

A SIMPLIFIED NONLINEAR DYNAMIC MODEL FOR SEISMIC ANALYSIS OF EARTH-RETAINING DIAPHRAGM-WALLS

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ABSTRACT

This paper introduces a simplified model for the dynamic analysis of diaphragm walls retaining dry cohesion-less soils with horizontal back-slope subjected to seismic excitation. The model is based on the well-known one-dimensional Winkler approximation and on the non-linear shear-beam model for the ground layers on both sides of the wall. The model can include anchor-ties and can account for non-linearity in all of its elements (retained soil, anchors and wall). A numerical application is presented, which includes a validation of the proposed model results versus those of a refined plane-strain nonlinear finite-element analysis carried out with a commercial code. Based on these preliminary results the model appears to yield quite accurate predictions of static and dynamic bending moment distributions and permanent wall displacements.

Keywords: dry cohesion-less soil, permanent cumulative displacements, Winkler model, one-dimensional wave-propagation.

INTRODUCTION

The seismic analysis of earth-retaining structures can be carried out, in principle, in a sufficiently rigorous way by means of available non-linear dynamic finite element/difference procedures (PLAXIS, DYNAFLOW, FLAC, etc). The use of such advanced numerical tools, however, is still restricted to particularly important structures, besides requiring specialised background and experience (Green and Ebeling, 2002)(Gazetas *et al*, 2004)(Siller *et al*, 1991). On the other hand, the methods in practical use have progressed very little from the early simplified proposals. Seismic design of earth-retaining gravity or cantilever walls is almost exclusively carried out using the pseudo-static approach by Mononobe-Okabe (Mononobe and Matsuo, 1929)(Okabe, 1926), as modified by Seed and Whitman (Seed and Whitman, 1970), accounting in some approximate fashion for the flexibility of the structure. This approach is still the one adopted in most, if not all, international seismic design codes.

More recently, elastic and visco-elastic dynamic solutions have been proposed, starting from the original one by Wood, arriving at the works by Veletsos and Younan. These studies consider the dynamic response of a visco-elastic soil stratum, infinitely extended in one direction and restrained in the other by a vertical wall, excited at the rigid base by a spatially homogeneous motion. Subsequent refinements have led from the analysis of a fixed-base rigid wall (Wood, 1975) to that of a flexible wall elastically restrained at its base (Veletsos and Younan, 1994 and 1997)(Younan and Veletsos, 2000). Though highly idealised and hardly representative of realistic design situations, these solutions have the great merit of capturing some fundamental aspects of the physics of the phenomenon, i.e. the dynamic response of the medium and the effect of the relative flexibility of the wall and the medium on their interaction. A recent study (Psarropoulos *et al*, 2005) presents a numerical assessment of the

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assumptions and results of the Veletsos model, showing how 2D finite element analyses converge to the Wood and the Veletsos solutions for rigid and flexible walls, respectively. Further, it is shown how, in the case of a flexible wall with compliant base, the dynamic visco-elastic solution becomes roughly equivalent to the static limit-equilibrium solution of Mononobe-Okabe. This supports the cited widespread adoption of the latter in seismic design.

Diaphragm walls represent in general a considerably more complex problem than gravity or cantilever walls. This increased complexity stems from several sources, such as the large variety of typologies (steel sheet-pile walls, drilled-pile concrete walls, with and without tiebacks, in single or multiple layers, etc), the fact that, due to their construction methods, they usually retain natural layered soil, rather than compacted granular backfill material, but most importantly, the essential role played by the dynamic soil-structure interaction in their stability. This last aspect makes the use of static limit-equilibrium approaches for their seismic design theoretically invalid. Yet, this is the only approach to be found in most international design codes (Kramer, 1996).

In-between limit-equilibrium approaches and non-linear dynamic finite element analyses (Siller *et al*, 1991), there is a number of intermediate pseudo-static possibilities adopted in professional practice, as reported e.g. in (Faccioli and Paolucci, 1996). The main feature of these approaches is that the seismic action is represented by equivalent static forces of assigned distribution. For what concerns soil-structure interaction, some of them adopt the well-known subgrade reaction method (SRM), others are based on the discretisation of a two-dimensional continuum with the finite element or difference method. The main aspect of arbitrariness in this group of approaches lies in the first mentioned aspect, i.e. the representation of the seismic action. An example of the consequences of such an arbitrariness is shown in Figure 1, adapted from (Faccioli and Paolucci, 1996).

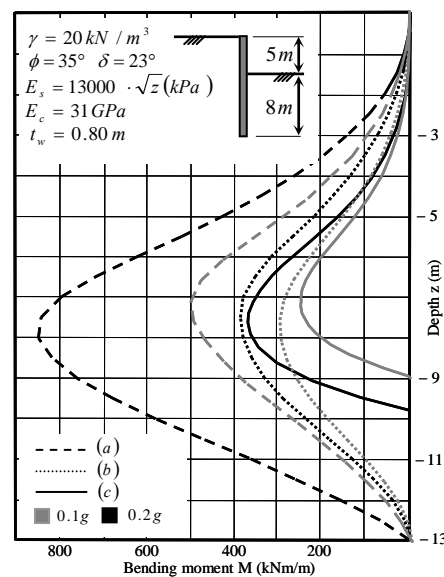


Figure 1. Bending moment distribution in an unanchored diaphragm wall in homogenous soil for PGA=0.1g and 0.2g. (a) FD analysis (b) SRM (c) limit-analysis (M-O). Adapted from (Faccioli and Paolucci, 1996).

