4th International Conference on
Earthquake Geotechnical Engineering
June 25-28, 2007
Paper No. 1215

SLOPE STABILITY ANALYSIS IN SEISMIC AREAS BY A RELIABILITY APPROACH

Dalia YOUSSEF ABDEL MASSIH[1], Abdul-Hamid SOUBRA[2] and Elias EL-HACHEM[3]

ABSTRACT

A reliability-based analysis of the stability of slopes in seismic areas is performed in this paper. Quasi-static representation of earthquake effects using the seismic coefficient concept is adopted. Two deterministic approaches are used. The first one provides upper-bound solutions in the framework of the kinematical method of the limit analysis theory. The second one is based on numerical modeling of the slope stability using the Lagrangian explicit finite difference code $FLAC^{3D}$. The random variables considered in the analysis are the soil shear strength parameters c and φ and the horizontal seismic coefficient K_h. The response surface methodology is used to find an approximation of the analytically-unknown performance function and the corresponding reliability index while using the numerical $FLAC^{3D}$ simulations. However, direct minimization of the reliability index is performed while using the limit analysis approach. The numerical probabilistic results obtained from these methods are presented and discussed.

Keywords: slope, earthquake load, limit analysis, simulations, reliability, response surface.

INTRODUCTION

The stability analysis of slopes in seismic areas by a pseudo-static approach has been extensively investigated in literature using deterministic approaches. In these methods, average values of the input parameters (angle of internal friction, cohesion and seismic coefficient) are used and a global safety factor is calculated (see for instance Chen & Sawada, 1983; Leshchinsky & San, 1994; Michalowski 2002 and Loukidis et al., 2003 among others). A reliability-based approach for the slope stability analysis in seismic areas is more rational than the traditional deterministic methods since it takes into account the inherent uncertainty of each input variable.

The reliability theory was introduced by several authors in the analysis of slope stability but without taking into consideration the seismic loading (see for instance Vanmarcke, 1977; Chowdhury & Xu, 1993; Christian et al., 1994; Low et al., 1998; Husein Malkawi et al., 2000; Auvinet & Gonzales, 2000; El-Ramly et al., 2002; and Sivakumar Babu & Mukesh, 2004 among others). Most of these studies are based on approximate deterministic models as the limit equilibrium method which is founded on *a priori* assumptions concerning (i) the form of the slip surface and (ii) the normal stress distribution along this surface or the inter-slice forces.

[1] PhD Student, University of Nantes & Lebanese University, BP 11-5147, Beirut, Lebanon. E-mail: Dalia.Youssef@univ-nantes.fr

[2] Professor, University of Nantes, Institut de Recherche en Génie Civil et Mécanique, UMR CNRS 6183, Bd. de l'université, BP 152, 44603 Saint-Nazaire cedex, France (Corresponding author). E-mail: Abed.Soubra@univ-nantes.fr

[3] Assistant Professor, Lebanese University, Faculty of Engineering, Roumieh, Lebanon. E-mail: Eliashachem@ul.edu.lb

The application of the reliability theory to the slope stability problems taking into account the pseudo-static seismic loading is much less investigated (see for instance Christian & Urzua, 1998 and Al-Hamoud & Tahtamoni, 2000). In this paper, a reliability-based analysis of the stability of earth slopes in seismic areas is performed. Quasi-static representation of earthquake effects using the seismic coefficient concept is adopted where the seismic loading is represented by horizontal inertial forces. Two deterministic models are used for the assessment of the reliability index of the slope. The first one is based on the upper-bound method of the limit analysis theory using rotational log-spiral failure mechanism (Chen, 1975). The second one is based on numerical modeling using the Lagrangian explicit finite difference code *FLAC^3D^*. The Hasofer-Lind reliability index is adopted here for the assessment of the slope reliability. The random variables considered in the analysis are the soil shear strength parameters c and φ and the horizontal seismic coefficient K_h. The response surface methodology is used to find an approximation of the analytically-unknown performance function and the corresponding reliability index while using the numerical $FLAC^{3D}$ simulations. However, direct minimisation of the reliability index is performed while using the limit analysis approach.

After a brief description of the basic reliability concept, the limit analysis failure mechanism and the numerical $FLAC^{3D}$ code are presented. Then, the probabilistic analysis and the corresponding numerical results are presented and discussed.

BASIC RELIABILITY CONCEPTS

Two different measures are commonly used in literature to describe the reliability of a structure: The r*eliability index* and the *failure probability*. The reliability index of a geotechnical structure is a measure of the safety that takes into account the inherent uncertainties of the input variables. The widely used reliability index is the one defined by Hasofer and Lind (1974). Its matrix formulation is given by:

