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ANALYTICAL INVESTIGATION OF DEPTH NON-HOMOGENEITY EFFECT ON DYNAMIC RESPONSE OFA SATURATED SOIL LAYER SUBJECTED TO BED MOTION

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ABSTRACT

An analytical solution for the dynamic response of a saturated continuously non-homogeneous soil layer laid on a moving bed rock is presented and the depth non-homogeneity effects on amplification of bed motion at the ground surface or any other points of the media is studied. The presented results of the paper clearly show how the dynamic response of media and the variation form of amplification factor including displacement profile along depth and resonant frequencies are affected by the depth non-homogeneity parameter, the characteristic parameter of soils, ignored in lots of the available studies where the soil layer is assumed to be a homogeneous media.

Keywords: Depth non-homogeneity, Site effect, Amplification factor, Shear wave propagation, Saturated media.

INTRODUCTION

Propagation of shear wave from the bed rock to the surface, illustrated in figure 1, depends on the bed rock motion, frequency content, soil properties and soil thickness. In most available analytical solutions, the soil layer is assumed to be a homogeneous media where the experimental evidences show that even for a non layered media, a constant depth profile for the shear modulus may be a rather poor approximation to the real sub-soil conditions since soil stiffness usually varies with depth.

There are some solutions, based on dividing the non-homogeneous layer into multi layer system that the mechanical properties are constant for each layer but sudden change for one layer to another one [8], but discretization of soil causes cumulative error in computation of transfer function, that sum of these errors in the whole layer may be not negligible and also sudden change in soil properties can yield to sensible effect on results.

In this research, the dynamic response of a saturated soil layer subjected to bed motion is studied analytically. The results of the research are used to analyze the effect of depth non-homogeneity on dynamic response of deep soil layers including the amplification factor of rigid bed rock motion at the ground surface (Site Effect) or any other points and also the results can be approximately used to analyze the dynamic response of a multi-layer media if the variation of shear modulus in different layers is estimated by an appropriate continuous function in the whole media. In the latter case, in compare of usual methods, the computations are mainly decreased.

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PROBLEM STATEMENT

As shown in figure 1, a bounded saturated media subjected to rigid bed motion is considered. The mass density, porosity and permeability of the media are constant but the shear modulus varies solely with depth.

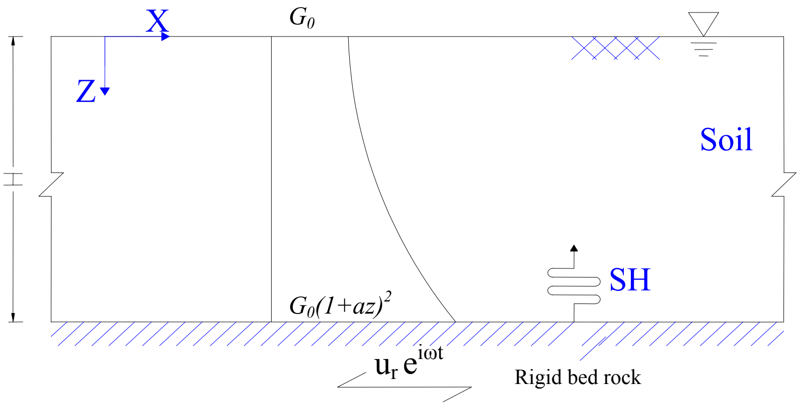


Figure1. Continuously non-homogeneous saturated media subjected to bed rock motion

The variation of shear modulus is described by a quadratic function; this type of variation was proposed by Gibson, for normally-consolidated saturated clays (Gibson [6]) :

$$G(z) = G_0 (1 + az)^2 \tag{1}$$

Where G_0 is the shear modulus at the surface and a is a constant with the dimension of inverse length, called coefficient of depth non-homogeneity or non-homogeneity parameter. By varying the parameters a and G_0, a wide range of real soil strata can be approximately described by equation (1).

