

## SEISMIC ANALYSIS OF GEOTECHNICAL STRUCTURES USING THE MATERIAL POINT METHOD

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### ABSTRACT

The Material Point Method, or MPM, a numerical simulation technique that combines features of particle methods and the finite element method, is used in this paper for the seismic analysis of geotechnical structures. With MPM, a body is discretized into a collection of lagrangian particles, which carry all the variables which are needed to define the body's state and the material behavior. Interaction between particles takes place in a background fixed mesh similar to those used in the finite element method. Using the explicit version of MPM there is no need to perform matrix inversion and the computational cost of every solution step is low. However, the explicit algorithm requires a large number of integration steps. Solutions obtained for some examples are presented, showing the ability of the method to accurately model plastic strain localization phenomena with development of sliding surfaces and large displacements.

Permanent deformations in a slope after a seismic event are frequently evaluated by Newmark's method. This kind of analysis requires the value of lateral pseudo-static acceleration that trigs collapse. An example of this type of analysis for a cohesive slope is shown in the paper. The lateral failure acceleration was evaluated using a limit equilibrium method and with a MPM particle model. The MPM analysis could identify a collapse mechanism that was later analyzed using the upper bound theorem of plastic collapse, in order to compare the yield acceleration obtained with both methods.

Some results for a particle model of a concrete faced gravel dam which is being built in San Juan Province, Argentina are also presented. This second example shows the capabilities of MPM to model large displacements and deformation with diffuse shear deformation.

The examples presented in this paper suggest that the MPM can be used to model seismic response of geotechnical structures that involve strain localization and large displacements.

Keywords: *Material Point Method, strain localization, collapse mechanism, earthdams, slopes*

### INTRODUCTION

Failure mechanisms of geotechnical structures during or after seismic loading involve strain concentrations and large displacements in materials with highly inelastic behavior: soil masses sliding along weak planes, slope failures, settlement of foundation structures, etc. All these mechanisms involve strain localization and therefore, displacement and velocity discontinuities. The prediction of these failure modes is a topic of great interest in earthquake geotechnical engineering, particularly for structural security assessment, which requires estimation of structural behavior during and after collapse.

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Although the prediction of strain localization phenomena is a fundamental issue in the analysis of collapse mechanisms of geotechnical structures, it is rather unusual to find accurate calculation methods in standard engineering practice. Generally, analysis cases involving strain localization are evaluated by using limit equilibrium methods, or less frequently by the application of the theorems of plastic collapse of structures (Lancellota, 1995). These tools, however, can predict neither the location nor the shape of the surfaces where plastic strain localization develops. Also in these methods, the loads, self weight and horizontal forces, are fixed and do not vary in time, therefore collapse mechanisms involved are also fixed during dynamic analysis. Moreover, these methods do not provide information on the strain level or displacements reached at failure, or at the end of an earthquake, which that are needed for estimation of structure's final configuration. For these reasons, numerical methods turn out to be an appropriate tool to accurately study the onset of strain localization and its evolution until structural failure.

Simulation of strain localization requires materials constitutive models to be mathematically consistent, that is, they should be able to describe the continuum behavior, avoiding pathological dependence of the solution with the size of the adopted discretization. This difficulty can be treated by applying regularization techniques, which introduce an internal characteristic length in the formulation. A second difficulty is a scale problem: the region where plastic strain localization develops is generally a narrow band compared to the dimensions of the modeled structure. The analysis strategy must be able to adequately represent the region where plastic deformation is concentrated while keeping the complexity of the discretization at a level suitable to the available computation capacity. Finite element models use several procedures to analyze localization problems. These procedures include the use of shape functions capable of representing a discontinuity in the displacement field, and the automatic generation of adaptive meshes which capture discontinuities by mesh refining in the zone of strain localization. Large strains in these zones produce high distortion levels in the mesh, introducing important errors in the analysis.

Since particle methods do not use a mesh to discretize the continuum, they are appropriate for modeling large displacements and large strain problems. One of the easiest methods to implement is the material point method or MPM. A brief description of the MPM is presented in this paper. In addition to this, examples are presented showing that MPM is suitable for modeling seismic analysis of geotechnical structures.