$$\beta_{HL} = \min_{x \in F} \sqrt{(x-\mu)^T C^{-1} (x-\mu)} \tag{1}$$

in which x is the vector representing the n random variables, μ is the vector of their mean values, C is their covariance matrix and F is the failure region. The minimisation of equation (1) is performed subject to the constraint $G(x) \leq 0$ where the limit state surface $G(x) = 0$, separates the n dimensional domain of random variables into two regions: a failure region F represented by $G(x) \leq 0$ and a safe region given by $G(x) > 0$. The classical approach for computing β_{HL} reliability index by equation (1) is based on the transformation of the limit state surface into the space of standard normal uncorrelated variates. The shortest distance from the transformed failure surface to the origin of the reduced variates is the reliability index β_{HL}. An intuitive interpretation of the reliability index was suggested in Low and Tang (1997) where the concept of an expanding ellipse led to a simple method of computing the Hasofer-Lind reliability index in the original space of the random variables. These authors stated that the minimization of the reliability index is equivalent to find the smallest dispersion ellipsoid that is tangent to the limit state surface. When the random variables are non-normal and correlated, the optimisation approach uses the Rackwitz-Fiessler equivalent normal transformation without the need to diagonalize the correlation matrix as shown in Low (2005). The computations of the equivalent normal mean μ^N and equivalent normal standard deviation σ^N for each trial design point are automatically found during the constrained optimization search. The method of computation of the reliability index using the concept of an expanding ellipse suggested by Low and Tang (1997) is used in this paper. From the First Order Reliability Method *FORM* and the Hasofer-Lind reliability index β_{HL}, one can approximate the failure probability as: $P_f \approx \Phi(-\beta_{HL})$, where $\Phi(\cdot)$ is the cumulative distribution function of a standard normal variable. In this method, the limit state function is approximated by a hyperplane tangent to the limit state surface at the design point.

DETERMINISTIC MODELS FOR SLOPE STABILITY ANALYSIS

A pseudo-static approach is adopted in this paper for the simulation of the seismic loading. Only horizontal seismic inertia forces are applied everywhere in the soil mass, the vertical seismic forces often being disregarded. The soil considered in the analysis is characterized by its cohesion c, its angle of internal friction φ and its unit weight γ. The factor of safety is defined as the ratio of the available shear strength of the soil to that required to maintain equilibrium. It is the factor by which the available shear strength parameters need to be reduced to bring the slope to failure.

Limit analysis model

A rigid rotational log-spiral failure mechanism that passes below the toe of the slope is considered here in the framework of the kinematical method of the limit analysis theory (*cf.* Figure 1). The mechanism passing below the toe may lead in certain cases to more critical results than the one passing through the toe. It reduces to the mechanism passing through the toe when $\beta' = \beta$. For further details on the theoretical formulation of the log-spiral limit analysis model, the reader may refer to Chen (1975).

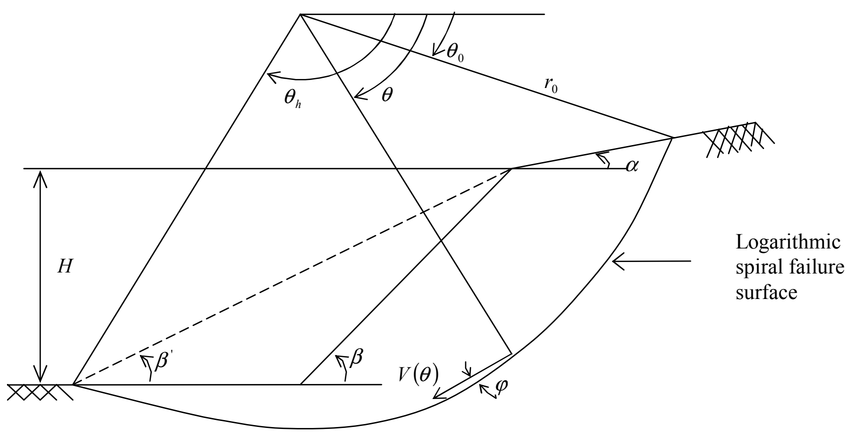

Figure 1: Failure mechanism

Numerical modeling using the Lagrangian explicit finite difference code *FLAC^{3D}*

FLAC^{3D} (Fast Lagrangian Analysis of Continua) is a commercially available three-dimensional finite difference code in which an explicit Lagrangian calculation scheme and a mixed discretization zoning technique are used. It should be mentioned that *FLAC^{3D}* includes an internal programming option (FISH) which enables the user to add his own subroutines. The soil domain is divided by the user into a 3D finite difference mesh of polyhedral zones. Constant strain-rate elements of tetrahedral shape whose vertices are the nodes of the mesh are used. This enables the velocity field to be linear inside the tetrahedrons. Each element behaves according to a prescribed linear or nonlinear stress/strain law in response to applied forces or boundary restraints. Several constitutive models are available. In this code, although a pseudo-static (*i.e.* non-dynamic) mechanical analysis is required, the equations of motion are used. The solution to a static problem is obtained through the damping of a dynamic process by including damping terms that gradually remove the kinetic energy from the system. For further details, one can refer to *FLAC^{3D}* manual.

The theoretical background of the *FLAC^{3D}* code can be summarized as follows: The equations of motion of an equivalent static problem involving inertial terms were written in a discrete form at the different nodes of the discretized medium. This enables one to transform the equations of motion of the continuum into a set of Newton's law at the nodes of the mesh. The later equations constitute a system of differential equations which are solved using the Lagrangian explicit finite difference scheme in time. The new idealized medium can be viewed as an assembly of point masses located at the nodes of the mesh and connected by linear springs since the system of ordinary differential equations obtained is similar to that describing the motion of a mass-spring system. The analogy with the idealized medium is immediate if one interprets the statically equivalent nodal force of all contributing tetrahedra and nodal applied loads (called hereafter out-of-balance force) as the resultant of spring reactions and external applied forces in the mass-spring system. The out-of-balance forces of all nodes are equal to zero when the medium has reached equilibrium. In the present numerical model, the inertial terms are used as means to reach, in a numerically stable manner, the steady state of static equilibrium or plastic flow. This is performed by replacing the mass involved in the inertial term by a fictitious nodal mass whose value ensures numerical stability of the system on its route to steady state.

