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AN APPROXIMATE ESTIMATION OF DYNAMIC INFLUENCE FACTOR IN ELASTIC SEMI-INFINITE GROUND

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ABSTRACT

In seismic analysis of a piled raft foundation using a simplified model where the soil is expressed by springs and dashpots connected to pile and raft nodes, estimation of interaction factor (influence factor) between the soil springs through the soil is needed. However, the Mindlin equation is not valid for dynamic loading and its dynamic equivalent does not exist in closed form. Hence, in this paper, an approximate method for estimating the dynamic influence factor is proposed based on superposition of dynamic Kelvin problem. The proposed method makes use of the analytical solutions for the dynamic Kelvin problem proposed by Matsuoka and Yahata (1980) and the static Kelvin solutions.

Keywords: Elastic semi-infinite ground, Dynamic influence factor, Superposition of Kelvin problem

INTRODUCTION

Simplified methods for analysing deformation of pile foundations have been proposed by Clancy and Randolph (1993), Kitiyodom and Matsumoto (2002, 2003), for examples, where the raft is modelled by thin plates, the piles by beam elements, and the soil is treated as interactive springs connected to pile and raft nodes. In the method proposed by Clancy and Randolph (1993), vertical soil resistance of the raft and the piles alone is considered, because analysis of the foundation structure subjected to vertical loading is of main concern. However, foundation structures are subjected to combined loads of vertical load, horizontal load and overturning moment in usual. Hence, Kitiyodom and Matsumoto (2002, 2003) developed a simplified analytical method (called PRAB) of three-dimensional deformation of a piled raft using a hybrid modelling as shown in Figure 1. In this modelling, two soil springs in the horizontal directions are added at each pile and raft node, so as to allow for horizontal resistance of the piles and the raft. In the above methods, Mindlin solutions (Mindlin 1936) are employed to estimate influence factors between soil springs through the soil. Note that the solutions of the first Mindlin problem where a vertical point force acts at an arbitrary point in the semi-infinite elastic ground are used in the method by Clancy and Randolph (1993), whereas not only the first solutions but also the solutions of the second Mindlin problem where a horizontal point force acts at an arbitrary point in the semi-infinite elastic ground are employed in PRAB.

In order to extend PRAB to accommodate dynamic analysis of a piled raft subjected to dynamic loading or earthquake, inertia of the raft and the piles has to be considered, and a dashpot expressing radiation damping has to be incorporated in parallel with the soil spring (see Figure 1(b)).

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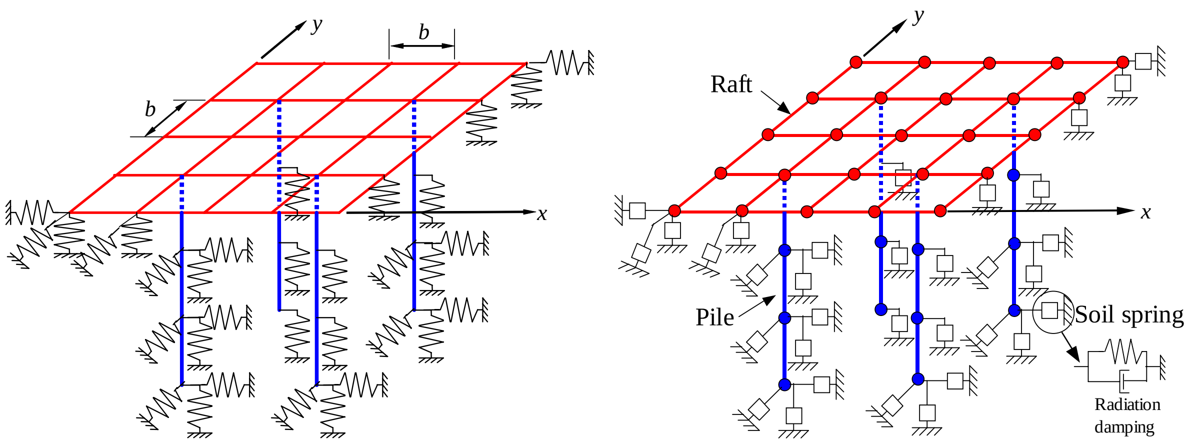


(a) Static analysis (b) Dynamic analysis

Figure 1. Modelling of a piled raft foundation

It is also important to introduce dynamic influence factors between the raft and pile nodes through the soil in the dynamic analysis. It should be noted that the Mindlin equation is not valid for dynamic loading and its dynamic equivalent does not exist in closed form, as suggested by Tabesh and Poulos (2001).

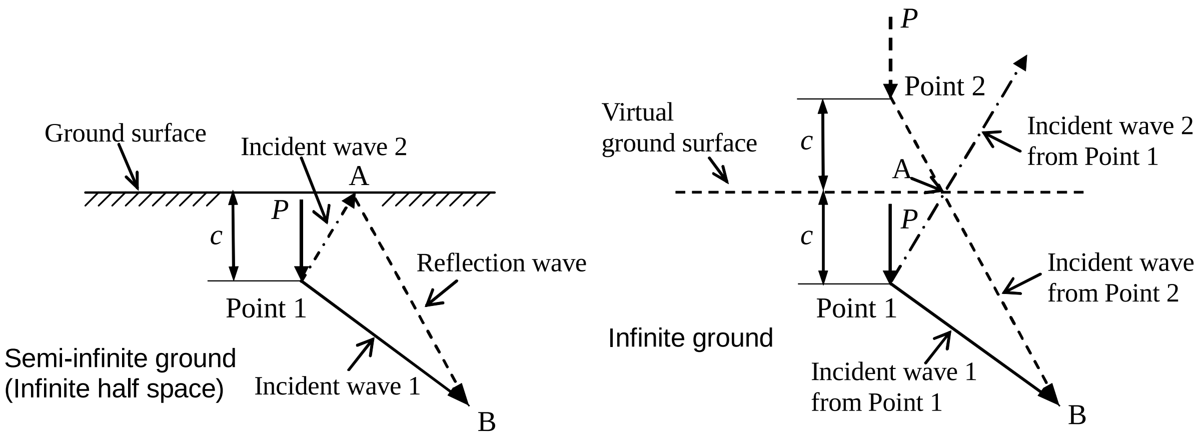
Matsuoka and Yahata (1980) presented analytical solutions for dynamic Kelvin problem where a pulse force acts at an arbitrary point in the infinite elastic ground. The objective of this paper is to propose an approximate method to estimate dynamic influence factors using combination of the solutions for dynamic Kelvin problem and static Kelvin problem along with principle of superposition. The validity of the proposed method is validated through the comparisons with results of dynamic FEM analyses.