The problem is that of an unanchored diaphragm wall in homogenous soil (dimensions in the figure) subjected to two levels of horizontal seismic acceleration of 0.1g and 0.2g. Diagram (a) is the result of an analysis carried out with FLAC, with an elasto-plastic soil model with Mohr-Coulomb failure criterion, perfect bond at the interface, under a uniform horizontal inertia force applied along the entire height of the wall; diagram (b) is the result of an elasto-plastic SRM analysis with an inverted triangular distribution of equivalent static forces on the wall terminating at the excavation level; finally, diagram (c) is obtained with Mononobe-Okabe earth pressures. The difference in the pattern of the applied forces is intentional and reflects the criteria associated to each of the methods in actual engineering practice. The difference in the results is beyond commenting.

The situation, as briefly outlined, clearly provides an incentive to devise solutions that, while remaining professionally practicable, adhere more closely to the mechanics of the problem. One such attempt is presented in the following of this paper.

THE PROPOSED MODEL

The problem under examination is characterised by the lack of symmetry introduced by the excavation. Qualitatively, the cut subdivides the soil stratum in two portions (regions A-B and D-E, in Figure 2.a) resting on a common base (region F) and interacting through the anchored/unanchored diaphragm wall (element C). Within each of these portions of soil two sub-regions can be further identified, one which is sensitive to the disturbance due to the interaction with the wall (regions B and D), and one which is not (regions A and E).

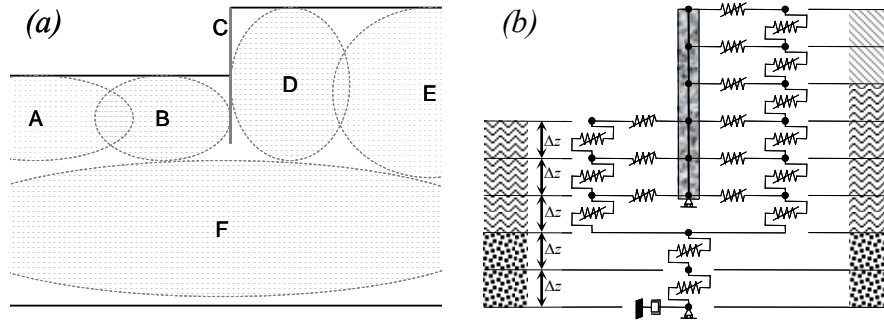


Figure 2. Model of the diaphragm-wall/soil system.

Figure 2.b shows the simplified one-dimensional model which attempts to simulate the response of the soil-wall system as described above. It includes a lower non-linear hysteretic soil column (F), continuing vertically with two distinct soil columns of unequal height (A and E, same type of nonlinearity as for F) connected through non-linear hysteretic springs (B and D) to the retaining structure (C). This system is excited at its base and includes a damper to account for a non reflecting boundary. Tiebacks, when present, are modelled as nonlinear horizontal springs, connecting the wall to the soil column at the same height, within the undisturbed (E) portion. The effect of inclination is accounted for in the stiffness/strength parameters.

The components of the model have the following properties. The wall is discretised into frame elements, that can have either elastic or any desired inelastic behaviour. Anchors are modelled as prestressed springs with tension-only elastic-plastic behaviour (Figure 3(a)). The parameters of the spring are the stiffness k , the ultimate force f_u , and the prestress f_0 . Since walls and anchors are generally designed for remaining elastic during and after strong ground motion, the interest on inelastic modelling of these components is limited.

The soil columns are discretised as mass-spring systems. Each node has a tributary mass m equal to the sum of half of the masses of the soil elements above and below. The hysteretic force-displacement relation for the soil springs is obtained from the local shear stress-strain law of the soil, and is modelled by the well-known and highly versatile Bouc-Wen model (Wen, 1975) (Song and Der Kiureghian, 2006) (Badoni and Makris, 1996), which gives the reaction force as a function of the displacement and of an internal variable ζ according to the equations:

$$f = \alpha k u + (1 - \alpha) k u_y \zeta \quad (1.a)$$

$$u_y \zeta + \gamma |u| \zeta |\zeta|^{n-1} + \beta u |\zeta|^n - A u = 0 \quad (1.b)$$

By setting $A = 1$ in the second expression, one recognises that k is the initial stiffness and α the hardening ratio. Further, by setting $\alpha = 0$ and $\beta + \gamma = 1$, the maximum resisting force is given by $f_{\max} = k u_y$. The values of n , β and γ determine the smoothness of the transition between the elastic and post-elastic branches, the ratio of unloading to loading stiffness and the amount of pinching,

respectively. The maximum force f_{\max} and the initial stiffness k are obtained from the soil properties according to:

$$f_{\max} = A_s \tau_{\max} = A_s \sigma_v \tan \phi \quad (2.a)$$

$$k = \frac{GA_s}{\Delta z} = \frac{A_s}{\Delta z} \rho V_s^2 \quad (2.b)$$

where A_s and Δz are the soil element cross-section area and height, σ_v is the average overburden pressure within the soil element, ρ , ϕ and V_s are the soil mass density, internal friction angle and the low-strain shear-wave velocity at the depth of the soil element.

