)	4.361	66.73%
10DOF (variable h)	0.034	30.20%	10DOF (variable h)	4.035	54.25%

### The Effect of Variable Damping

At first, the filtering effect of the soil is depicted in Fig. 4, where both free surface displacements and accelerations have been plotted for a constant velocity layer exhibiting three types of damping, namely constant, increasing and decreasing with frequency (see Eq. (5)), and the input are the synthetic accelerations of Fig. 3. Furthermore, Tables 4 and 5 give maximum values for these two time histories, the former pertaining to the reference homogeneous layer and the latter to the layer with increasing wave velocity. The effect of variable damping is noticeable, given that it is possible to observe overall reductions of up to 12% for the displacements and up to 18% for the accelerations.

**Table 4. Maximum displacement and acceleration values for a deposit with fixed  $c_s = 300$  m/sec and variable damping values**

SDOF					
Displacement (m)			Acceleration (m/s <sup>2</sup> )		
<i>Model</i>	<i>Value</i>	<i>Deviation</i>	<i>Model</i>	<i>Value</i>	<i>Deviation</i>
$C_0$	0.032	-	$C_0$	1.809	-
$C_{INCREASE}$	0.030	5.40%	$C_{INCREASE}$	1.814	0.32%
$C_{DECREASE}$	0.032	0.63%	$C_{DECREASE}$	1.808	0.06%

**Table 5. Maximum displacement and acceleration values for a deposit with variable  $c_s = 300 - 900$  m/sec and variable damping values**

3DOF, equal height layers					
Displacement (m)			Acceleration (m/s <sup>2</sup> )		
<i>Model</i>	<i>Value</i>	<i>Deviation</i>	<i>Model</i>	<i>Value</i>	<i>Deviation</i>
$C_0$	0.049	-	$C_0$	2.616	-
$C_{INCREASE}$	0.045	8.16%	$C_{INCREASE}$	2.502	4.35%
$C_{DECREASE}$	0.050	2.04%	$C_{DECREASE}$	2.676	2.29%
10DOF, equal height layers					
Displacement (m)			Acceleration (m/s <sup>2</sup> )		
<i>Model</i>	<i>Value</i>	<i>Deviation</i>	<i>Model</i>	<i>Value</i>	<i>Deviation</i>
$C_0$	0.031	-	$C_0$	4.361	-
$C_{INCREASE}$	0.028	11.86%	$C_{INCREASE}$	3.794	13.01%
$C_{DECREASE}$	0.033	5.77%	$C_{DECREASE}$	4.703	7.84%

### The Effect of Discontinuities

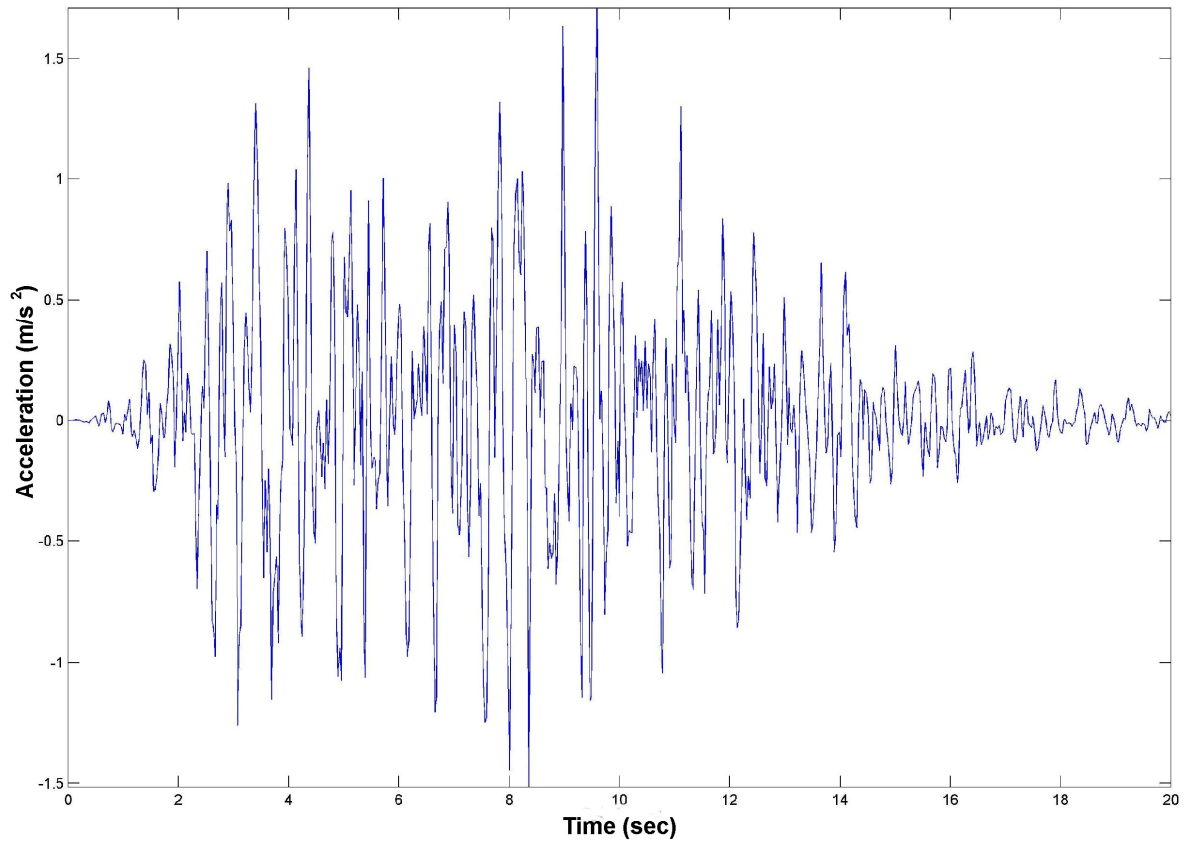
The presence of a discontinuity, either in the form of an unlined or a lined tunnel with shallow burial depth ( $d = 6$  m to the crown), causes an increase in the maximum values of the kinematic fields recovered at the free surface. Specifically, Tables 6 and 7, pertaining to the homogeneous and to the inhomogeneous deposit, respectively, predict changes up to 80% in the displacements and up to 25% in the accelerations for the unlined tunnel. These values drop to 15% and 11%, respectively, when a liner replaces the tunnel, because part of the drop in layer integrity is compensated for by the presence of the relatively stiff liner.