REASONS FOR USING SUPERPOSITION OF KELVIN PROBLEM IN ESTIMATION OF DYNAMIC INFLUENCE FACTOR

In this paper, elastic homogeneous semi-infinite space is considered. Let us imagine wave propagation in the semi-infinite ground due to a point force acting in the semi-infinite ground (Figure 2(a)). If a point force P acts at point 1, longitudinal wave and shear wave start to propagate in any direction in the ground. For simplicity, let us consider the behaviour of point B due to longitudinal waves. First, incident wave 1 reaches point B directly from loading point (point 1). Incident wave propagating to the direction of point A is named incident wave 2. Incident wave 2 is reflected at point A and the reflection wave will reach point B, delaying behind the arrival of incident wave 1. Similar wave propagation phenomena occur for shear waves, although wave propagation speeds of longitudinal and shear waves are different. Hence, in the case that step loading is applied at point 1, it is inferred that displacement of point B changes in 4-step manner with time. However, as mentioned, the Mindlin equation is not valid for dynamic loading and its dynamic equivalent does not exist in closed form.

Therefore, the idea of the proposed method is use of superposition of dynamic Kelvin problem as shown in Figure 2(b). Reflection of incident wave 2 does not occur in dynamic Kelvin problem, since infinite ground is assumed. In order to allow for the influence of the reflection wave in an approximate way, another load is applied at point 2 that is located at mirrored position in respect to the ground surface. That is, the influence of the reflection wave is replaced by that of incident wave from point 2.

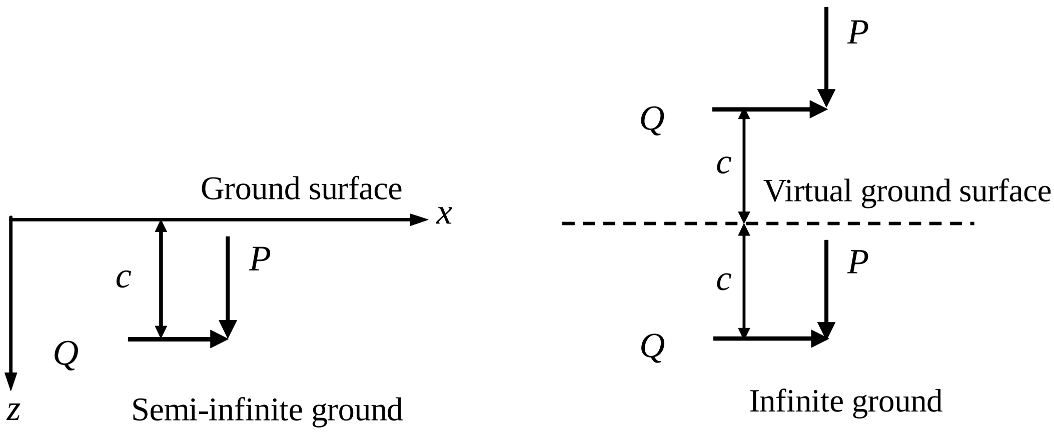
The authors are aware that the proposed method is an approximate method, because the proposed method does not strictly satisfy the boundary conditions at the ground surface. For examples, let us consider the cases where static vertical force, P, or static horizontal force, Q, acts at a point in semi-infinite ground (Figure 3(a)).



(a) Mindlin problem (b) Superposition of Kelvin problem

Figure 2. Wave propagation in the ground due to point force in the ground

In these Mindlin problems, vertical stresses, σ_z, and shear stresses, τ_{zx}, along the ground surface are zero. If the Mindlin problems are approximated by superposition of Kelvin problem as shown in Figure 3(b), $\sigma_z = 0$ is satisfied while $\tau_{zx} = 0$ is not satisfied along the ground surface for the vertical force, P. In contrast, for the horizontal force Q, $\sigma_z = 0$ is not satisfied while $\tau_{zx} = 0$ is satisfied along the ground surface. Nevertheless, superposition of dynamic Kelvin problem is used in the proposed method to estimate dynamic influence factor, because analytical solutions are available for dynamic Kelvin problem.



(a) Mindlin problem (b) Superposition of Kelvin problem to express Mindlin problem

Figure 3. An approximate method for expressing Mindlin problem by superposition of Kelvin problem

APROXIMATE METHOD FOR ESTIMATING DYNAMIC INFLUENCE FACTOR

Solutions for impulsive loading in infinite elastic media

Matsuoka and Yahata (1980) gave a close-form solution of displacements, u_i, due to impulsive forces, F_i, acting at the origin of Cartesian coordinates (x, y, z):

$$u_i = \frac{1}{4\pi G}\left[\frac{1}{R}\delta\left(t-\frac{R}{c_2}\right)F_i^{'} + c_2^2\frac{\partial}{\partial x_i}\nabla \bullet \boldsymbol{F}\,\frac{1}{R}\left\{\left(t-\frac{R}{c_1}\right)U\left(t-\frac{R}{c_1}\right)-\left(t-\frac{R}{c_2}\right)U\left(t-\frac{R}{c_2}\right)\right\}\right] \tag{1}$$

where

$$u_i = u_i(x, y, z, t) \quad (i = x, y, z) \tag{2a}$$

$$F_i(x, y, z, t) = F_i^{'}\delta(x, y, z)\delta(t) \quad (i = x, y, z) \tag{2b}$$

$$R = \sqrt{x^2 + y^2 + z^2} \tag{2c}$$

$$\nabla \bullet \boldsymbol{F} = \frac{\partial}{\partial x_i} F_i^{'} = \frac{\partial}{\partial x} F_x^{'} + \frac{\partial}{\partial y} F_y^{'} + \frac{\partial}{\partial z} F_z^{'} \tag{2d}$$

$$c_1 = \sqrt{\frac{E(1-\nu)}{(1+\nu)(1-2\nu)\rho}} \text{ : Longitudinal wave velocity} \tag{2e}$$

$$c_2 = \sqrt{\frac{G}{\rho}} \text{ : Shear wave velocity} \tag{2f}$$

$$\delta\left(t - \frac{R}{c}\right) = \begin{cases} 0 & t \neq R/c \\ \infty & t = R/c \end{cases} \text{ : Dirac function} \tag{2g}$$

$$U\left(t - \frac{R}{c}\right) = \begin{cases} 0 & t < R/c \\ 1 & t > R/c \end{cases} \text{ : Heaviside unit function} \tag{2h}$$

in which E, G, ν and ρ are Young's modulus, shear modulus, Poisson's ratio and density of the ground.