The calculation scheme invokes equations of motion in their discretized forms (*i.e.* Newton's law at the different nodes) to derive new velocities and displacements from stresses and forces. Then, strain rates are derived from velocities, and new stresses from strain rates. The stresses and deformations are calculated at several small timesteps (called hereafter cycles) until a steady state of static equilibrium or plastic flow is achieved. The convergence to this state may be controlled by a maximal prescribed value of the unbalanced force for all elements of the model. It should be mentioned that in $FLAC^{3D}$, the application of prescribed displacements or stresses on the soil and/or the structure creates unbalanced forces in the system. Damping is introduced in order to remove these forces or to reduce them to very small values compared to the initial ones.

In this paper, $FLAC^{3D}$ is used for the determination of the slope safety factor determined by the strength reduction method. This method is implemented in $FLAC^{3D}$ through the "SOLVE fos" command. This command enables an automatic search for factor of safety using the bracketing approach, as described in Dawson et al. (1999): Simulations are run for a series of trial values of the safety factors F_s^{trial}. The cohesion c and the angle of internal friction φ at each trial are adjusted according to the equations:

$$c^{trial} = \frac{1}{F_s^{trial}} c \tag{2}$$

$$\varphi^{trial} = \arctan\left(\frac{1}{F_s^{trial}} \tan\varphi \right) \tag{3}$$

The value of F_s^{trial} at which failure occurs is found using a bracketing and bisection approach. Upper and lower brackets are first established. The initial lower bracket corresponds to any trial factor of safety for which the system is stable. The initial upper bracket corresponds to any trial factor of safety for which the system is unstable. Next, a new value, midway between the upper and lower brackets, is tested. If the system is stable for this midway value, the lower bracket is replaced by this new trial factor of safety. If the system does not reach equilibrium, the upper bracket is then replaced by the midway value. The process is repeated until the difference between upper and lower brackets is less than a specific tolerance.

RELIABILITY ANALYSIS OF SLOPE STABILITY

The aim of this paper is to perform a reliability-based analysis of slope stability in a seismic area. The failure or unsatisfactory performance mode considered in the analysis involve the ultimate limit state and emphasis on the slope stability failure. The two deterministic models presented above are used. The response surface methodology is utilized to find an analytical approximation of the unknown performance function and the corresponding critical Hasofer-Lind reliability index while using the numerical *FLAC^{3D}* simulations. However, direct minimization of the Hasofer-Lind reliability index is performed while using the limit analysis model since the closed form solution of the performance function is known in this case. Due to uncertainties in soil shear strength parameters and horizontal seismic coefficient, the cohesion *c*, the angle of internal friction φ, and the seismic coefficient K_h are considered as random variables. For the probability distribution of the random variables, three cases are studied. In the first case, referred to as *normal* variables, *c,* φ and K_h are considered as normal variables. In the second case referred to as *non-normal (EVD)* variables, *c* is assumed to be lognormally distributed and φ is assumed to be bounded and a beta distribution is used (Fenton and Griffiths 2003). The parameters of the beta distribution are determined from the mean value and standard deviation of φ (Haldar & Mahadevan, 2000). For the seismic coefficient an Extreme Value type II Distribution (*EVD*) (Haldar & Mahadevan, 2000) is used. The third case, named as *non-normal (Exp D)* variables is similar to the second one except that an Exponential Distribution (*Exp D*) (Haldar & Mahadevan, 2000) is considered for the seismic coefficient. For *normal* and *non-normal* distributions, both correlated and uncorrelated variables are considered.

After a brief description of the performance function used in the present analysis, the response surface methodology and its numerical implementation are presented. Then, the probabilistic numerical results are presented and discussed.

Performance function

The performance function corresponding to the unsatisfactory performance mode (slope stability failure) used in this reliability analysis is given as follows:

$$G = F_s - 1 \tag{4}$$

where F_s is the safety factor calculated by the limit analysis model or *FLAC^{3D}* simulations.

Response surface method

If the performance function is an explicit function of the random variables, the reliability index can be calculated easily. In *FLAC^{3D}* model, the closed form solution of the performance function is not available and the determination of the reliability index is then not straightforward. Therefore, an algorithm based on the response surface methodology proposed by Tandjiria et al. (2000) is used in this paper in the aim to calculate the reliability index and the corresponding design point. The basic idea of this method is to approximate the performance function by an explicit function of the random variables, and to improve the approximation *via* iterations. The approximate performance function used in this study has a quadratic form. It uses a second order polynomial with squared terms but no cross terms. The expression of this approximation is given by:

$$G(x) = a_0 + \sum_{i=1}^{n} a_i . x_i + \sum_{i=1}^{n} b_i . x_i^{\,2} \tag{5}$$

where x_i are the random variables, *n* is the number of the random variables and, a_i, b_i are the coefficients to be determined. In this paper, three random variables are considered (*i.e.* $n = 3$). They are characterized by their mean values μ_i and their coefficients of variation σ_i. A brief explanation of the used algorithm is as follows:

1- Evaluate the performance function $G(x)$ at the mean value point μ and the *2n* points each at $\mu \pm k\sigma$ where $k = 1$ in this paper.