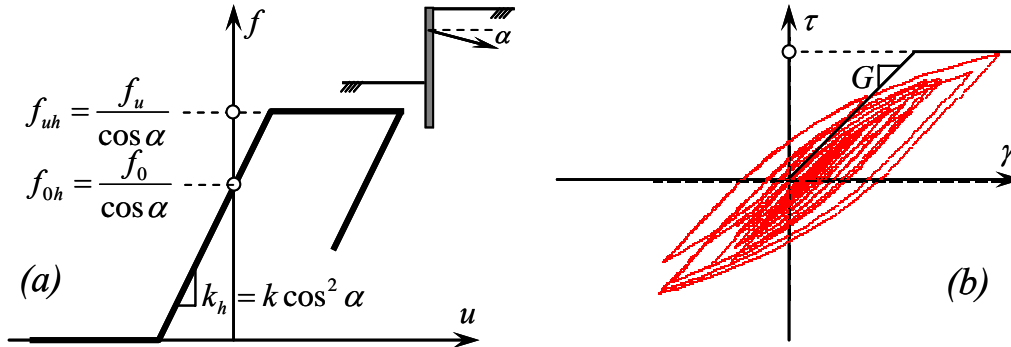


Figure 3. The constitutive law for the tiebacks and the soil columns.

The establishment of an appropriate constitutive law for the interface Winkler springs is central to the accuracy of the proposed model. Theoretically one could attempt to derive the properties of the uniaxial springs starting from a multiaxial cyclic material constitutive law. Since available multiaxial cyclic soil models are still rather limited, and considering also the limitation already implied by the use of a Winkler model, recourse to experimental data appears as the more reasonable solution. However, experimental results of soil cyclic tests under the stress-paths of interest, characterised by loading and unloading in compression, under a vertical confining pressure, are scarcely available.

Some indications on cyclic soil behaviour for these stress conditions can be found in (England *et al*, 2000), where the effect of cyclic horizontal displacements imposed by the thermal deformations of an integral bridge deck on the soil in contact with the abutment-walls is studied; also, recent numerical simulations of the ratcheting strain behaviour of granular soils can be found e.g. in (Alonso-Marroquin and Herrmann, 2004).

The constitutive law for the Winkler interaction springs is established based on the references above. Figure 4(a) illustrates the normalised-stress/strain paths for two soil elements located on opposite sides of the wall. It is noted the strain accumulation under stress cycling up to the active/passive thresholds (strain ratcheting) with stable loading/unloading stiffness and low dissipated energy per cycle. The simplified model adopted in this study is shown in Figure 4(b), and consists of a shifted non-symmetric elasto-plastic law characterised by four parameters, the shift σ_{h0} , the active and passive thresholds σ_{ha} and σ_{hp} , and the elastic stiffness E_h . Figure 4(b) does not show a complete cycle with strain reversal which would produce the large energy dissipation associated with an elasto-plastic model. However, for the typology of diaphragm walls considered the situation of complete strain reversal should occur infrequently, if at all, and has never arisen in all the analyses performed.

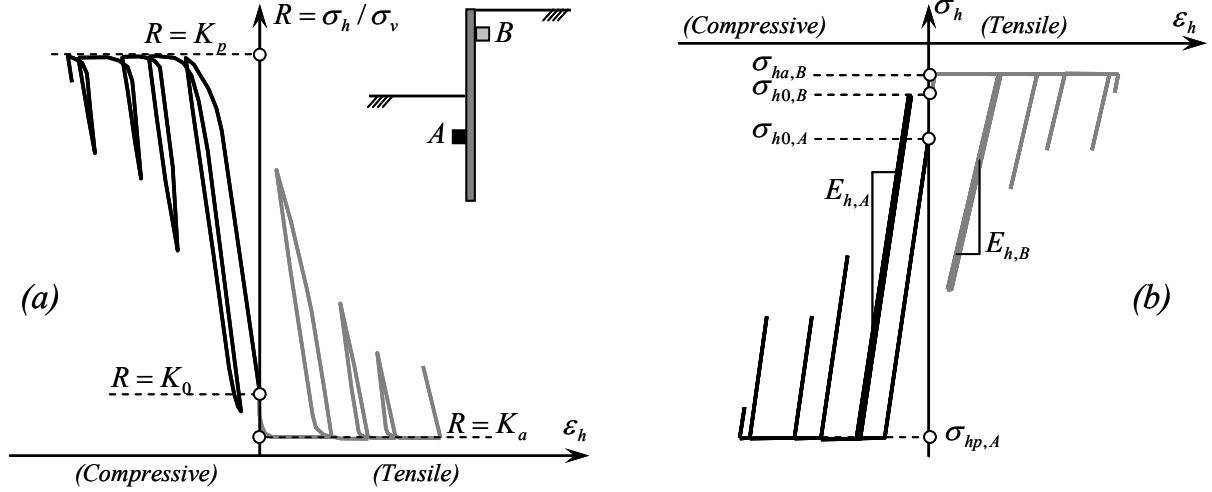


Figure 4. The nonlinear interface spring constitutive law.

The values of the interface parameters σ_{h0} , σ_{ha} , σ_{hp} , and E_h , are to be determined based on the soil type and stress history. This study covers only dry cohesion-less materials, governed by the internal friction angle ϕ . The horizontal stresses σ_{h0} , σ_{ha} , σ_{hp} are obtained as the product of the vertical stress σ_v times the usual coefficients K_o , K_a and K_p . The actual expressions employed are those from the Muller-Breslau theory of earth pressures (horizontal component), which account for the wall-soil friction angle δ :

$$K_o = 1 - \sin \phi \quad (3.a)$$

$$K_a = \frac{\cos^2 \phi}{(1 + \kappa)^2} \quad \text{with} \quad \kappa = \sqrt{\frac{\sin(\phi + \delta) \sin \phi}{\cos \delta}} \quad (3.b)$$

$$K_p = \frac{\cos^2 \phi}{(1 - \kappa)^2} \quad (3.c)$$

For values of the soil-wall friction angle $\delta > \phi/3$ the above expressions increasingly underestimate K_a and overestimate K_p .