In order to obtain u_i from Equation (1), numerical calculations are required. The functions, $U(t - R/c)$ and $\delta(t - R/c)$, are expressed approximately as follows:

$$U\left(t - \frac{R}{c}\right) = \lim_{\varepsilon \to 0} \frac{1}{\pi}\left\{\tan^{-1}\left(\frac{t - R/c}{\varepsilon}\right) + \frac{\pi}{2}\right\} \approx \frac{1}{\pi}\left\{\tan^{-1}\left(\frac{t - R/c}{\varepsilon}\right) + \frac{\pi}{2}\right\} \text{ for small } \varepsilon \tag{3a}$$

$$\delta\left(t - \frac{R}{c}\right) = \lim_{\varepsilon \to 0} \frac{1}{\pi}\left\{\frac{\varepsilon}{\varepsilon^2 + (t - R/c)^2}\right\} \approx \frac{1}{\pi}\left\{\frac{\varepsilon}{\varepsilon^2 + (t - R/c)^2}\right\} \text{ for small } \varepsilon \tag{3b}$$

For example, Dirac function δ is expressed as shown in Figure 4.

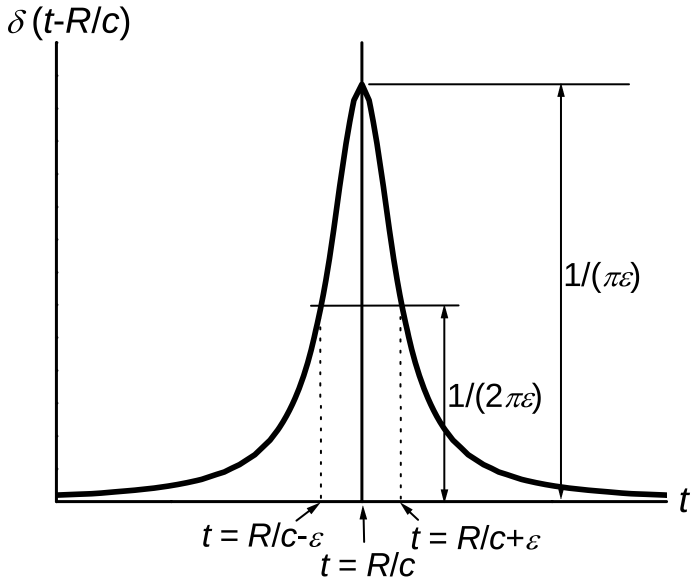


Figure 4. Approximation of Dirac function δ

Table 1. Soil conditions assumed in the example calculation shown in Figure 5

Young's modulus of soil, E (kPa)	48000
Poisson's ratio of soil, ν	0.333
Shear modulus of soil, G (kPa)	18000
Density of soil, ρ (ton/m^3)	1.8
Longitudinal wave velocity, c_1 (m/s)	200
Shear wave velocity, c_2 (m/s)	100

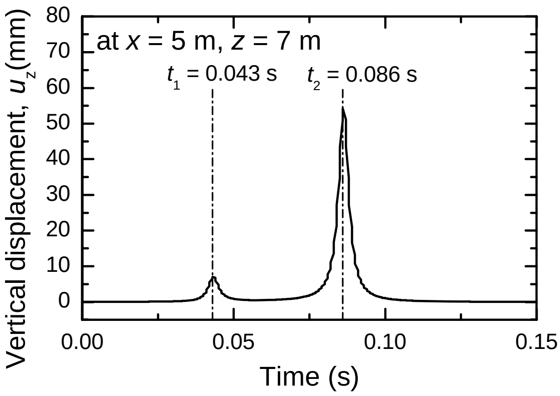


Figure 5. Change in vertical displacement due to vertical impulsive force

Figure 5 shows an example of change in the vertical displacement calculated from Equations (1), (2) and (3). In this example calculation, vertical force, F_z' = 1000 kN was applied at the origin, and the soil conditions were assumed as shown in Table 1. The parameter, ε = 1/500 s, was used in numerical calculation. In Figure 5, the change in vertical displacement, u_z, with time at x (or y) = 5 m and z = 7 m is shown. Theoretically, the longitudinal and shear waves reach that point at t_1 = 0.043 s and t_2 = 0.086 s. The calculated results in Figure 5 simulate this theoretical behaviour well. It is seen that the vertical displacement is generated by the arrivals of the longitudinal and shear waves, separately. Here, it should be noted that the values of the displacements induced by the longitudinal and shear waves depend on the value of ε employed in the numerical calculation. However, the ratio of the vertical displacement induced by the longitudinal wave to that induced by the shear wave can be regarded to be correct. Hence, another method should be used to determine the amplitudes of displacements induced by the longitudinal and shear waves, along with the theoretical solutions by Matsuoka and Yahata (1980).

Estimation of Dynamic Influence Factor for Step Load from Combination of Solutions for Dynamic and Static Kelvin Problems

Determination of the amplitudes of displacements induced by longitudinal and shear waves due to a step prolonged unit load (Figure 6) is proposed. The displacements at an arbitrary point due to the step load occur on the arrivals of waves, then increase with time and finally approach to the theoretical values from the static Kelvin problem.

Based on the solutions of Matsuoka and Yahata (1980) for impulse load, the change in the displacement with time has 2-step stepwise form for the step load, as illustrated in Figure 7. As mentioned earlier, the amplitude of displacement cannot be determined from the solutions of Matsuoka and Yahata (1980). Therefore, the final value of the displacement is determined from the static Kelvin theory in the proposed method. Finally the change in displacement with time can be drawn as the bold line in Figure 7, because the ratio of increments of displacement induced by longitudinal and shear waves can be determined from the solutions by Matsuoka and Yahata (1980).

The displacement (vertical axis) in Figure 7 is normalised by the theoretical value from the static Kelvin problem, and time (horizontal axis) is normalised by time t_1 where t_1 is time instant when the longitudinal wave starts to reach the considered point.