The media is assumed to obey Biot's dynamic poroelastic theory (Biot [2]), So the following equations which are total dynamic equilibrium equation, the mean of effective stress, linear elastic constitutive law, dynamic equilibrium for the fluid phase (generalized Darcy law) and the mass conservation law respectively, could be written as:

$$\sigma_{ij.j} + f_i = \rho \ddot{u}_i + \rho_f \ddot{w}_i \tag{2}$$

$$\sigma'_{ij} = \sigma_{ij} + \delta_{ij} p \tag{3}$$

$$\sigma'_{ij} = \lambda^* u_{k,k} \delta_{ij} + G^* (u_{i,j} + u_{j,i}) \tag{4}$$

$$- p_{,i} = \rho_f \ddot{u}_i + \frac{\alpha \rho_f}{n} \ddot{w}_i + b \dot{w}_i \tag{5}$$

$$\dot{u}_{i,i} + \dot{w}_{i,i} + \frac{\dot{P}}{Q_f} = 0 \tag{6}$$

where σ_{ij} represents the total stress tensor, σ'_{ij} represents the effective stress tensor, f_i denotes body force, ρ denotes mass density of saturated media, ρ_f denotes mass density of the fluid phase, u_i denotes the displacement of the solid phase, w_i denotes the displacement field of fluid phase, P denotes the pore water pressure, δ_{ij} is the Kroncker delta, λ^* and G^* are complex Lame coefficients, α denotes the additive mass coefficient, n denotes the porosity of saturated media, b denotes diffusive coefficient, Q_f denotes compressibility modulus of saturated media.

In the classic mechanics of porous media, the parameters α, b, and Q_f are defined as:

$$\alpha = \frac{1}{2}(1+\frac{1}{n}) \tag{7}$$

$$b = \frac{g\rho_f n}{k} \tag{8}$$

$$\frac{1}{Q_f} = \frac{n}{Q} + \frac{1-n}{k_s} \tag{9}$$

Where g is the acceleration of gravity, k is the permeability of the media, Q is the compressibility modulus of the fluid phase and k_s is the bulk module of solid grains, usually assumed as ∞. Also ρ, the mass density of the saturated media, is calculated by the following formula:

$$\rho = (1-n)\rho_s + n\rho_f \tag{10}$$

For one dimensional shear wave propagation, as illustrated in figure 1, because σ_{xz} is the only non-zero stress and also because the effective shear stress is always equal to total shear stress(eq. 3), if the body forces are neglected the equation (2) is reduced to:

$$\frac{d\sigma'_{xz}}{dz} = \rho\ddot{u}_x + \rho_f\ddot{w}_x \tag{11}$$

On the other hand, because u_x and w_x are the only non-zero displacements of solid and fluid phases respectively and also they solely have differentiations with respect to z-direction, so the equation (4) is reduced to:

$$\sigma'_{xz} = G^* \frac{du_x}{dz} \tag{12}$$

Where G^*, the complex shear modulus, is defined as:

$$G^* = G(1+2\delta i) \tag{13}$$

Where δ is the hysteretic damping coefficient. The combination of equations (11) and (12) is as follows:

$$G^* \frac{d^2u_x}{dz^2} + \frac{dG^*}{dz}\frac{du_x}{dz} = \rho\ddot{u}_x + \rho\ddot{w}_x \tag{14}$$

Because the problem is investigated in frequency domain and also as mentioned above the volumetric strains, $u_{i,i}$ and $w_{i,i}$, are zero, according to equation (6), the pore water pressure equals zero. So the equation (5) can be written as:

$$\rho_f \ddot{u}_x + \frac{\alpha \rho_f}{n} \ddot{w}_x + b\dot{w}_x = 0 \tag{15}$$

If the x subscripts are omitted for simplicity, then equations (14) and (15) are rewritten as:

$$G^* \frac{d^2u}{dz^2} + \frac{dG^*}{dz}\frac{du}{dz} = \rho \ddot{u} + \rho_f \ddot{w} \tag{16}$$

$$\rho_f \ddot{u} + \frac{\alpha \rho_f}{n} \ddot{w} + b\dot{w} = 0 \tag{17}$$

Because the motion is time harmonic i.e.:

$$u(z,t) = u(z)e^{i\omega t} \tag{18}$$

$$w(z,t) = w(z)e^{i\omega t} \tag{19}$$

The equations (18) and (19) can be rewritten as:

$$\rho\omega^2 u + \frac{dG^*}{dz}\frac{du}{dz} + G^* \frac{d^2u}{dz^2} + \rho_f \omega^2 w = 0 \tag{20}$$

$$\rho_f \omega^2 u + \frac{\alpha \rho_f}{n} \omega^2 w - ib\omega w = 0 \tag{21}$$