The actual spring force-displacement constants follow by multiplying the above parameters by the corresponding tributary area.

As it regards the elastic spring stiffness, this is set equal to $E_h = 1.2E_s$ where:

$$E_s = 2G(1 + \nu) = 2\rho V_s^2(1 + \nu) \quad (4)$$

is the Young modulus and ν the Poisson coefficient of the soil medium. The increase in stiffness due to the plane-strain condition and the overburden pressure is accounted for by means of the 1.2 factor. The sensitivity of the model to this parameter has been investigated to a limited extent and found to be quite limited. It is stressed that in this inelastic model the initial stiffness of the springs is less important than the secant stiffness, which depends on the limit active and passive stresses.

NUMERICAL EXAMPLE

The proposed model is applied to the analysis of the unanchored diaphragm wall shown in Figure 5. For this example the ground is assumed to be spatially homogeneous dry sand with the mechanical properties in the figure. The concrete wall thickness is 0.5m. The case study does not conform to common thumb-rules for the proportioning of unanchored walls of this type. In particular, the embedment length (5m) is smaller than the excavation height (6m). A verification with conventional design methods (static sub-grade reaction method) has shown the wall to be safe with safety margins

just slightly lower than usual. Internal forces are compatible with the resistance of the concrete wall. The reason for selecting this example is that significant displacements are expected under moderate seismic action and this allows a verification and validation of the model well in the nonlinear range.

A moderately large value of the soil-wall friction has been chosen ($\delta = 0.67\phi$). As already mentioned, the employed expression for K_a and K_p underestimate and overestimate, respectively, the corresponding “true” values from “log-spiral” theory. For this value of δ , however, the error is within -10% and +20% for K_a and K_p , respectively.

The values adopted for the Bouc-Wen parameters of the soil columns are $n = 3$ and $\beta = \gamma = 0.5$.

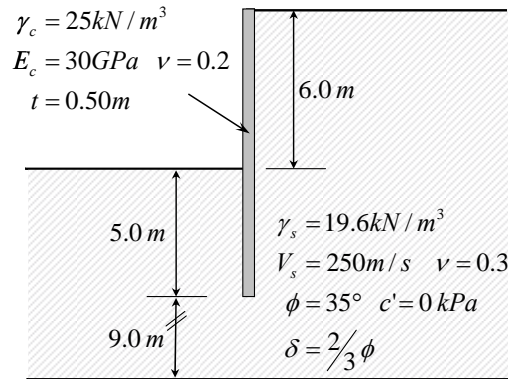


Figure 5. Case-study: geometry and material properties.

Two different excitations are considered. The first one is a short duration artificial signal, a 2nd order Ricker wavelet (Ricker, 1943) with central frequency $f = 2\text{Hz}$, chosen to represent a short duration displacement pulse as in (Gazetas *et al*, 2004). The second one is a natural ground motion from the 1997 Umbria-Marche (Italy) $M = 5.5$ earthquake, recorded in Colfiorito, on stiff soil, at 7km from the epicenter. This ground motion has a duration of 15s and a peak ground acceleration of about 0.12g. For the purpose of the analyses to follow it is scaled up to the same PGA of 0.2g used for the Ricker wavelet. Figure 6 shows the two signals.

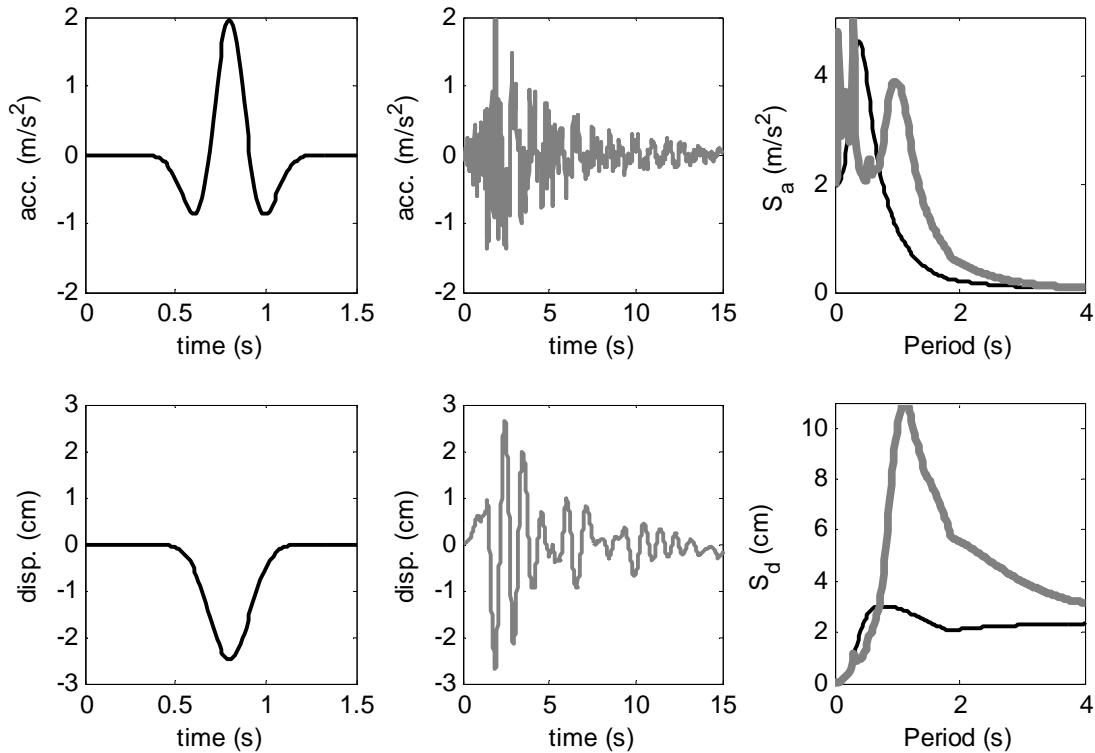


Figure 6. Selected excitations: acceleration (top) and displacement (bottom) time-histories and response spectra of the Ricker wavelet (black) and of the Colfiorito record (red).