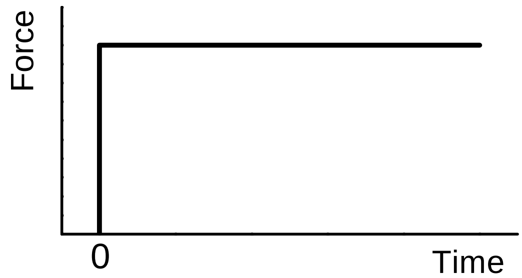


Figure 6. Step load considered in the proposed method

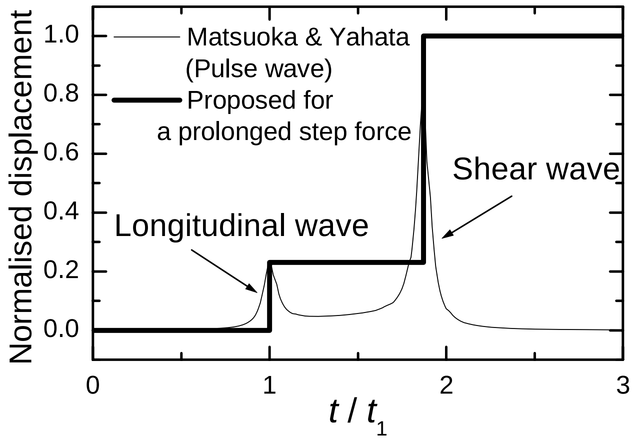


Figure 7. Change in displacement with time calculated using the analytical solutions by Matsuoka & Yahata (1980) and static Kelvin theory

The approach shown in Figure 7 is superposed according to superposition of dynamic Kelvin problem that has been shown in Figure 2(b). Figure 8 shows an example of change in normalised displacement (dynamic influence factor) with time calculated using the proposed method. In Figure 8, R_1 is the distance between point 1 and the considered point (point B), R_2 is that between point 2 (mirrored position) and the considered point. Due to the superposition, the change with time in displacement has 4-step stepwise form. The final value is the sum of displacements due to static forces at point 1 and point 2. The increment of displacement due to each wave (longitudinal and shear waves from point 1, and those from point 2) can be calculated using the solutions by Matsuoka and Yahata (1980).

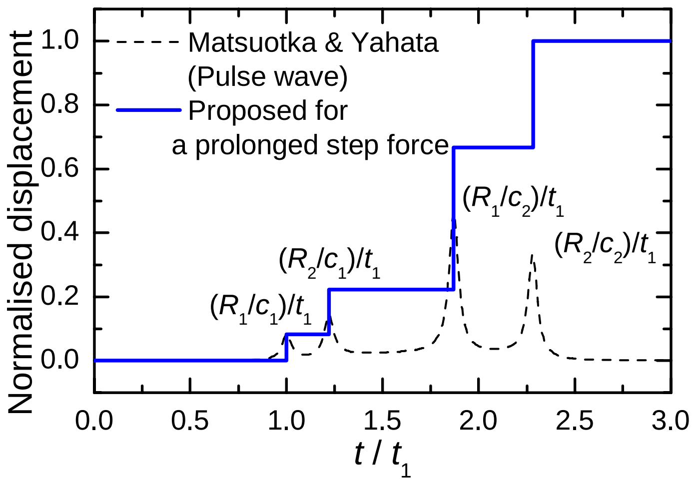


Figure 8. Example of dynamic influence factor calculated using the proposed method

VALIDATION OF THE PROPOSED METHOD

The proposed method for estimating the dynamic influence factor is validated below. First, the superposition of Kelvin problem to simulate Mindlin problem is validated for static problems where the theoretical solutions (Mindlin's and Kelvins's solutions) are available. Then, validity of the superposition of Kelvin problem to simulate Mindlin problem is examined for dynamic problems. As the close-form solution does not exist for dynamic problems, the validity will be discussed through comparison with results of dynamic FEM analyses. Finally, the validity of the proposed method to simulate the dynamic Mindlin problem will be discussed and demonstrated.

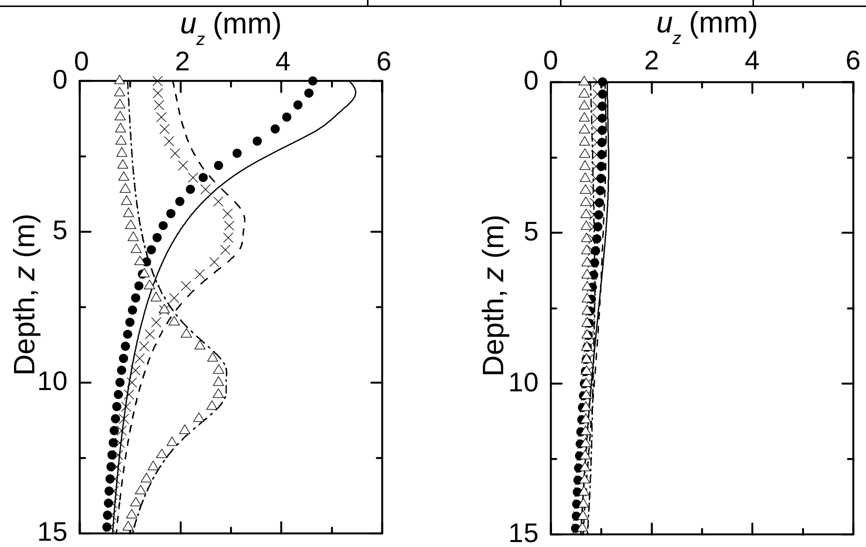
Validation for Static Loading

Although the proposed method is not used for static loading, it is useful to investigate how the superposition of Kelvin problem can simulate the exact solutions by Mindlin because the final value of displacement in dynamic loading is estimated in the proposed method using the theoretical value from the superposition of static Kelvin problem.

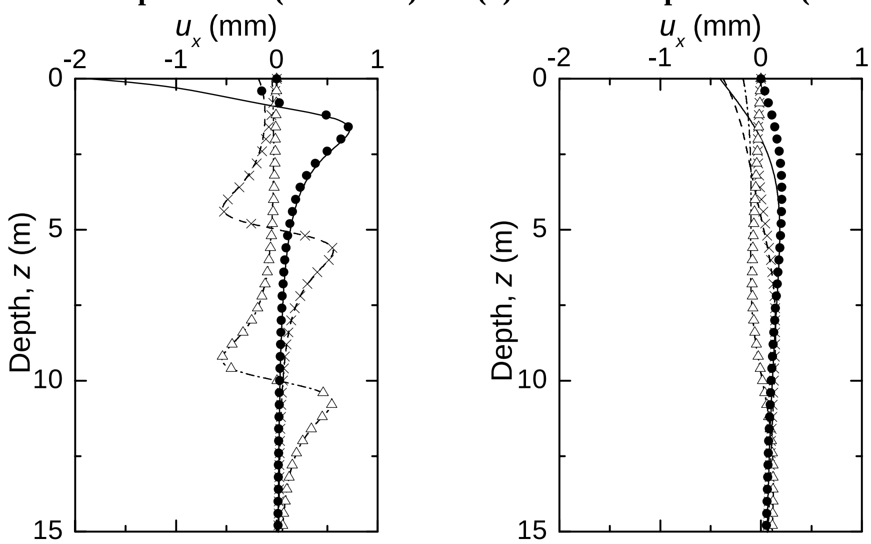
Table 2. Calculation conditions

Property	Value
Vertical load, P (kN)	100
Horizontal load, Q (kN)	100
Depth of loading point, c (or z) (m)	1, 5, 10
Shear modulus of soil, G (kPa)	2000
Poisson's ratio of soil, ν	0.30