From equation (21), the relation between w and u can be written as:

$$w = -\frac{\rho_f \omega}{\frac{\alpha \rho_f}{n}\omega - ib} u \tag{22}$$

Inserting the above relation into equation (20) yields to governing differential equation of shear wave propagation as follows:

$$G^* \frac{d^2u}{dz^2} + \frac{dG^*}{dz}\frac{du}{dz} + \theta u = 0 \tag{23}$$

Where

$$\theta = \rho\omega^2 - \frac{\rho_f^2 \omega^3}{\frac{\alpha \rho_f}{n}\omega - ib} \tag{24}$$

The boundary conditions of problem are:

$$@\ z=0 \qquad \sigma_{xz} = G_0 \frac{du}{dz} = 0 \tag{25}$$

$$@\ z=H \qquad u = u_r \qquad \textit{(input motion)} \tag{26}$$

The second order differential equation (23), subjected to above boundary conditions defines the boundary value problem for the dynamic response of a continuously saturated layer in frequency domain.

GENERAL SOLUTION

To solve the boundary value problem, it is convenient to introduce a subsidiary depth variable:

$$\xi = 1 + az \tag{27}$$

Which transforms the interval $0 \le z \le H$ onto $1 \le \xi \le 1 + aH$. The shear modulus variation reduces to:

$$G = G_0 \xi^2 \tag{28}$$

Inserting the above transformations into differential equation (23) yields to:

$$G_0^* a^2 \xi^2 u'' + 2G_0^* a^2 \xi u' + \theta u = 0 \tag{29}$$

( )´denotes differentiation with respect to ξ. The equation (29) can be simplified to:

$$\xi^2 u'' + 2\xi u' + \eta u = 0 \tag{30}$$

Where

$$\eta = \frac{\theta}{G_0^* a^2} \tag{31}$$

The equation (30) is in the form of classic Cauchy-Euler equation. The general solution of the equation is [5]:

$$u = A\xi^{R_1 + I_1 i} + B\xi^{R_2 + I_2 i} \tag{32}$$

Where A and B are arbitrary coefficients, $R_1+I_1 i$ and $R_2+I_2 i$ are complex roots of the following distinctive equation:

$$\lambda^2 + \lambda + \eta = 0 \tag{33}$$

Two arbitrary coefficients A and B are determined by satisfying boundary conditions, mentioned in equations (25) and (26). It is necessary to rewrite the boundary conditions by using subsidiary depth variable ξ. So the boundary conditions are converted to:

$$@\ \xi = 1 \qquad \frac{du}{d\xi} = 0 \tag{34}$$

$$@\ \xi = 1 + aH \qquad u = u_r \tag{35}$$

If the available solution for u, based on equation (32), is inserted into the above boundary condition, we have:

$$A(R_1 + I_1 i) + B(R_2 + I_2 i) = 0 \tag{36}$$

$$A(1+aH)^{R_1+I_1 i}+B(1+aH)^{R_2+I_2 i}=u_r \tag{37}$$

Solving the system of simultaneous equations for A and B, we obtain:

$$A=\frac{-u_r(R_2+I_2 i)}{(R_1+I_1 i)(1+aH)^{R_2+I_2 i}-(R_2+I_2 i)(1+aH)^{R_1+I_1 i}} \tag{38}$$

$$B=\frac{u_r(R_1+I_1 i)}{(R_1+I_1 i)(1+aH)^{R_2+I_2 i}-(R_2+I_2 i)(1+aH)^{R_1+I_1 i}} \tag{39}$$

Using above solutions for A and B into equation (32) results explicit solution for displacement function at any point within the domain.