Validation versus 2D nonlinear finite element analysis

Plane strain model set-up

The validation of the model is carried out by comparing results in terms of wall bending moment distributions and top displacement time-history against those obtained by a refined plane-strain analysis employing the finite element code PLAXIS.

As it regards the soil, the basic inelastic material model is employed, i.e. the elasto-plastic model with Mohr-Coulomb failure criterion. Parameters have been assigned according to Figure 5. As advised in the program documentation, a small cohesion $c = 1\text{kPa}$ is adopted for improved numerical stability. The soil-wall interface can be modeled as rigid (perfect compatibility) or non-rigid (a gap with friction). The latter option is adopted and the interface parameter $R = \tan\delta/\tan\phi$ is set to 0.62 in order to match the $\delta = 0.67\phi$ used in the proposed model.

Available absorbing boundaries in PLAXIS (Lysmer and Kuhlemeyer, 1969) do not work for the seismic case. These dashpots are automatically connected to a fixed reference and hence can only be used to absorb waves coming from a source inside the meshed domain as it occurs, for instance, in a forced-vibration problem on a half space (in the seismic case they should work on the relative velocity between free field and model along the lateral boundaries of the latter). To avoid excessive and uncontrollable energy absorption, these boundaries are not used and the model is extended for 100m on both sides of the diaphragm wall (see geometry and mesh in Figure 7, elements are 6-node triangular with quadratic displacement, i.e. linear strain, interpolation; the mesh is a non-structured one).

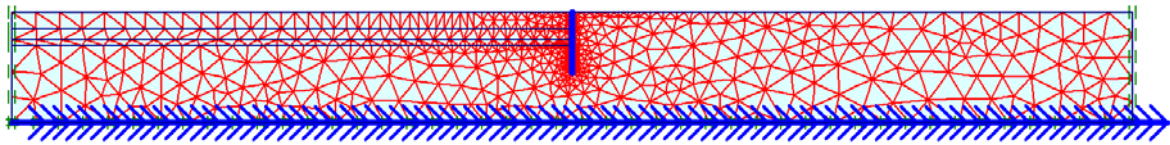


Figure 7. Plane-strain finite element model in PLAXIS (the mesh extends for 100m on both sides of the diaphragm, the mesh is shown at the beginning of the analysis before excavation starts).

Damping is present in the model in two forms. The default settings for the Newmark integration algorithm in PLAXIS are $\alpha = 0.6$ and $\beta = 0.3025$, which correspond to a damped Newmark scheme. These settings have been used also in the proposed model to replicate the same numerical damping. Further dissipation is added in the form of initial-stiffness proportional Rayleigh damping with viscous damping ratio $\xi = 0.01$ at the natural periods of the left (short) and right (tall) soil columns, as computed in the proposed model. The same mass and stiffness coefficients $\alpha_M = 0.23$ and $\beta_K = 0.0004$ are used in PLAXIS for the soil. The overall amount of damping in the model is kept intentionally small for this comparison in order to highlight differences between the models that might otherwise be masked by larger damping.

Finally, in PLAXIS ground motion can be input to the model only in the form of a time-varying multiplication factor for prescribed imposed displacements. The program accepts both acceleration and displacement time-histories for the multiplication factor, performing internally the double integration with base-line correction in the former case. Application of a time-varying prescribed displacement at the base corresponds to the case of a fully-reflecting boundary, i.e. an infinitely rigid bedrock. Hence the motion is applied in both programs as an acceleration time-history to a fixed-base model.

Results

Figures 8 and 9 show the results of the comparison for the Ricker wavelet and the Colfiorito record, respectively. Both figures show wall bending moment and top-displacement, main design quantities for these structures, from the proposed 1D model and the 2D finite element analysis. The match is quite satisfactory for both quantities, with the proposed model yielding a slight overestimation of the maximum bending moment and corresponding underestimation of the maximum displacement.

The agreement between the predictions is almost unexpectedly good and it is confirmed in other examples not reported in the paper. A more conclusive assessment of the model performance, however, requires a wider parametrical exploration, which is currently under way.