Depth of loading point	c = 1.0 m	c = 5.0 m	c = 10 m
Mindlin problem	———	-------	-·-·-·-·-
Superposition of Kelvin problem	●	×	△



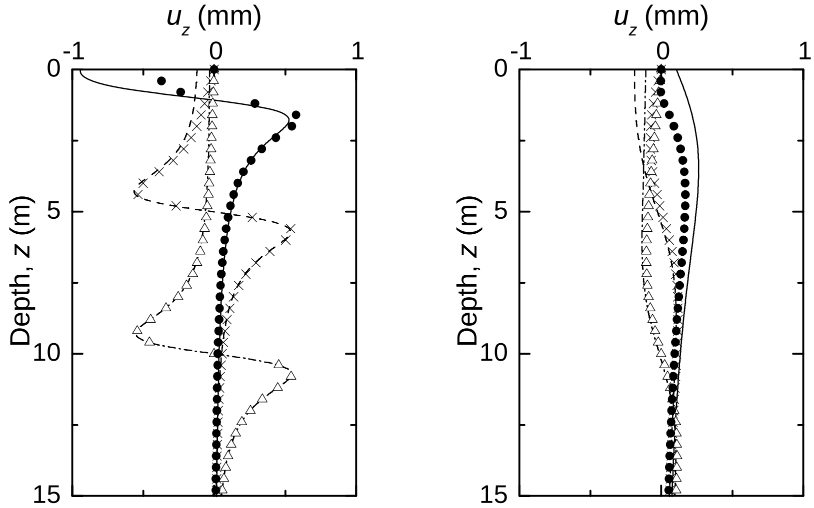
(a) Vertical displacement (at x = 1 m) **(b) Vertical displacement (at x = 5 m)**



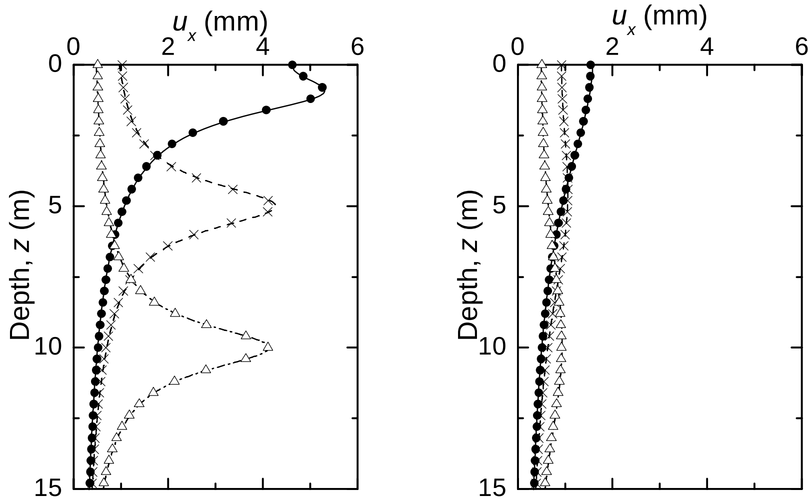
(c) Horizontal displacement (at x = 1 m) **(d) Horizontal displacement (at x = 5 m)**

Figure 9. Cases of static vertical loads (c =1 m, 5 m, 10 m)

Depth of loading point	c = 1.0 m	c = 5.0 m	c = 10 m
Mindlin problem	———	- - - - - - -	-·-·-·-·-
Superposition of Kelvin problem	●	×	△



(a) Vertical displacement (at x = 1 m) **(b) Vertical displacement (at x = 5 m)**



(c) Horizontal displacement (at x = 1 m) **(d) Horizontal displacement (at x = 5 m)**

Figure 10. Cases of static horizontal loads (c =1 m, 5 m, 10 m)

Table 2 shows the calculation conditions including loading conditions and soil properties. Comparisons of theoretical results from the superposed Kelvin problem and Mindlin problem are made in Figure 9 for vertical loads and in Figure 10 for horizontal loads. Distributions of vertical displacements, u_z, and horizontal displacements, u_x, with depths at x = 1 and 5 m are indicated in Figures 9 and 10.

When vertical loads are applied, vertical displacements from the superposed Kelvin problem are good approximations for the exact solutions (Figures 9(a), (b)). Horizontal displacements from the superposed Kelvin problem are approximations for the exact solutions except for shallow depths less than about 2 m (Figures 9(c), (d)). In contrast, when horizontal loads are applied, horizontal displacements from the superposed Kelvin problem are good approximations for the exact solutions (Figures 10(c), (d)). Vertical displacements from the superposed Kelvin problem are approximations for the exact solutions except for shallow depths less than about 2 m (Figures 10(a), (b)). It may be concluded from Figures 9 and 10 that displacements in the same direction as the loading direction and the displacements perpendicular to loading direction except for at shallow locations less than 2 m can be approximated by the superposition of Kelvin theory irrespective of loading direction and location.

Validation for Dynamic Loading

Comparisons of FEM Results of Dynamic Mindlin Problem and Superposed Dynamic Kelvin Problem

In this section, validity of superposed Kelvin problem is examined through comparisons of FEM analyses of superposed dynamic Kelvin problems and FEM analyses of dynamic Mindlin problems.

FEM meshes used for the dynamic Mindlin problem and the superposed Kelvin problem are shown in Figures 11and 12, respectively. Table 3 shows the properties of the elastic soil used in the FEM analyses. In the Mindlin problem, the force shown in Figure 13 was applied at $x = y = 0$ and $z = 2$ m in vertical or horizontal direction. The force had a rise time of 0.05 s and thereafter was kept constant at 500 kN. In the superposed Kelvin problem, the force shown in Figure 13 was applied at $z = 2$ m (point 1) and at $z = -2$ m (point 2) in vertical or horizontal direction. Calculation time was limited to 0.35 s. Within this calculation time, the influence of waves reflected at side and bottom boundaries did not return to $x = y = z = 10$ m.