2- The above $2n+1$ values of $G(x)$ can be used to solve equation (5) for the coefficients (a_i, b_i). This obtains a tentative response surface function which is based on the values of the $2n+1$ sampled points near the mean value point.
3- Solve equation (1) to obtain a tentative design point and a tentative β_{HL} subject to the constraint that the tentative response surface function of step 2 be equal to zero.
4- Repeat steps 1 to 3 until convergence. Each time step 1 is repeated, the $2n+1$ sampled points are centred at the new tentative design point of step 3.

Numerical implementation of the response surface method
As described in the previous section, the determination of the Hasofer-Lind reliability index requires (i) the determination of the coefficients (a_i, b_i) of the tentative response surface *via* the resolution of equation (5) for the $2n+1$ sampled points and (ii) the minimisation of the Hasofer-Lind reliability index subject to the constraint that the tentative response surface function of step 2 be equal to zero. These two operations which constitute a single iteration were done using the optimization toolbox available in *Matlab 7.0* software. Several iterations are performed until convergence of the Hasofer-Lind reliability index. Notice that the determination of the performance function at the $2n+1$ sampled points is performed using deterministic $FLAC^{3D}$ calculations. The results of these computations constitute the input data for the determination of the coefficients of the tentative response surface (a_i, b_i) using *Matlab 7.0*. Also, the value of the design point determined using the minimization procedure in *Matlab 7.0* is an input data for the determination of the performance function at the $2n+1$ sampled points in $FLAC^{3D}$. Therefore, an exchange of data between $FLAC^{3D}$ and *Matlab 7.0* in both directions was necessary to enable an automatic resolution of the iterative algorithm for the determination of the Hasofer-Lind reliability index. The link between $FLAC^{3D}$ and *Matlab 7.0* was performed using text files and *FISH* language commands.

NUMERICAL RESULTS

The numerical results presented in this paper consider the case of a slope with an angle equal to 63.4^{o} (2V to 1H slope). The soil has a unit weight of 20 kN/m^3. The illustrative values used for the statistical moments (*i.e.* mean μ and coefficient of variation COV) of the shear strength parameters and their coefficient of correlation $\rho_{c,\varphi}$ are given as follows: $\mu_c = 20$ kPa, $\mu_\varphi = 30^\circ$, $COV_c = 20\%$, $COV_\varphi = 10\%$ and $\rho_{c,\varphi} = -0.5$.

Limit analysis results
The determination of the reliability index is performed by minimization of equation (1) not only with respect to the random variables (c, φ, K_h), but also with respect to the geometrical parameters of the failure mechanism $(\theta_0, \theta_h, \beta')$ shown in figure (1). The obtained surface corresponding to the minimum reliability index is referred to here as the critical probabilistic surface. The reliability index obtained using this surface is smaller (*i.e.* more critical) than the one calculated by using the critical deterministic surface.

Reliability index, design point and failure probability
For a horizontal seismic coefficient equal to 0.3, the critical deterministic slope height that leads to failure was found equal to 7.278 m. This value was used in all subsequent calculations.

Table (1) presents the Hasofer-Lind reliability index and the corresponding design point for different values of the mean of K_h for *normal* and *non-normal (EVD)* variables when $COV_{K_h} = 30\%$. Both correlated and uncorrelated shear strength parameters are considered. The reliability index decreases with the increase of the mean of K_h (*i.e.* with the decrease of the slope safety factor F_S) until it vanishes when the ultimate deterministic state of failure (*i.e.* $F_S = 1$) is reached. This ultimate state is the one for which the design point is equal to the mean point for *normal* variables and equivalent normal mean point for *non-normal* variables. The corresponding failure probability is equal to 50%.

Table 1: Reliability results for different values of the mean of K_h

a) *Normal* variables – Uncorrelated shear strength parameters

μ_{K_h}	c^* (kPa)	φ^* (°)	K_h^*	β_{HL}	P_f (%)	F_c	F_φ	γ_{K_h}
0.05	12.65	28.01	0.052	1.96	2.48	1.58	1.09	1.06
0.1	14.18	28.37	0.110	1.59	5.61	1.41	1.07	1.10
0.15	15.83	28.80	0.168	1.18	11.81	1.26	1.05	1.12
0.2	17.43	29.24	0.220	0.77	22.03	1.15	1.03	1.10
0.25	18.85	29.65	0.265	0.37	35.55	1.06	1.01	1.06
0.3	20.00	30.00	0.3	0.00	50.00	1.00	1.00	1.00

b) *Normal* variables – Correlated shear strength parameters

μ_{K_h}	c^* (kPa)	φ^* (°)	K_h^*	β_{HL}	P_f (%)	F_c	F_φ	γ_{K_h}
0.05	11.16	31.26	0.054	2.36	0.91	1.79	0.95	1.08
0.10	13.09	30.89	0.114	1.91	2.81	1.53	0.97	1.14
0.15	15.18	30.55	0.175	1.41	7.90	1.32	0.98	1.17
0.20	17.16	30.28	0.228	0.90	18.28	1.17	0.99	1.14
0.25	18.79	30.11	0.270	0.43	33.50	1.06	1.00	1.08
0.30	20.00	30.00	0.300	0.00	50.00	1.00	1.00	1.00

c) *Non-normal (EVD)* variables for $COV_{K_h} = 30\%$ – Uncorrelated shear strength parameters