RESULTS

The presented analytical solution in the previous sections has been applied to investigate the depth non-homogeneity effect on dynamic response of the media and shear wave propagation utilizing the dimensionless variables as follows:

$$\overline{\varpi}=\frac{\omega H}{v_s} \tag{40}$$

$$\overline{u}=\frac{\|u_{z=0}\|}{\|u_{z=H}\|} \tag{41}$$

$$\overline{a}=aH \tag{42}$$

$$\overline{k}=\frac{k}{v_s} \tag{43}$$

Where $\overline{\varpi}$ is dimensionless frequency, $\overline{u}$ is dimensionless displacement (amplification factor of bed motion), $\overline{a}$ is the dimensionless coefficient of depth non-homogeneity, $\overline{k}$ is dimensionless permeability and v_s is the shear wave velocity in solid phase which is defined as:

$$v_s=\sqrt{\frac{G_0}{\rho_s}} \tag{44}$$

In order to study the effect of depth non-homogeneity on amplification of bed rock motion at ground surface (site effect), variation of amplification factor versus frequency is illustrated in figure 2 for the various magnitudes of non-homogeneity parameter and the selected values of porosity and permeability. The other model's properties are given in table 1. As shown in the figure, the variation form of amplification factor including the maximum values and resonant frequencies depends on the magnitude of non-homogeneity parameter.

Table1. Selected models' properties

No.	Models' Parameter	Value	Unit
1	Shear modulus at surface (G_0)	18	MPa
2	Compressibility modulus of fluid (Q)	2068	MPa
3	Mass density of grains (ρ_s)	2650	Kg/m^3
4	Mass density of fluid (ρ_f)	1000	Kg/m^3
5	Height of the layer (H)	3	m

On the other hand, in the figure 3, the variation of amplification factor at the ground surface versus depth non-homogeneity parameter is illustrated for the selected values of frequency (sensitivity analysis). As it is clear from the figure, for large values of depth non-homogeneity parameter, the amplification factor converges to 1, implying the soil layer tends to a rigid medium.

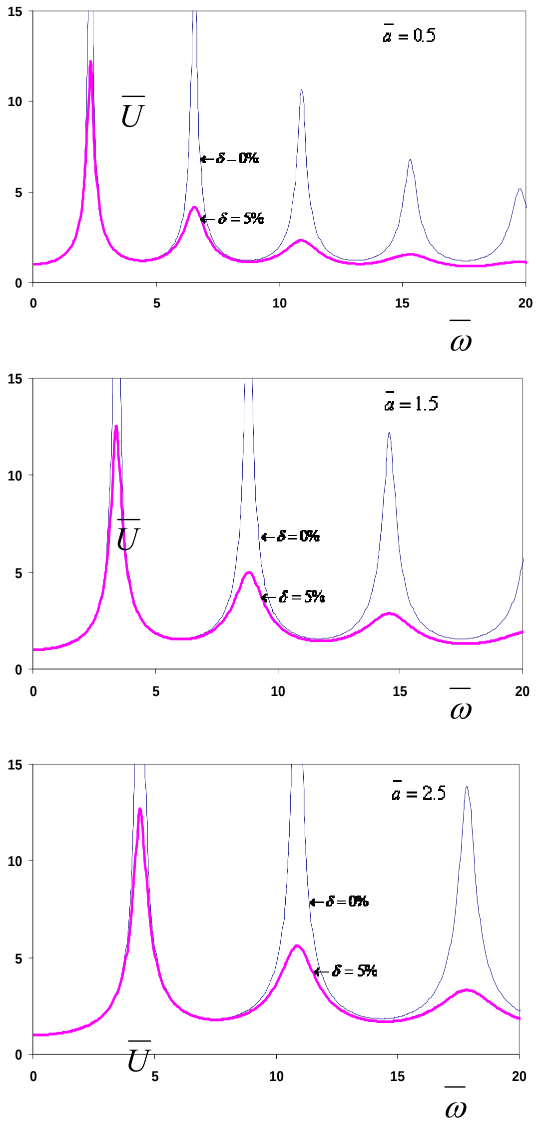


Figure2. Amplification factor at the ground surface surface versus frequency for different values of depth different homogeneity parameter ($n = .30, \bar{k} = 10^{-5}$)