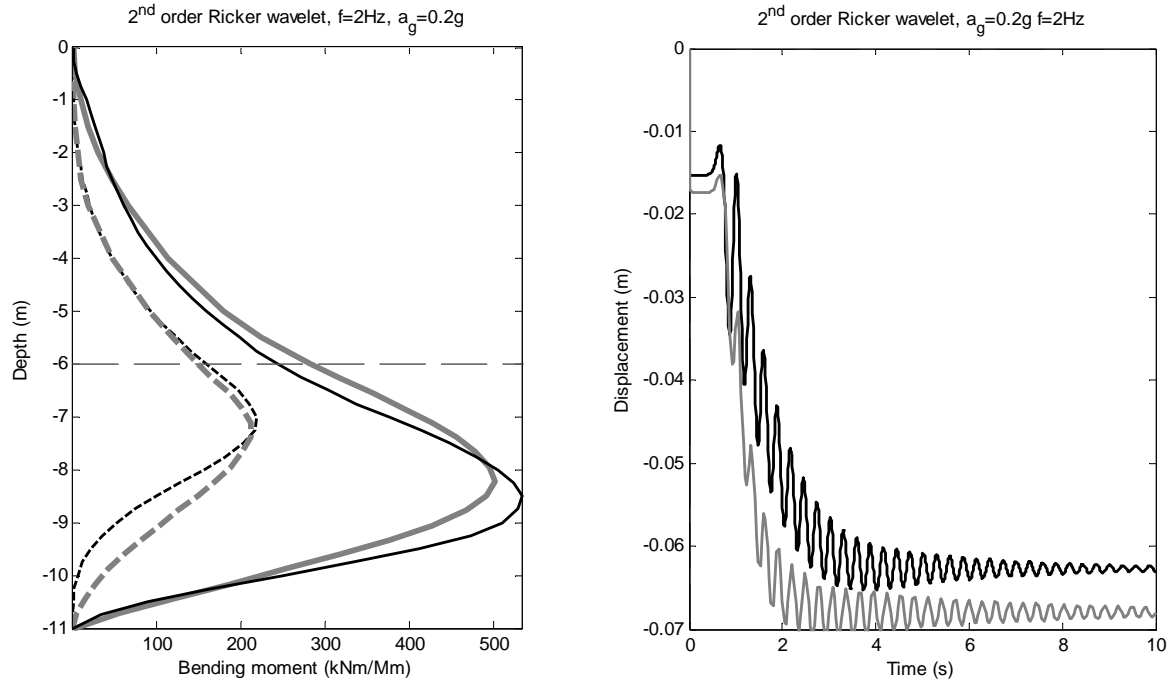


Figure 8. Comparison of model (black) versus PLAXIS (grey) results: left, bending moment distribution in the diaphragm at the end of the excavation (dashed) and dynamic envelope (solid); right, horizontal displacement time-history of the diaphragm wall top.

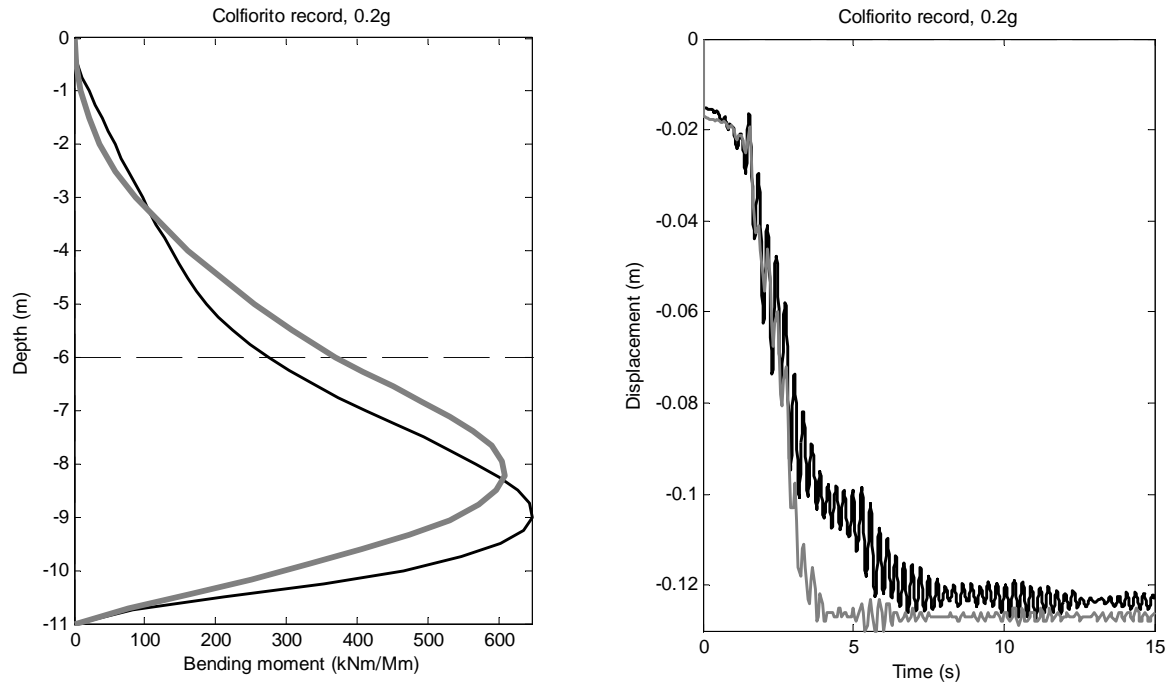


Figure 9. Comparison of model (black) versus PLAXIS (grey) results for the Colfiorito record: left, dynamic envelope of bending moment in the diaphragm; right, horizontal displacement time-history of the diaphragm wall top.

Additional results

The model employed in the previous paragraph is recognisedly unrealistic especially due to the reduced ground thickness between the wall lower tip and the fully-reflecting lower boundary. This choice is motivated by reasons of computational economy, given that the purpose of the analyses is one of comparison, rather than of realistic response assessment, for which the finite element model should have been extended much further below.

This is not an issue with the proposed model, to which ground motion can be input either as an acceleration time-history at a fixed base (infinitely rigid bedrock) or as force time-history proportional to the ground motion velocity (compliant base) as in:

$$f(t) = c_{base} v(t) = 2A_s \rho_b V_{sb} v(t) \quad (5)$$

where $v(t)$ is the ground velocity corresponding to the acceleration obtained by deconvolving the target outcropping signal to the lower boundary of the model. The depth of this boundary needs not to be larger than the depth of the region sensitive to the interaction with the wall. In (5) ρ_b and V_{sb} are the mass density and the shear wave-velocity of the supposedly uniform half-space below the model base, while A_s is the area of one of the two soil columns. The procedure is schematically illustrated in Figure 10.