**Table 6. Maximum displacement and acceleration values for a 10DOF deposit with fixed  $c_s = 300$  m/sec, fixed damping and the presence of discontinuities**

Displacement (m)			Acceleration (m/s <sup>2</sup> )		
<i>Model</i>	<i>Value</i>	<i>Deviation</i>	<i>Model</i>	<i>Value</i>	<i>Deviation</i>
Continuous	0.043	-	Continuous	2.418	-
Unlined Tunnel	0.067	56.05%	Unlined Tunnel	2.993	23.79%
Liner	0.049	14.42%	Liner	2.677	10.70%

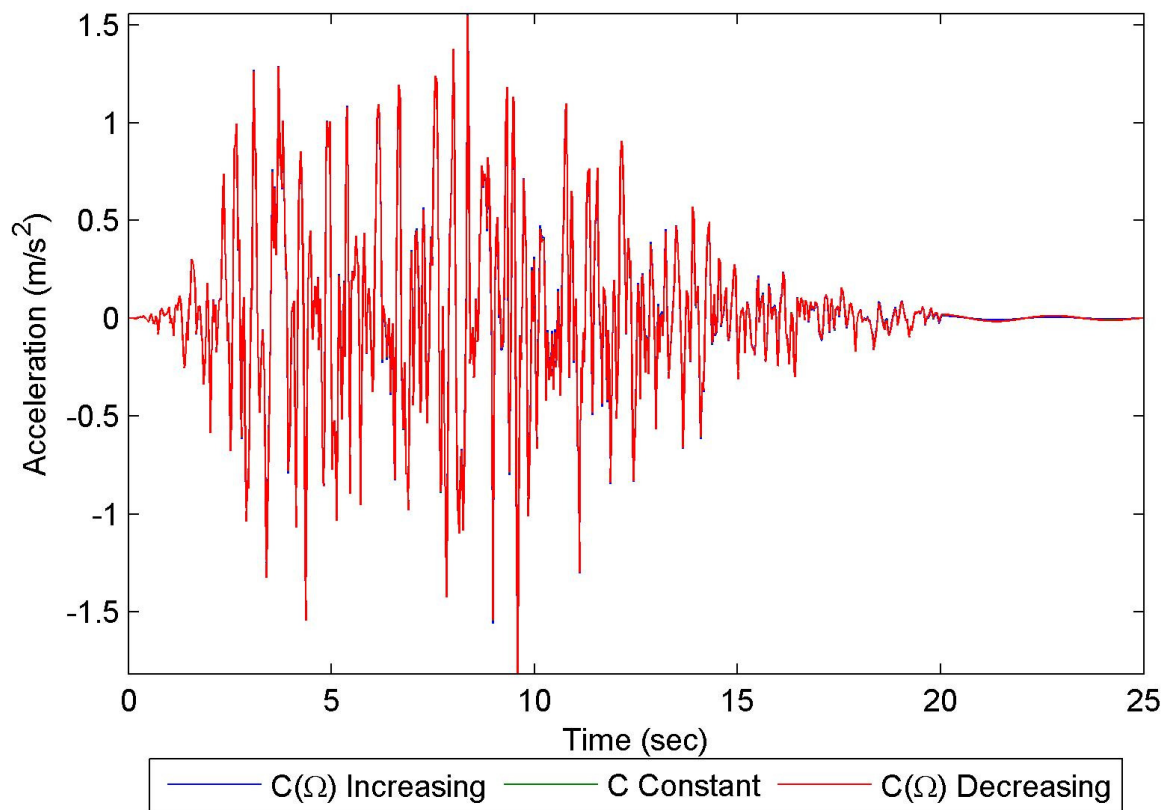
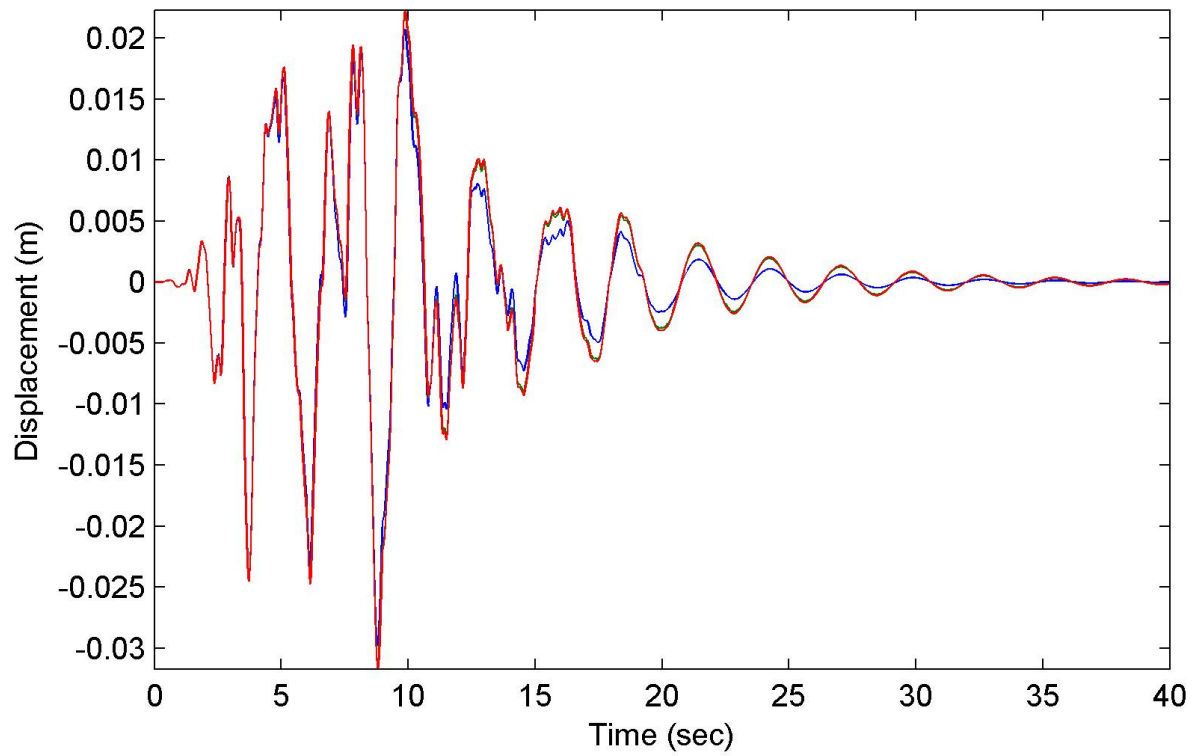
**Table 7. Maximum displacement and acceleration values for a 10DOF deposit with variable  $c_s = 300 - 900 \text{ m/sec}$ , fixed damping and the presence of discontinuities**

Displacement (m)			Acceleration ( $\text{m/s}^2$ )		
<i>Model</i>	<i>Value</i>	<i>Deviation</i>	<i>Model</i>	<i>Value</i>	<i>Deviation</i>
Continuous	0.034	-	Continuous	4.035	-
Unlined Tunnel	0.062	80.41%	Unlined Tunnel	3.866	4.19%
Liner	0.038	9.65%	Liner	4.475	10.91%



**Figure 3. Synthetic ground acceleration history with  $\text{PGA}=0.16\text{g}$  at bedrock**





**Figure 4. Free surface displacements and accelerations for a homogeneous layer with  $c_s = 300\text{m/s}$  for a synthetic ground motion input from the specific barrier model**

## CONCLUSIONS

Free-field motions in the horizontally-layered inhomogeneous half-plane have been computed by the simple shear-column model subject to the following refinements: (a) frequency-dependent viscoelastic damping and (2) layer degradation due to the presence of cavities and tunnels. Input at the bottom of the layered structure consists of synthetic accelerations stemming from fault-induced tremors moving through competent rock, and output is given in the form of filtered accelerations at the horizontal free-surface. The mathematical formulation employs Duhamel's integral for a MDOF model, whereby the synthetic ground motion input is convoluted with the impulse response vector function of the system that includes the effect of variable damping. The impulse response function itself derives from the complex frequency response vector function of the MDOF system in the frequency domain through the use of the inverse FFT. All numerical integrations are carried out using the simple Simpson's rule, which turns out to be quite accurate provided the integration step is kept small compared with the dominant period of the system. A series of numerical results serve to validate the method and to yield data that is useful within the context of soil-structure-interaction studies. Finally, future improvements include the use of dimensionless correction functions that will account for the presence of (1) geological cracks and (2) surface topography in the layered soil structure.

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