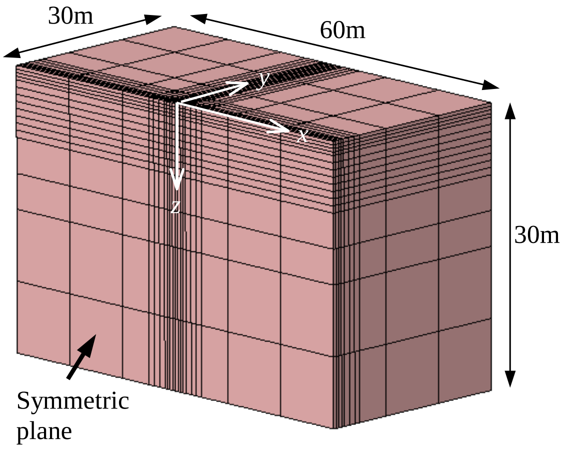


Figure 11. FEM mesh used for Mindlin problem

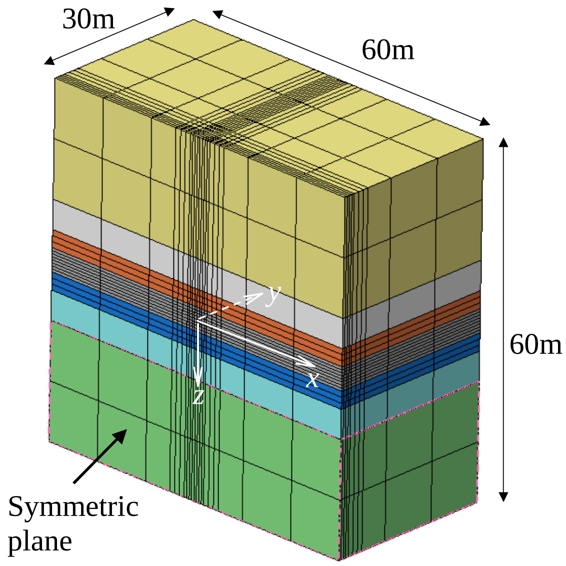


Figure 12. FEM mesh used for superposed Kelvin problem

Table 3. Properties of the soil used in analyses

Property	Value
Young's modulus, E (kPa)	11700
Poisson's ratio, ν	0.30
Shear modulus, G (kPa)	4500
Density, ρ (ton/m^3)	1.8
Longitudinal wave velocity, c_1 (m/s)	93.5
Shear wave velocity, c_2 (m/s)	50

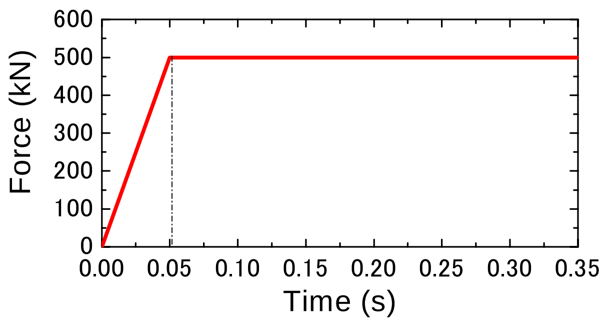


Figure 13. Dynamic load used in the dynamic FEM analyses

Figures 14 to 17 show comparisons of vertical and horizontal displacements from the Mindlin and superposed Kelvin problems. It can be seen from the figures that results of the superposed Kelvin problem are good approximations for the results from the dynamic Mindlin problem, and that the calculated displacements converge to the theoretical value of the static Mindlin problem. Symbols of ${}_1t_1$ and ${}_1t_2$ in the figures denote time instants when longitudinal and shear waves, respectively, from point 1 reach that point. Similarly, ${}_2t_1$ and ${}_2t_2$ denote time instants when longitudinal and shear waves, respectively, from point 2 reach that point. It is interesting that the calculated displacements become almost constant with time about 0.05 s after the last wave reaches that point. Note that superposed dynamic Kelvin problems did not simulate well displacements perpendicular to the loading direction at locations shallower than 2 m.

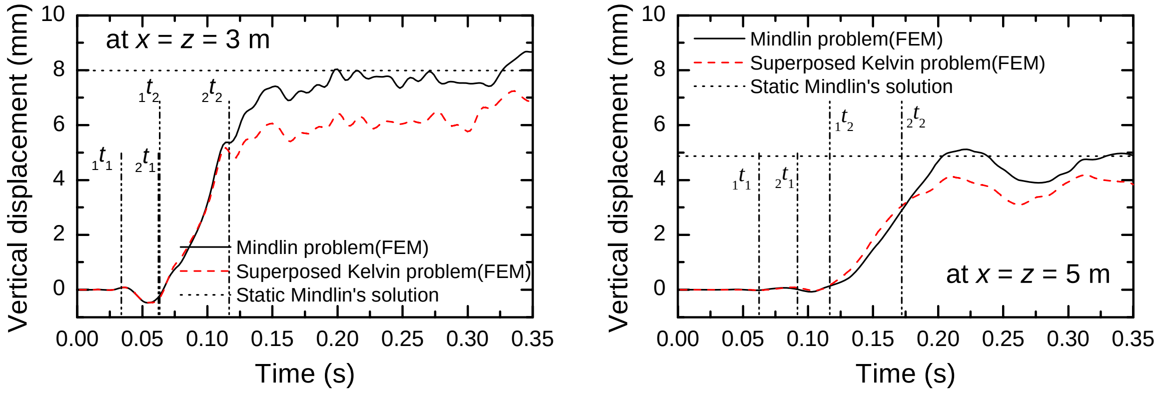


Figure 14. Vertical displacements in Mindlin problem and superposed Kelvin problem due to vertical force