μ_{K_h}	c^* (kPa)	φ^* (°)	K_h^*	β_{HL}	P_f (%)	F_c	F_φ	γ_{K_h}
0.05	13.41	26.76	0.06	2.22	1.31	1.49	1.19	1.16
0.1	14.86	27.81	0.12	1.66	4.84	1.35	1.15	1.23
0.15	16.38	28.67	0.18	1.12	13.16	1.22	1.09	1.21
0.2	17.78	29.32	0.23	0.62	26.69	1.12	1.06	1.16
0.25	19.10	29.84	0.27	0.17	43.31	1.05	1.03	1.10
0.3	19.61	30.01	0.32	0.00	50.00	1.02	1.01	1.08

d) *Non-normal (EVD)* variables for $COV_{K_h} = 30\%$ – Correlated shear strength parameters

μ_{K_h}	c^* (kPa)	φ^* (°)	K_h^*	β_{HL}	P_f (%)	F_c	F_φ	γ_{K_h}
0.05	12.46	29.18	0.07	2.92	0.18	1.60	1.07	1.32
0.1	14.69	29.90	0.15	2.06	1.96	1.36	1.03	1.50
0.15	16.31	30.07	0.20	1.34	8.97	1.23	1.00	1.34
0.2	17.71	30.10	0.24	0.73	23.14	1.13	1.00	1.20
0.25	19.07	30.04	0.28	0.20	42.20	1.05	1.00	1.11
0.3	19.61	30.01	0.32	0.00	50.00	1.02	1.00	1.08

The values (c^*, φ^* and K_h^*) of the design points corresponding to different mean values of the horizontal seismic coefficient can give information about the resistance and load factors of the different random variables as follows: $F_c = \mu_c / c^*$, $F_\varphi = \tan(\mu_\varphi) / \tan\varphi^*$, $\gamma_{K_h} = K_h^* / \mu_{K_h}$. For uncorrelated shear strength parameters, the values of c^* and φ^* at the design point are smaller than their respective mean values and increase with the increase of the horizontal seismic load. They tend to the equivalent normal mean value when $\mu_{K_h} = 0.3$. However, K_h^* is found to be higher than its mean value since the horizontal seismic coefficient is a driving parameter, and it tends to the equivalent normal mean when $\mu_{K_h} = 0.3$. The resistance factors F_c and F_φ decrease with the increase of the seismic coefficient. However, the load factor γ_{K_h} first increases and then decreases with the increase of μ_{K_h}. The later result can be explained as follows: for small values of the seismic coefficient, the slope stability is not highly affected by the seismic coefficient and thus a small load factor is obtained. Notice finally that for negatively correlated shear strength parameters, φ^* slightly exceeds the mean for some values of the seismic coefficient.

Figures (2) shows the variation of the Hasofer-Lind reliability index β_{HL} with the safety factor F_S for the case of *normal* and *non-normal (EVD)* variables when $COV_{K_h} = 30\%$. The results are presented for both correlated and uncorrelated shear strength parameters. The reliability index corresponding to uncorrelated shear strength parameters was found smaller than the one of negatively correlated variables for both types of distributions. One can conclude that assuming uncorrelated shear strength parameters is conservative in comparison to assuming negatively correlated shear strength parameters (see Mostyn & Li, 1993). From the same figure, it can be shown that the reliability index of *normal* variables is higher than that of *non-normal (EVD)* variables for values of the safety factor varying from 1 to about 1.23. For higher values of the safety factor, the reliability index of *normal* variables becomes smaller. As a conclusion, the assumption of *normal* variables is conservative for high values of the safety factor.

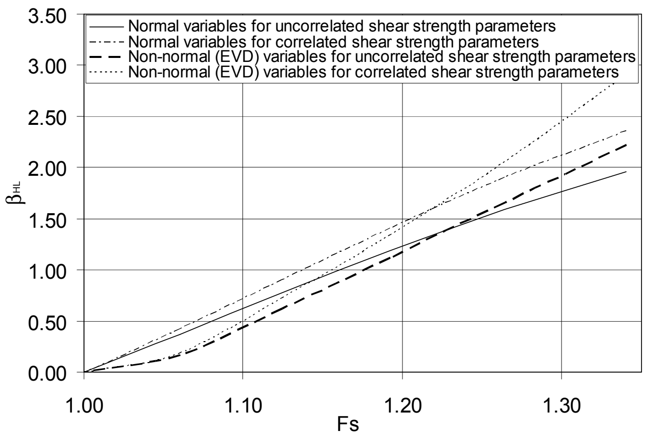

Figure 2: Reliability index versus safety factor

Figure (3) shows that the assumption made on the probability distribution of the seismic coefficient may largely affect the probability distribution of the slope safety factor. A larger dispersion of this factor is noted when one considers an exponential distribution. As a conclusion, the assumption of an *Exp D* for the seismic coefficient is highly conservative in comparison to the *EVD*.

Effect of the variability of each random variable on the CDF of the safety factor

Figure (4) shows the effect of a change in the coefficient of variation of each random variable on the CDF of the safety factor. The mean value of the seismic coefficient is set equal to 0.1. The assumption of *normal* variables and correlated shear strength parameters is considered here. Four cases are analyzed. The first case, referred to as "reference case", considers the values of the coefficients of variation as given in the introduction of the section named "*Numerical results*" with $COV_{K_h} = 30\%$. The other cases correspond to an increase by 10% of the coefficient of variation of each variable. It can be seen that a small variation in the coefficient of variation of c highly affects the

CDF curve of the safety factor. However, the CDF is less sensitive to the variability of φ. For K_h, one can notice that the variation of its uncertainties does not significantly affect the failure probability.