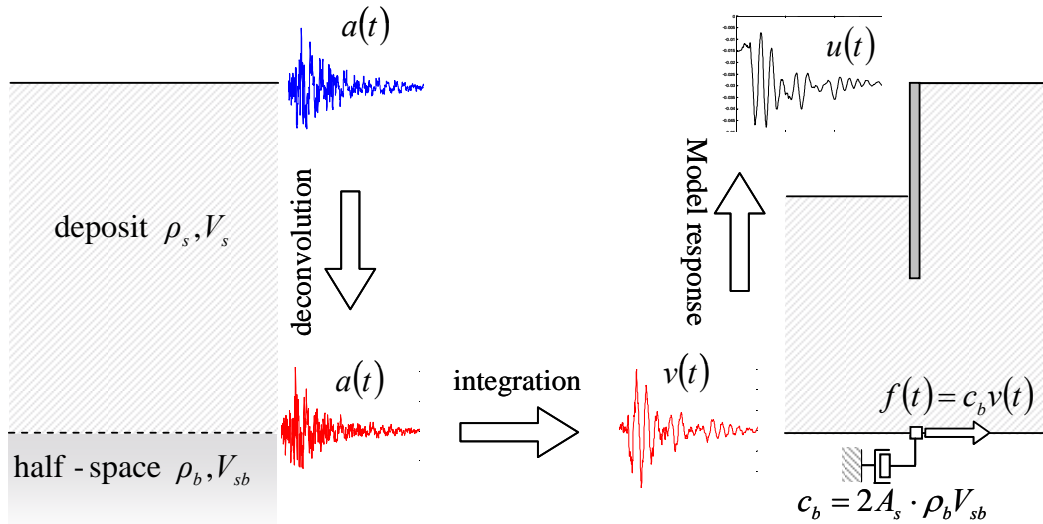


Figure 10. Application of the excitation to the model base for a non-rigid supporting half-space.

Figure 11 shows the target input motion (the Colfiorito record) and the corresponding signal at the depth of -20m, obtained by deconvolution with the program SHAKE91 (Idriss and Sun, 1992), assuming that the half-space has the same properties of the overlying ground layer and using Seed and Idriss average curves for sand for equivalent damping and modulus degradation curves.

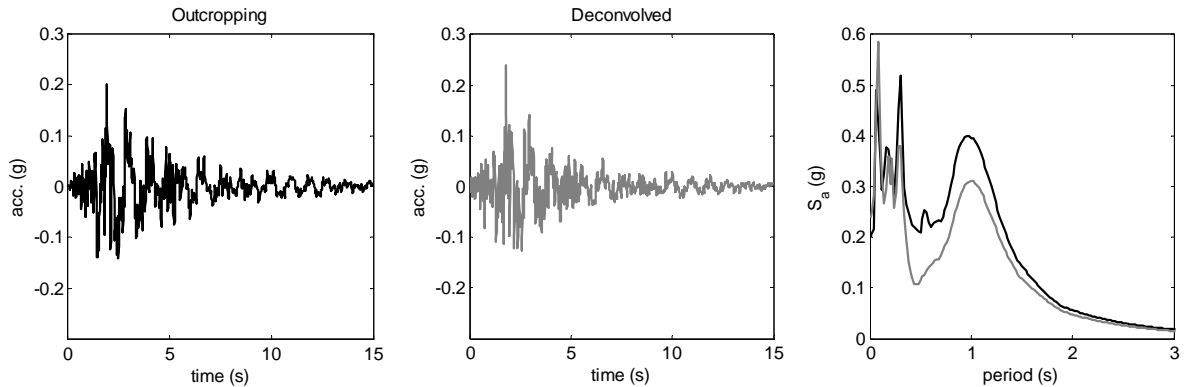


Figure 11. Outcropping and deconvolved acceleration THs and corresponding response spectra.

Given the apparent similarity of the two acceleration time-histories, and considering the overall approximation intrinsic in all available approaches, it can be of interest to compare the responses obtained by applying at the base the deconvolved signal or, more simply, the outcropping signal, thus avoiding the extra task of deconvolution. The results are shown in Figure 12, in terms of maximum absolute acceleration profile along the height of the right (tall) soil column, and of bending moment and top displacement of the wall. As expected, the original outcropping signal applied directly at the base yields a larger response, since it incorporates the amplification occurring in the upper layers. In the present case of uniform soil properties, however, this effect is not pronounced and, in particular, the bending moments in the wall are practically coincident.

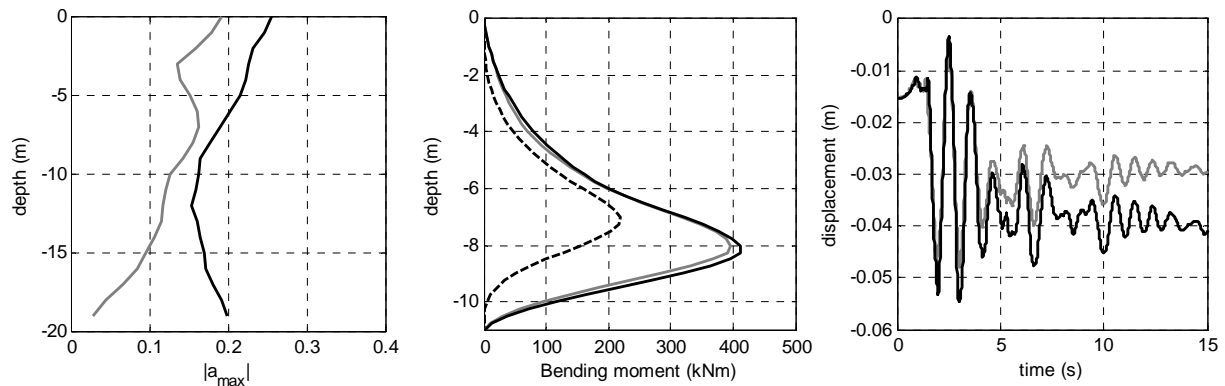


Figure 12. Comparison of results using the original (black) versus the deconvolved record (grey): max. acceleration profile (left); moment in the wall (centre); wall top displacement TH (right).