## THE MATERIAL POINT METHOD

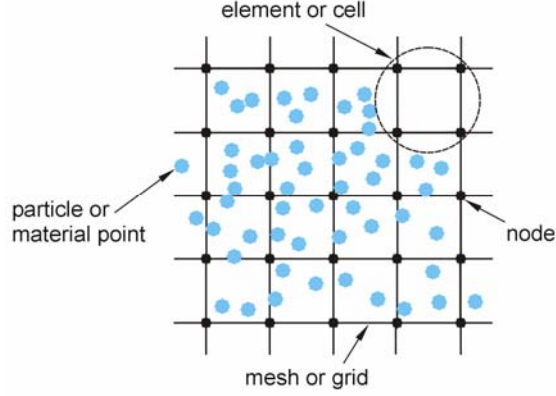
The material point method has evolved from a "particle-in-cell" method for fluid dynamics problems called Fluid Implicit Particle (Sulsky et al., 1995)

Implementation of the MPM is easy because several of its fundamental assumptions and mathematical technologies are similar to those that support the widely extended finite element method. The MPM is appropriate to model phenomena in which large displacements take place. Also, it can handle contact/separation problems "naturally", i.e. there is no need for a separate contact algorithm. MPM has been exhaustively applied to problems of impact, penetration, and other phenomena of contact between bodies, transmission of shock waves and analysis of high frequency vibrations.

The material point method represents the material contained in a region  $\Omega_0$  as a collection of unconnected particles. An initial mass is assigned to each particle. Particle masses remain fixed throughout the calculation process, thus insuring global mass conservation. Other initial quantities, such as velocities, strains and stresses, are also assigned to the material points.

The discrete motion equations are not solved at the material points. Instead a support mesh, built to cover the domain of the problem, is used (Figure 1). This mesh is composed of elements of the same type as those used in the finite element method. For the sake of simplicity, it is common to use bilinear regular quadrilateral elements. The variables required to solve the motion equations in the mesh at any step of the analysis are transferred from the particles to the nodes of the mesh by using mapping functions (York et al., 2000). These are the typical shape functions used in the finite element method.

The boundary conditions are imposed at the mesh nodes and the motion equations are solved by using an incremental scheme. Then the quantities carried by the material points are updated through the interpolation of the mesh results, using the same shape functions. The information associated to the mesh is not required for the next step of the analysis; therefore it can be discarded provided that the boundary conditions that may have been established are preserved. The discrete form of the governing equations can be found in Sulsky et al, 1995.



**Figure 1: Components of a material point method model**

## NUMERICAL EXAMPLES

The ability of the material point method to perform geotechnical seismic analyses including strain localization and large displacements is shown in this section. Two examples are presented, both using an elastoplastic, non-associated constitutive equation with strain hardening/softening. The constitutive equation was originally developed by Abbo and Sloan (Abbo & Sloan, 1995) as a smooth approximation to the Mohr – Coulomb failure criterion. The examples were modeled assuming plane strain conditions and fully drained behavior. In both cases, the time integration step selected was less than 25% of the critical step imposed by the Courant-Friedrichs–Levy condition. This critical time-step is equal to the time that a pressure wave takes to travel a distance equal to the element size of the supporting mesh. No numerical damping was added to any of the models. Some figures refer to the effective plastic strain, given in rate form by