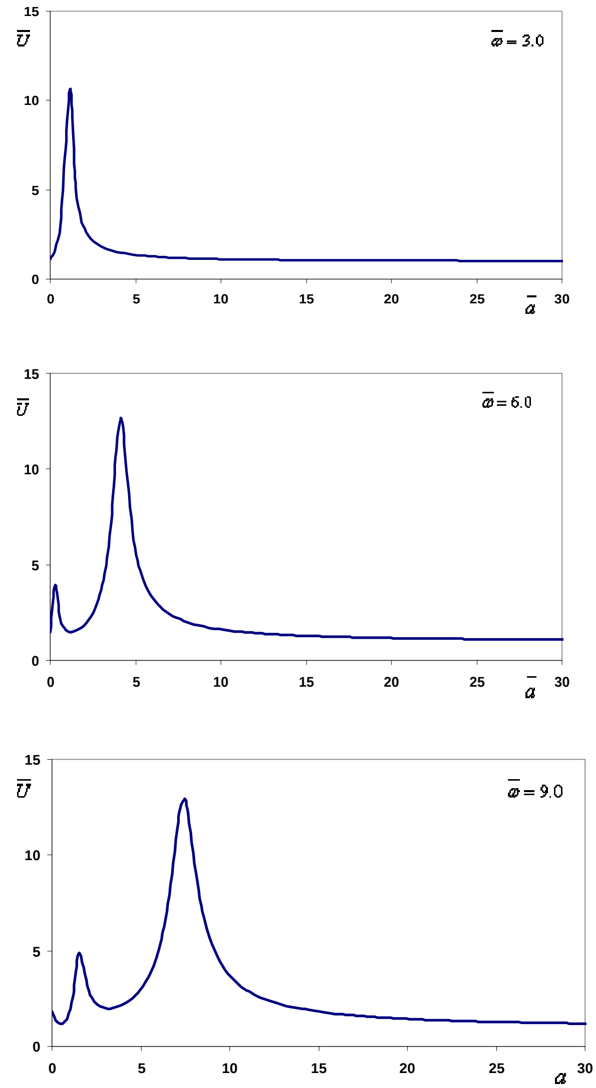
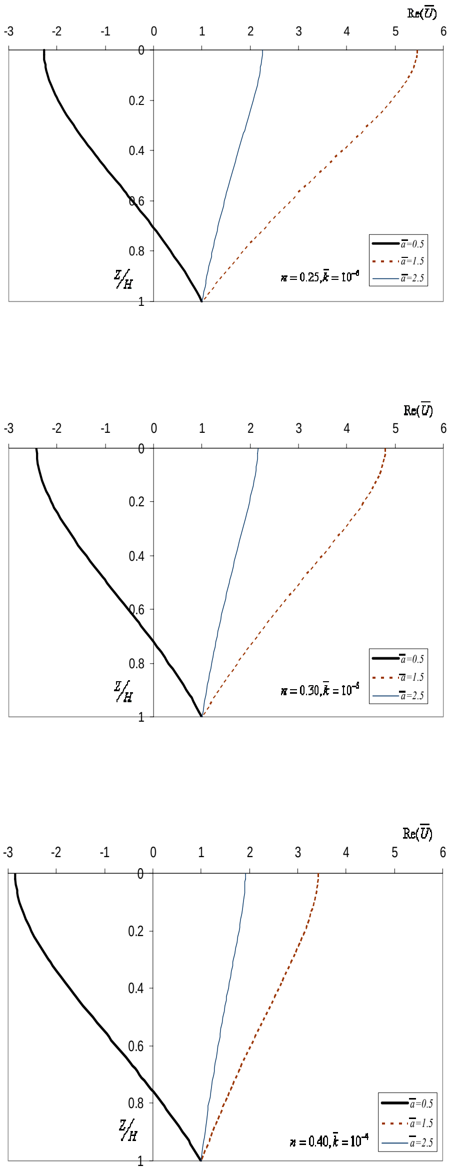


Figure3. Amplification factor at the ground versus depth non-homogeneity parameter for values of frequency ($n = .30, \bar{k} = 10^{-5}, \delta = 0.05$)

Displacement profiles along depth at various magnitudes of depth non-homogeneity parameter are illustrated in figure 4 for the different values of porosity and permeability. As shown in the figure the displacement profile is strongly dependant to depth non-homogeneity parameter.