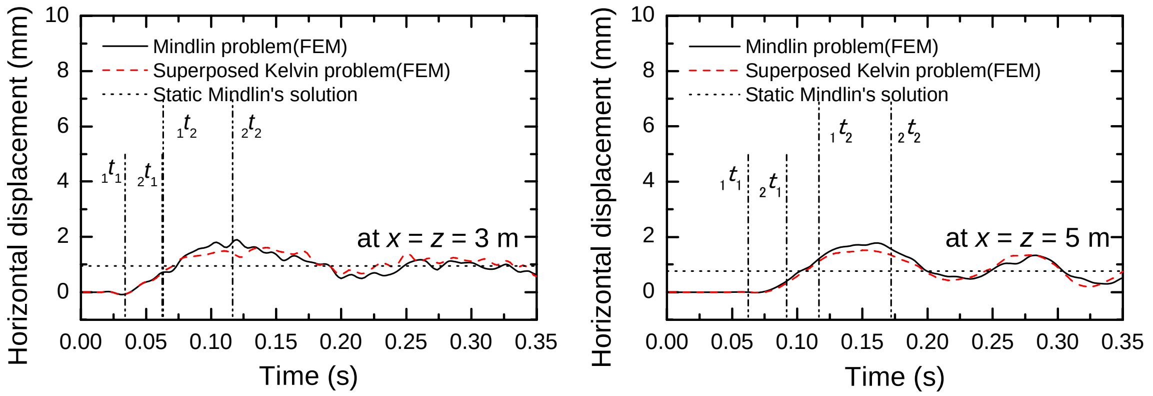


Figure 15. Horizontal displacements in Mindlin problem and superposed Kelvin problem due to vertical force

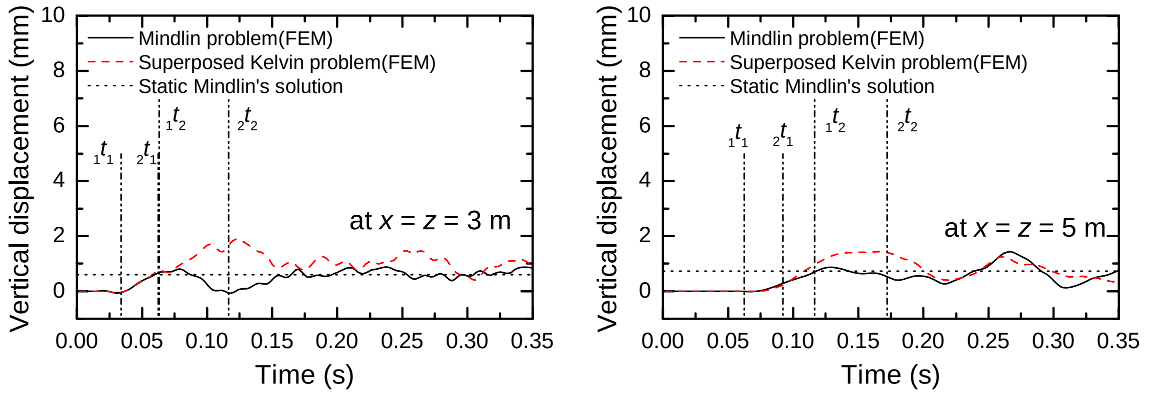


Figure 16. Vertical displacements in Mindlin problem and superposed Kelvin problem due to horizontal force

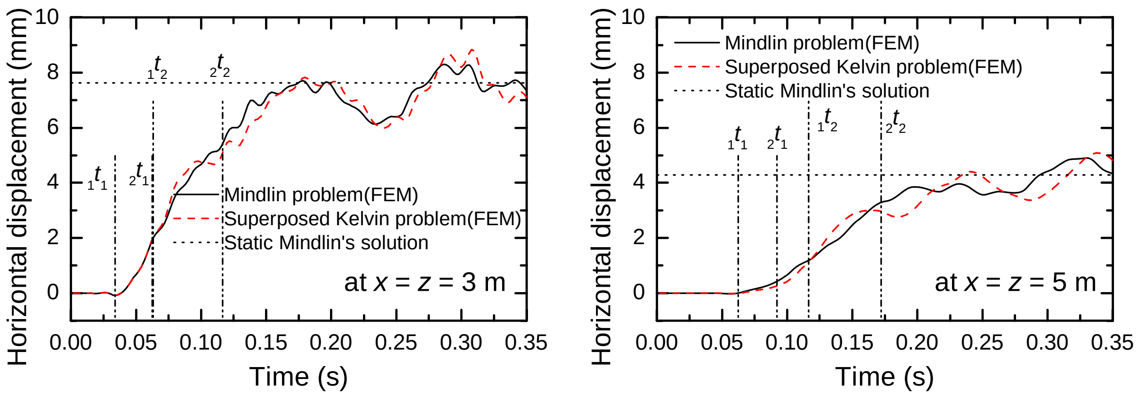


Figure 17. Horizontal displacements in Mindlin problem and superposed Kelvin problem due to horizontal force

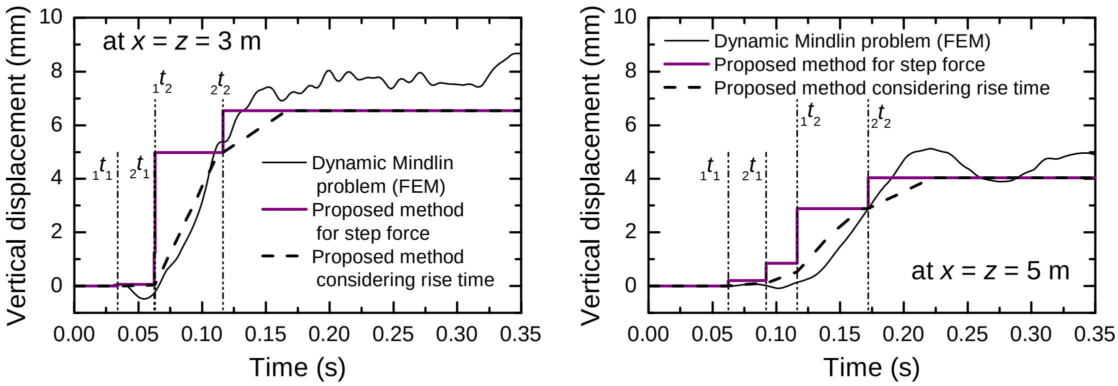


Figure 18. Vertical displacements from the proposed method and from FEM analysis of dynamic Mindlin problem due to vertical force