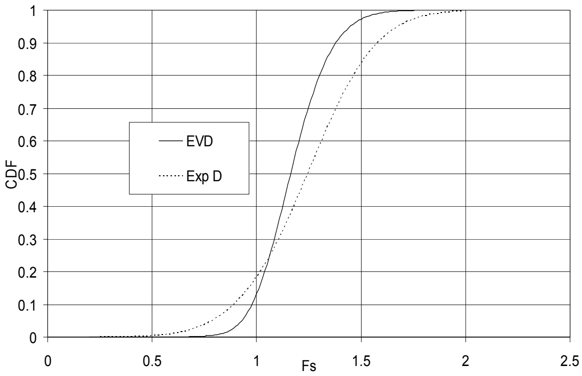

Figure 3: Effect of the probability distribution of K_h on the CDF of F_s

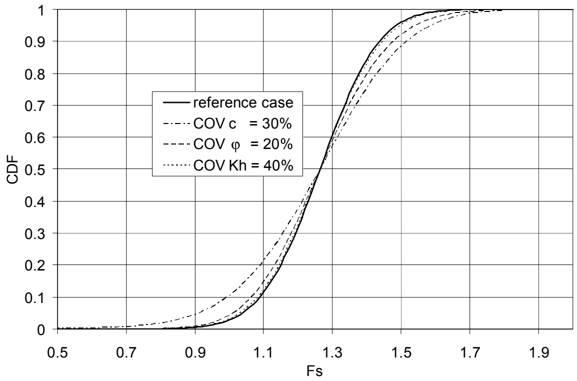

Figure 4: Effect of the variability of the random variables on the CDF of F_s

FLAC^{3D} results

Deterministic results

This section focuses on the determination of the safety factor defined with respect to the soil shear strength characteristics c and $\tan\varphi$ of a slope in the presence of pseudo-static earthquake loading using numerical *FLAC^{3D}* simulations. The critical deterministic slope height found in limit analysis for a seismic coefficient of 0.3 is used here (*i.e.* $H = 7.284\,m$). As a result of several verifications runs, the soil domain and the mesh shown in figure (5) are used in the analysis. The region was divided horizontally into 30 zones and vertically into 20 zones.

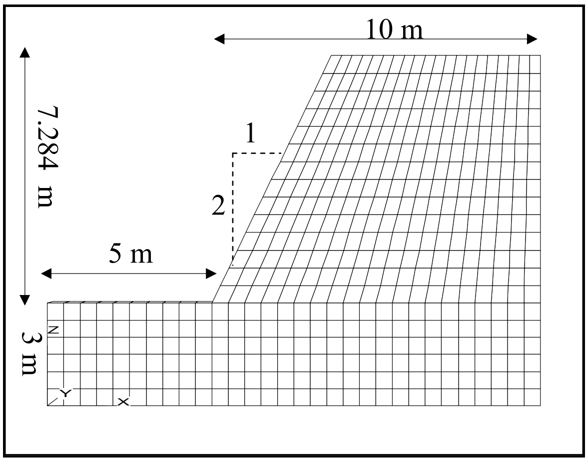

Figure 5: Slope geometry and mesh used in *FLAC^{3D}*

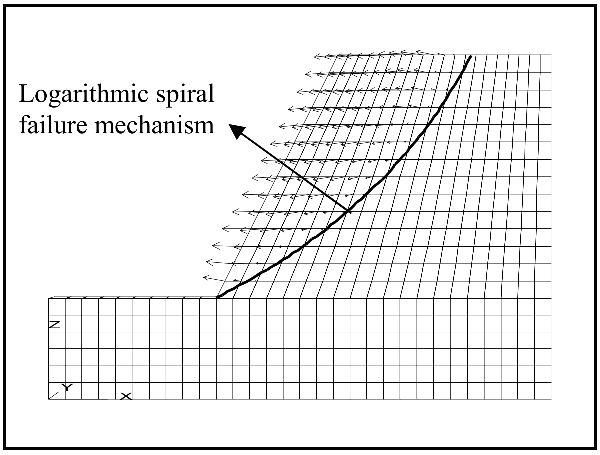

Figure 6: Logarithmic spiral failure mechanism and *FLAC^{3D}* velocity field

Since this is a *2D* case, all displacements in the *y* direction are fixed. For the displacement boundary conditions in the (X, Z) plan, the bottom boundary was assumed to be fixed and the right and left vertical boundaries were constrained in motion in the horizontal direction. A conventional elastic-perfectly plastic model based on the Mohr-Coulomb failure criterion is used to represent the soil. The soil elastic properties used are the shear modulus $G = 100MPa$ and the bulk modulus $K = 133MPa$

(for which the equivalent Young's modulus and Poisson's ratio are respectively $E = 240MPa$ and $\nu = 0.2$). The values of the soil shear strength parameters used in the analysis are: $c = 20kPa$ and $\varphi = \psi = 30^o$. The corresponding safety factor was found equal to 0.96 which is slightly lower than that of the limit analysis results (*i.e.* $F_s = 1$). This good agreement between the two solutions may be explained as follows: the soil mass in motion is nearly similar in both elasto-plastic (finite difference) and rigid-plastic (limit analysis) approaches (*cf.* Figure 6). It was checked that a more refined mesh improves the result by only 1% (*i.e.* only a 1% reduction in the safety factor) with a very high increase in the calculation time (by 285%). Thus, the mesh presented above will be used in all subsequent calculations.