Finally in figure 5, the three dimensional diagrams of amplification factor versus frequency and depth non-homogeneity are illustrated for the different values of porosity and permeability. This figure shows the dependency of the am plification factor curve to shear modulus distribution and depth non-homogeneity parameter in the general form



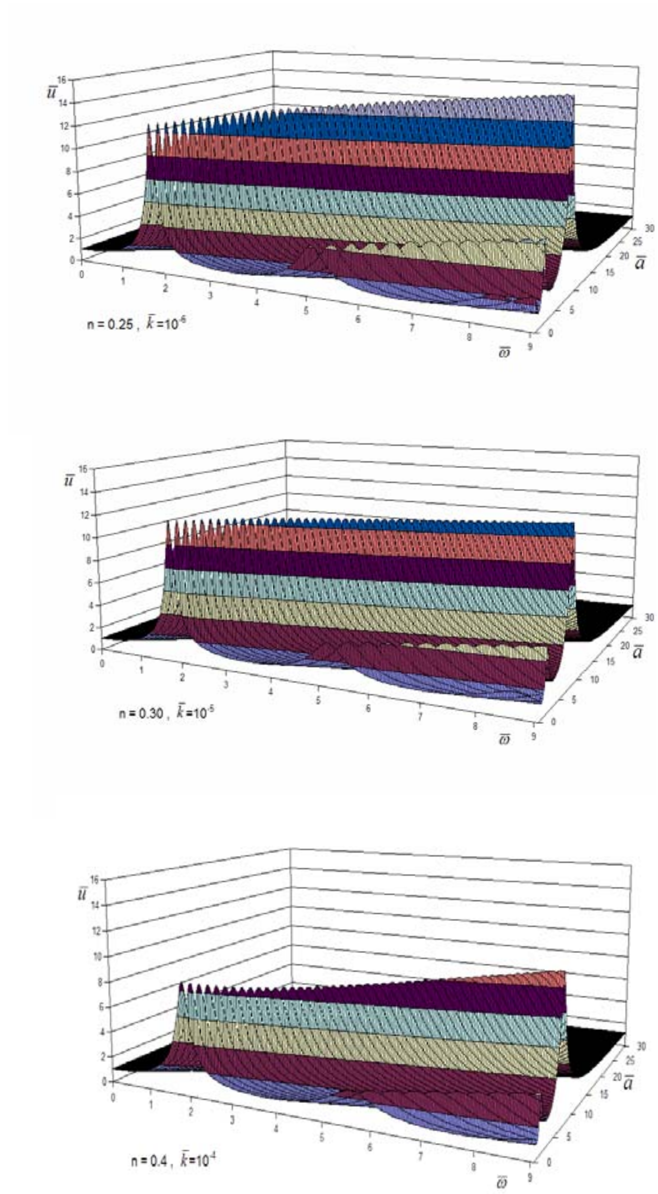


Figure4. Dimensionless displacement profile for different values of porosity and permeability and various depth non-homogeneity parameters ($\overline{\omega} = 3$, $\delta = 0.05$)

Figure5. Variation of amplification factor versus frequenc and coefficient of depth non-homogeneity for different values of porosity and permeability ($\delta = 0.05$)

To validate the formula derived above , in figure 6, the variation of amplification factor versus excitation frequency when $a\to 0$ and $n\to 0$ is compared to available analytical function of amplification factor for the homogeneous state as follows(Kramer[8]):

$$\bar{u} = \frac{1}{\sqrt{\cos^2 \bar{\varpi} + (\delta \bar{\varpi})^2}} \tag{45}$$