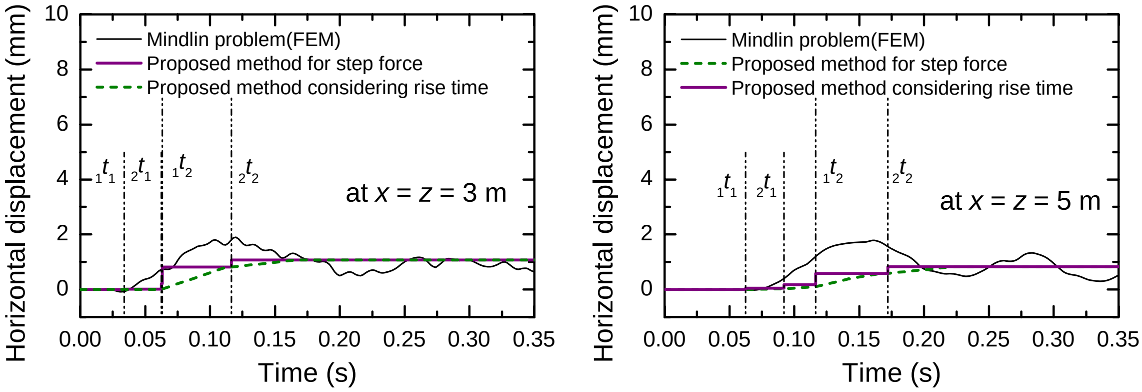


Figure 19. Horizontal displacements from the proposed method and from FEM analysis of dynamic Mindlin problem due to vertical force

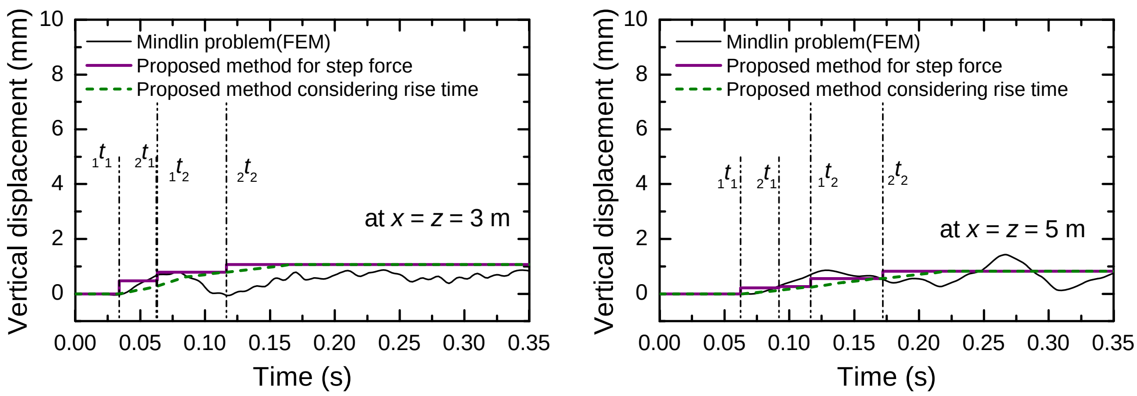


Figure 20. Vertical displacements from the proposed method and from FEM analysis of dynamic Mindlin problem due to horizontal force

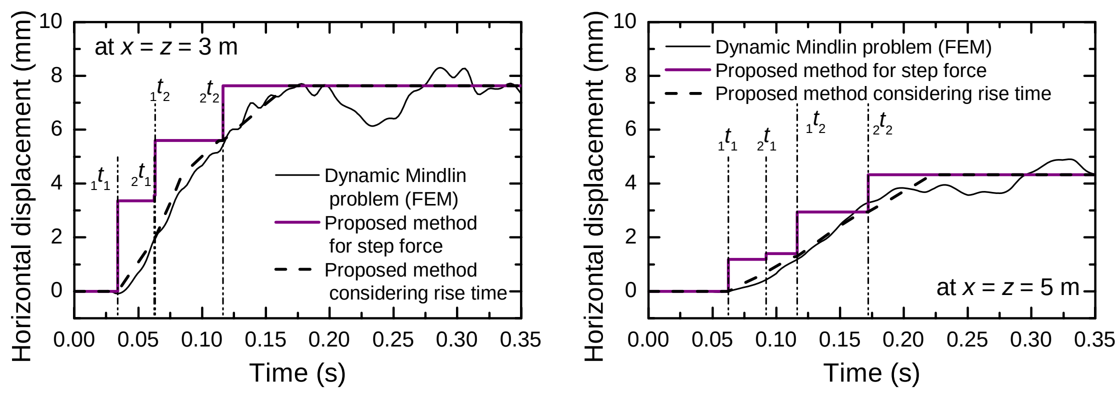


Figure 21. Horizontal displacements from the proposed method and from FEM analysis of dynamic Mindlin problem due to horizontal force

Comparisons of Results from the Proposed Method and Dynamic Mindlin Problem

In Figures 18 to 21, the displacements calculated using the proposed analytical method are compared with those calculated from the previous FEM analyses of the dynamic Mindlin problem. In the figures, two types of displacements from the proposed analytical methods are indicated. One of them is displacement for step load without rise time, and the other is displacement for the force having a rise time of 0.05 s (see Figure 13). Note here that vertical displacement was calculated from the proposed method, but horizontal displacement was calculated by multiplying a factor to the calculated dynamic vertical displacement for simplicity, where the factor was assumed to be equal to the ratio of static theoretical horizontal displacement to theoretical vertical displacement. It is clearly seen from the figures that the proposed analytical method for estimating dynamic displacement (dynamic influence factor), can be used with sufficient accuracy.

Note that dynamic influence factor for step force calculated using the proposed analytical method can be used in dynamic analysis for any form of dynamic force, because any form of dynamic force can be expressed by superposition of step forces.

CONCLUDING REMARKS

An approximate method to estimate the dynamic influence factor has been proposed based on superposition of dynamic Kelvin problem for a step prolonged force. The proposed method makes use of the analytical solutions for the dynamic Kelvin problem for an impulse load proposed by Matsuoka and Yahata (1980) and the static Kelvin solutions.

The validity of superposition of Kelvin problem was proved for static loading first by comparison with the theoretical results of the Mindlin problem.. Then, the validity of superposed dynamic Kelvin problem to simulate dynamic Mindlin problem was examined through FEM analyses of both problems. It was demonstrated that superposed dynamic Kelvin problem can simulate well the dynamic Mindlin problem. Finally, the results calculated using the proposed method where was compared with results of FEM analyses of the dynamic Mindlin problem, and reasonable agreement between the two solutions was obtained. However, soil displacements perpendicular to loading direction were not well simulated by superposed Kelvin problems at shallow locations less than a depth of about 2 m. Through the above comparisons, it was confirmed that the proposed method can be used to estimate dynamic influence factor in semi-infinite elastic ground except for at shallow locations less a depth of about 2 m.

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