Reliability index, design point and failure probability

Table (2) presents the Hasofer-Lind reliability index and the corresponding design point for different values of the mean of K_h using the response surface methodology presented above. The cases of correlated and uncorrelated shear strength parameters for *normal* distributions are considered in this section. The same conclusions found in limit analysis remain valid here. By comparing the present results to the limit analysis ones [*cf.* table 1 a) and b)], one can see that the reliability index calculated by *FLAC*^{3D} simulations is higher than that determined by the limit analysis model for small values of μ_{K_h}. For higher μ_{K_h} values, the reliability index calculated using *FLAC*^{3D} simulations becomes smaller (*i.e.* more conservative) than that of the log-spiral mechanism.

Table 2: Reliability results for different values of the mean of K_h

a) *Normal* variables – Uncorrelated shear strength parameters

μ_{K_h}	c^* (kPa)	φ^* (°)	K_h^*	β_{HL}	P_f (%)	F_c	F_φ	γ_{K_h}
0.05	12.24	27.48	0.054	2.13	1.67	1.63	1.11	1.07
0.1	14.3	28.09	0.117	1.66	4.85	1.40	1.08	1.17
0.2	17.90	29.18	0.2226	0.70	24.19	1.12	1.03	1.11
0.3	20.00	30.00	0.3000	0.00	50.00	1.00	1.00	1.00

b) *Normal* variables – Correlated shear strength parameters

μ_{K_h}	c^* (kPa)	φ^* (°)	K_h^*	β_{HL}	P_f (%)	F_c	F_φ	γ_{K_h}
0.05	10.39	31.00	0.056	2.63	0.43	1.93	0.96	1.11
0.10	12.82	30.42	0.119	2.10	1.80	1.56	0.98	1.19
0.20	17.89	29.93	0.2318	0.82	20.72	1.11	1.003	1.16
0.30	20.00	30.00	0.3000	0.00	50.00	1.00	1.00	1.00

Sensitivity of the failure probability to the variability of each random variable

To study the effect of the variability of the soil shear strength parameters and the seismic coefficient on the failure probability, Figure 7 shows the *FORM* failure probability versus the coefficient of variation of *c*, φ and K_h. For each curve, the coefficients of variation of two parameters are set equal to the values given by the "reference case" and the coefficient of variation of the third parameter is varied over the range 10% - 30%. The case of *normal* variables and correlated shear strength parameters is considered. The mean value of the seismic coefficient is taken equal to 0.1. The results show that the failure probability is highly influenced by the coefficient of variation of the cohesion. The greater the scatter in *c*, the higher the failure probability of the slope. This means that the accurate determination of the distribution of this parameter is very important in obtaining reliable probabilistic results. In contrast, the failure probability is less sensitive to the variation of COV_φ. For K_h, one can notice that the variation of its uncertainties has a minor effect on the failure probability. These findings are in conformity with the results of the limit analysis model.

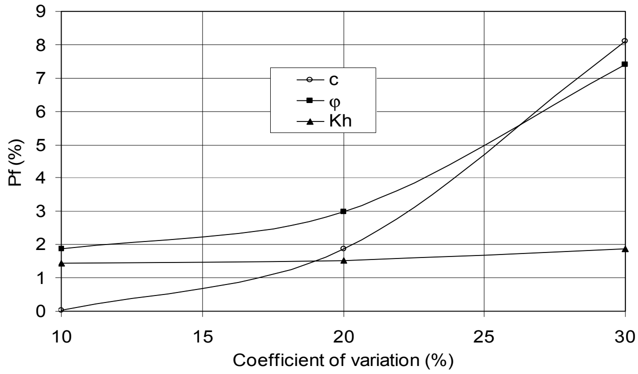

Figure 3: Failure probability versus the coefficient of variation

CONCLUSION

This paper presents a reliability-based analysis of the stability of slopes in seismic areas. Quasi-static representation of earthquake effects using the seismic coefficient concept is adopted. Two deterministic approaches are used. The first one provides upper-bound solutions in the framework of the kinematical method of the limit analysis theory. The second one is based on numerical modeling of the slope stability using the Lagrangian explicit finite difference code $FLAC^{3D}$. The random variables considered in the analysis are the soil shear strength parameters c and φ, and the horizontal seismic coefficient K_h. The response surface methodology was used to find an approximation of the analytically-unknown performance function and the corresponding reliability index while using $FLAC^{3D}$ simulations. However, direct minimization of the reliability index was performed while using the limit analysis approach. The main conclusions of the paper can be summarized as follows:

- The reliability index obtained using the probabilistic surface was found smaller (*i.e.* more critical) than the one calculated by using the critical deterministic surface;
- For *normal* and *non-normal* distributions, correlated and uncorrelated shear strength parameters, the reliability index decreases with the increase of the mean of K_h (*i.e.* with the decrease of the slope safety factor F_S). For uncorrelated shear strength parameters, the values of c^* and φ^* at the design point were found smaller than their respective mean values (resisting parameters). However, $K_h{}^*$ was found to be higher than its mean value since K_h is a driving parameter. The resistance factors F_c and F_φ decrease with the increase of the seismic coefficient. However, the load factor γ_{K_h} first increases and then decreases with the increase of μ_{K_h}. The later result can be explained as follows: for small values of the seismic coefficient, the slope stability is not highly affected by the seismic coefficient and thus a small load factor was obtained;
- The assumption of uncorrelated shear strength parameters was found conservative in comparison to the assumption of negatively correlated shear strength parameters;
- The assumption of *normal* variables was found conservative for high values of the safety factor;
- The assumption of an *Exp D* for the seismic coefficient is highly conservative in comparison to the *EVD*;
- A small variation in the coefficient of variation of *c* has a significant effect on the *CDF* curve of the safety factor and consequently on the failure probability;
- The comparison of $FLAC^{3D}$ results with the limit analysis ones has shown that the reliability index calculated by $FLAC^{3D}$ simulations was higher than that determined by the limit analysis model for small values of μ_{K_h}. However, for higher values of μ_{K_h}, the reliability index determined using $FLAC^{3D}$ simulations has became smaller (*i.e.* more conservative) than that of the log-spiral mechanism.

REFERENCES

Al-Hamoud, A.S., and Tahtamoni, W.W. "Reliability analysis of three-dimensional dynamic slope stability and earthquake-induced permanent displacement," *Soil Dynamics and Earthquake Engineering*, Vol. 19, 91-114, 2000.

Auvinet, G., and Gonzalez, J.L. "Three–dimensional reliability analysis of earth slopes," *Computers and Geotechnics*, Vol. 26, 247-261, 2000.

Chen, W.F. Limit Analysis and Soil Plasticity, Book published by *Elsevier*, Amsterdam, 638 p., 1975.

Chen, W.F., and Sawada, T. "Earthquake-induced slope failure in nonhomogeneous, anisotropic soils," *Soils and Foundations*, Vol. 23, N° 2, 125-139, 1983.

Chowdhury, R.N., and Xu, D.W. "Rational polynomial technique in slope-reliability analysis," *Journal of Geotechnical Engineering*, ASCE, Vol. 119, N° 12, 1910-1928, 1993.

Christian, J., Ladd, C., and Baecher, G. "Reliability applied to slope stability analysis," *J. of Geotech. Engrg.*, ASCE, Vol. 120, N° 12, 2180-2207, 1994.

Christian, J.T., and Urzua, A. "Probabilistic evaluation of earthquake-induced slope failure," *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, Vol. 124, N° 11, 1140-1143, 1998.

Dawson, E.M., Roth, W.H., and Drescher, A. "Slope Stability Analysis by Strength Reduction," *Géotechnique*, Vol. 49, N° 6, 835-840, 1999.

El-Ramly, H., Morgenstern, N. R., and Cruden, D. M. "Probabilistic slope stability analysis for practice," *Can. Geotech. J.*, Vol. 39, 665-683, 2002.

Fenton, G. A., and Griffiths D. V. "Bearing capacity prediction of spatially random C-φ soils," *Can. Geotech. J.*, Vol. 40, 54-65, 2003.

Haldar, A., and Mahadevan, S. Probability, reliability and statistical methods in Engineering Design, *John Wiley*, New York, 304p., 2000.

Hasofer, A. M., and Lind, N. C. "Exact and invariant second-moment code format," *J. of Engrg. Mech.*, ASCE, Vol. 100, N° 1, 11-121, 1974.

Husein Malkawi, A.H., Hassan, W., and Adbulla, F. "Uncertainty and reliability analysis applied to slope stability," *Structural Safety*, Vol. 22, 161-187, 2000.

Leshchinsky, D., and San, K.-C. "Pseudostatic seismic stability of slopes: Design charts," *Journal of Geotechnical Engineering*, ASCE, Vol. 120, N° 9, 1514-1532, 1994.

Loukidis, D., Bandini, P., and Salgado, R. "Stability of seismically loaded slopes using limit analysis," *Géotechnique*, Vol. 53, N° 5, 463-479, 2003.

Low, B. K. "Reliability-based design applied to retaining walls," *Géotechnique*, Vol. 55, N° 1, 63-75, 2005.

Low, B. K., and Tang, W. H. "Efficient reliability evaluation using spreadsheet," *J. of Engrg. Mech.*, ASCE, Vol. 123, 749-752, 1997.

Low, B. K., Gilbert, R. B., and Wright, S. G. "Slope reliability analysis using generalized method of slices," *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, Vol. 124, N° 4, 350-362, 1998.

Michalowski, R.L. "Stability charts for uniform slopes," *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, Vol. 128, N° 4, 351-355, 2002.

Mostyn, G.R., and Li, K.S. "Probabilistic slope analysis – State-of-play," *Probabilistic Methods in Geotechnical Engineering*, Balkema, Rotterdam, 89-109, 1993.

Sivakumar Babu, G.L. and Mukesh, M.D. "Effect of soil variability on reliability of soil slopes," *Géotechnique*, Vol. 54, N°5, 335-337, 2004.

Tandjiria, V., Teh, C.I., and Low, B.K. "Reliability analysis of laterally loaded piles using response surface methods," *Structural Safety*, Vol. 22, 335-355, 2000.

Vanmarcke, E. "Reliability of earth slopes," *Journal of the Geotechnical Engineering*, ASCE, Vol. 103, N° GT11, 1247-1265, 1977.

ACKNOWLEDGMENTS

The authors would like to thank the Lebanese National Council for Scientific Research (CNRSL) and the French organization EGIDE for providing the financial support for this research.