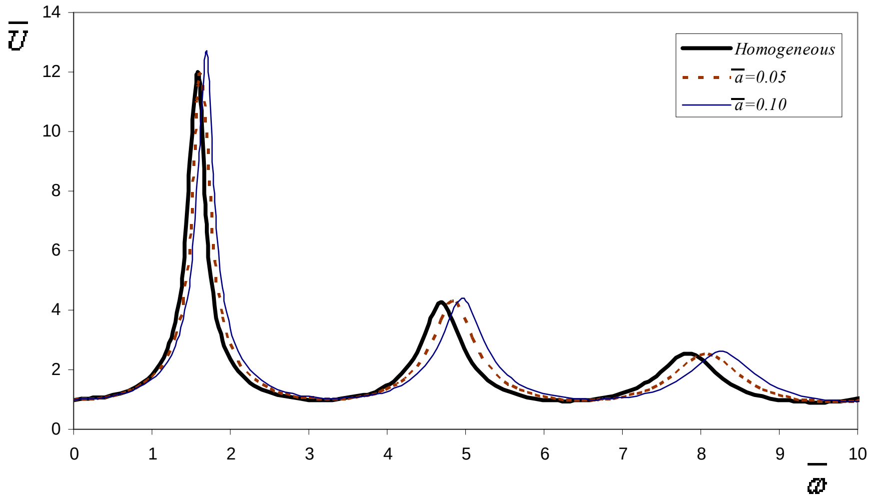


Figure6. The comparison between the presented amplification factor when *a*,*n*→0 and the available classic function for the homogeneous state $(\delta = 0.05)$

The agreement of the obtained results with the above function is found to be excellent.

CONCLUSION

In order to study the depth non-homogeneity effects on amplification of bed motion, the dynamic response of a saturated continuously non-homogeneous soil layer is investigated analytically. A quadratic function is assumed for the variation of shear modulus along the depth of the layer, according to Gibson's famous model. Selected results including three dimensional diagrams of amplification factor versus bed rock excitation frequency and depth non-homogeneity parameter for different values of media's porosity and permeability show the layer's dynamic response including the variation form of amplification factor, resonant frequencies and displacement profile along depth is strongly dependant to shear modulus distribution and depth non-homogeneity.

REFERENCES

Achenbach JD "Wave propagation in elastic solids," North-Holland, Amsterdam, 1973.

Biot MA " Theory of propagation of elastic waves in a fluid saturated porous solid: I. Low frequency range." The journal of Acoustical Society of America, 28,168-178, 1956.

Boresi AP "Elasticity in Engineering Mechanics," Elsevier Science, Amsterdam, 1987.

Chen L. and Wang G. "Torsional vibration of elastic foundation on saturated media," Soil Dyn. and Earthquake Eng. 22,223-227, 2002.

Boyce WE and Diprima RC "Elementary differential equations and boundary value problems," John Wiley & Sons, New York, 1992.

Gibson R.E. "Some results concerning displacement and stresses in a non homogeneous elastic half space," Géotechnique 17, 58-67, 1967.

Gatmiri B "A simplified finite element analysis of wave induced effective stress and pore pressures in permeable sea beds," Géotechnique 40 ,15-30, 1990.

Kramer SL, "Geotechnical Earthquake Engineering," Prentice Hall, New Jersey, 1996.

Noorzad A " Dynamic interaction between a rigid boy and the surrounding semi-infinite poroelastic medium," Ph.D. Thesis, Department of Civil Eng., University of Tokyo,1994.

Philippacopoulos A.J. "Lamb's problem for fluid saturated porous media." Bulletin of Seismological Society of America, 78, 908-923 , 1988.

Vrettos C "In-plane vibration of soil deposits with variable shear modulus: I. Surface waves," Int. J. Numer. Anal. Methods in Geomech. 14,209-222, 1990.

Yang J and Sato T. "Influence of water saturation on horizontal and vertical motion at porous soil interface induced by incident SV wave," Soil Dyn. and Earthquake Eng. 19 ,339-346 , 2000.

Yang J "Influence of water saturation on horizontal and vertical motion at porous soil induced by incident P wave," Soil Dyn. and Earthquake Eng. 19 ,575-581, 2000.

Yang J "Saturation effects on horizontal and vertical motions in a layered soil-bedrock system due to inclined SV waves," Soil Dyn. and Earthquake ,5275-536, Eng. 21, 2001.