CONCLUSIONS

The seismic analysis of diaphragm walls is quintessentially dynamic in nature, since the pressures in the wall depend on the dynamic response of a geometrically irregular soil profile and on the dynamic, inelastic, interaction between the soil and the diaphragm. The pseudo-static approaches in use for free-standing retaining walls have no way of capturing the above phenomena, hence their extension for the design of diaphragm walls is conceptually base-less, and has been shown to lead to widely different results depending on the criteria adopted in their application.

The attempt made with the model described in the paper is that of incorporating, in the simplest yet physically realistic way, the two salient features of the problem: the (nonlinear) dynamic response of the soil profile and the inelastic cyclic interaction between soil and diaphragm. The comparison with the results obtained from a specialized nonlinear finite element code for a selected case study shows that the accuracy of the model is remarkably good, with a computing time only a tiny fraction of the needed by the finite element code. Conceptual simplicity, allowing easier control on the mechanics of the problem, and computational efficiency, removing all computing-time related difficulties, are believed to represent significant advantages of the proposed model. The model is currently being tested in a number of cases differing in the geometry of the wall, mode of variation of soil properties along the height, and presence of tie-backs.

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REFERENCES

- Alonso-Marroquin F and Herrmann HJ. "Ratcheting of granular materials" *Phys. Rev. Lett.*, 92:054301-1 –054301-4, 2004.
- Badoni D and Makris N. "Nonlinear response of single piles under lateral inertial and seismic loads" *Soil Dynamics & Earthquake Engng*, 15:29-43, 1996.

- England GL, Tsang NCM, and Bush DI. "Integral bridges: a fundamental approach to the time-temperature loading problem" Thomas Telford, London, 2000.
- European Committee for Standardization (CEN). Eurocode 8 - Design Provision for Earthquake Resistance of Structures. prEN 1998:2001 E, 2003.
- Faccioli E and Paolucci R. "Seismic behaviour and design of foundations and retaining structures" Technical report, European Commission - Human Capital and Mobility Program: prenormative research in support of Eurocode 8 (ECOEST-PREC8), July 1996.
- Gazetas G, Psarropoulos PN, Anastasopoulos I, and Gerolymos N. "Seismic behaviour of flexible retaining systems subjected to short-duration moderately strong excitation" Soil Dynamics & Earthquake Engng, 24:537-550, 2004.
- Green RA and Ebeling RM. "Seismic analysis of cantilever retaining walls, phase i." Technical Report ERDC/ITL TR-02-3, US Army Corps of Engineers - Engineer Research and Development Center, September 2002.
- Kramer SL. "Geotechnical Earthquake Engineering" Prentice Hall, Upper Saddle River, NJ, 1996.
- Lysmer J and Kuhlemeyer RL. "Finite dynamic model for infinite media" ASCE Jnl Eng. Mech. Div., 4:859-977, 1969.
- Mononobe N and Matsuo H. "On the determination of earth pressures during earthquakes" volume 9, Tokyo, 1929. World Engineering Congress.
- Okabe S. "General theory of earth pressures" Jnl Japan. Soc. Civil Eng., 12, 1926.
- Psarropoulos PN, Klonaris G, and Gazetas G. "Seismic earth pressures on rigid and flexible retaining walls" Soil Dyn. & Earthquake Engng, 25:795-809, 2005.
- Ricker N. "The computation of output disturbances from amplifiers for true wavelet inputs" Bull. Seism. Soc. America, 33(3): 197-228. 1943.
- Seed HB and Whitman RV. "Retaining structures for dynamic loads" Cornell University, Ithaca, NY, 1970. ASCE Specialty Conf. of Lateral stresses in the ground and the design of earth-retaining structures.
- Siller TJ, Christiano PP, and Bielak J. "Seismic response of tied-back retaining walls" Earth. Eng. Struct. Dyn., 20:605-620, 1991.
- Song J and Der Kiureghian A. "Generalized Bouc-Wen Model for Highly Asymmetric Hysteresis" ASCE Jnl Eng Mech, 132(6), 2006.
- Veletsos AS and Younan AH. "Dynamic modeling and response of soil-wall systems" ASCE Jnl Geotech Engng, 120(12), 1994.
- Veletsos AS and Younan AH. "Dynamic response of cantilever retaining walls" ASCE Jnl Geotech. & Geoenv. Engng, 123(2), 1997.
- Wen YK. "Approximate method for nonlinear random vibration" ASCE Jnl Eng. Mech. Div., 101:389-401, 1975.
- Wood JH. "Earthquake-induced pressures on a rigid wall structure" Bull.NZ Natl Earthquake Engng, 8:175-186, 1975.
- Younan AH and Veletsos AS. "Dynamic response of flexible retaining walls" Earth. Eng. Struct. Dyn., 29:1815-1844, 2000.