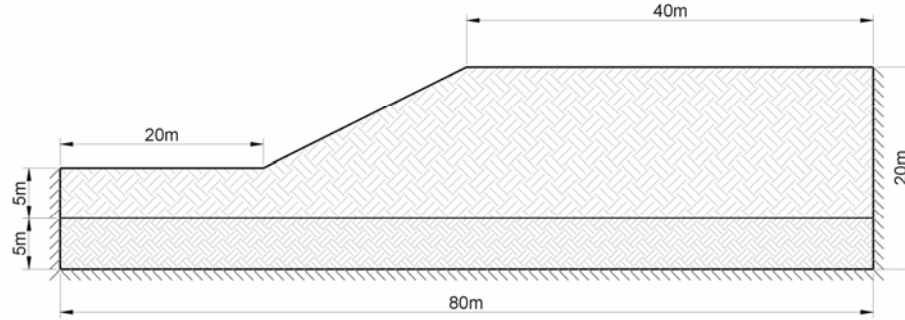
$$\dot{\varepsilon}_{ef}^p = \left( \frac{2}{3} \dot{\mathbf{e}}_{ij}^p \dot{\mathbf{e}}_{ij}^p \right)^{1/2} \quad (18)$$

where  $\mathbf{e}_{ij}$  are the deviatoric strains  $\mathbf{e}_{ij} = \varepsilon_{ij} - \frac{1}{3} \delta_{ij} \varepsilon_{kk}$ .

### Yield acceleration in a cohesive slope.

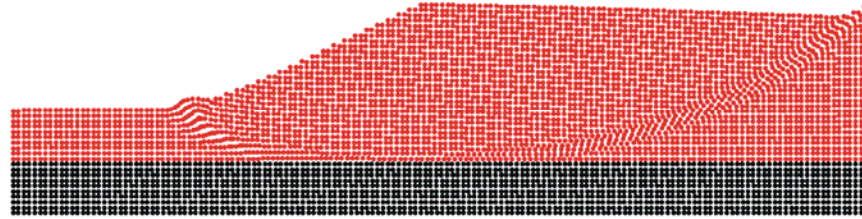
Approximate methods for slope displacement analysis under seismic actions require as input data the value of the horizontal yield acceleration. Newmark's sliding block method (Newmark & Rosenblueth, 1976) considers that the soil wedge generated by the failure surface has rigid-plastic behavior and displacements, relative to the slope, occur when the earthquake response acceleration exceeds the magnitude of the yield static acceleration. Under these assumptions, the displacement of the wedge relative to the slope can be estimated through double integration of the base acceleration history only for accelerations exceeding the yield acceleration magnitude and acting in the sliding direction. To estimate the yield acceleration it is a common practice to use slope limit equilibrium methods. These methods establish force balance, but they do not impose any restriction on the kinematic possibility of the failure mechanism stated; therefore, conservative values are generally obtained for the yield pseudo-static acceleration.

The example presented is aimed at finding the failure mechanism induced in a cohesive slope when subjected to a horizontal pseudo-static acceleration. The geometry and boundary conditions of the slope are shown in Figure 2. Materials were modeled by using Abbo and Sloan smooth approximation to Mohr-Coulomb failure criteria, with  $\phi = 0^\circ$  and associated plasticity without strain softening. The cohesion of the top layer was set to  $50\text{kN/m}^2$  while the base layer cohesion was equal to  $100\text{kN/m}^2$ . Both materials have identical specific weight ( $20\text{kN/m}^3$ ) and Young modulus ( $2 \cdot 10^5 \text{ kN/m}^2$ ).

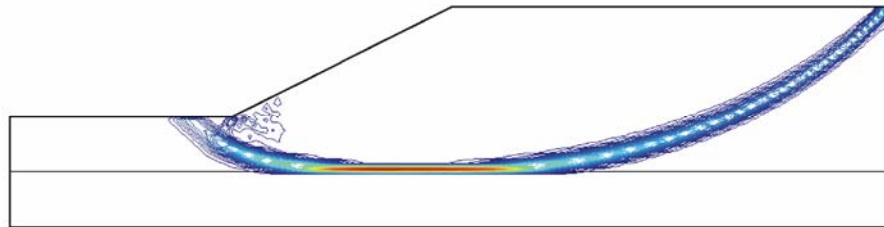


**Figure 2: Yield acceleration in a cohesive slope. Geometry and boundary conditions.**

The slope model has one particle per cell and was analyzed in two stages. In the first stage stresses and strains due to self weight loads were obtained. The second stage of the analysis starts from the conditions reached at the end of the first stage, and evaluates the effect of a slowly increasing lateral acceleration. Results of the calculations are shown in Figure 3 and Figure 4. The horizontal pseudo-static acceleration that triggers the sliding mechanism was found to be  $0.26g$ .

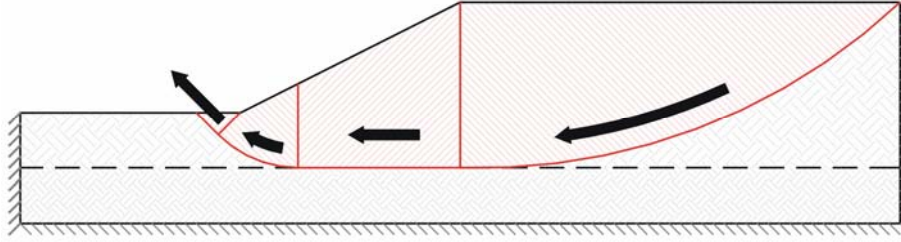


**Figure 3: Yield acceleration in a cohesive slope. Deformed configuration of material points.**



**Figure 4: Yield acceleration in a cohesive slope. Effective plastic strain contours.**

From the deformed configuration with displacements scaled by 10 of Figure 3 and the plastic strain contour plot of Figure 4, a failure mechanism was deduced. The mechanism includes a passive wedge and sliding blocks which are shown in Figure 5. The position and shape of the sliding blocks are clearly influenced by the interface between the two materials and by the rightmost boundary, following the typical failure pattern of pure-cohesive slopes, i.e. the volume of the sliding block tends a maximum within the constraints imposed by the problem boundaries.



**Figure 5: Yield acceleration in a cohesive slope. Failure mechanism.**

An analysis of the mechanism shown in Figure 5 was carried out by using the upper bound theorem of plastic collapse (Lancellota, 1995) as a way to check the results obtained with the MPM. The analysis resulted in yield acceleration equal to  $0.31g$ , which is close to that obtained from the MPM analysis. Obviously this yield acceleration is an upper bound of the actual acceleration value. An analysis of the slope using Carter's limit equilibrium method (Siegel, 1975) resulted in yield acceleration equal to  $0.14g$ , which is smaller than the magnitudes previously found. This is reasonable since limit equilibrium methods do not account for kinematical compatibility and therefore they usually yield conservative results.

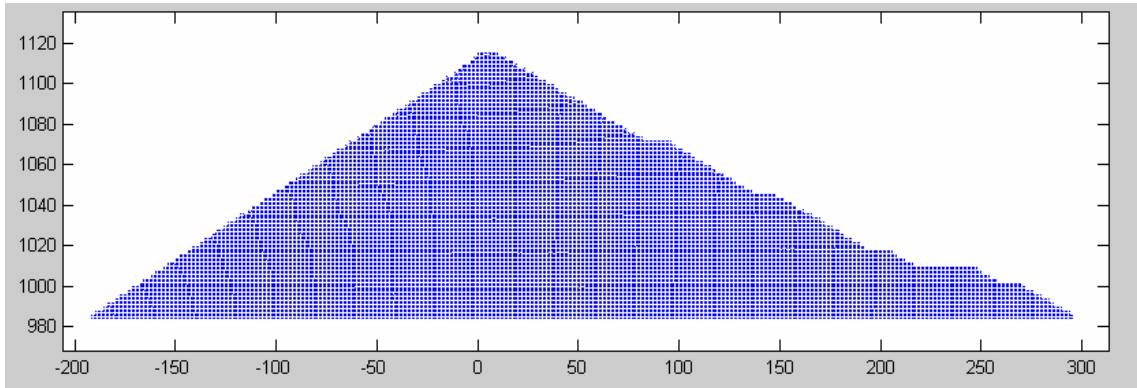
### Concrete faced gravel dam.

In this example some results for a model of Caracoles dam, a concrete faced gravel dam which is being built in San Juan Province, Argentina, are presented. Figure 6 shows the initial state of the particle model. The model has 31156 particles, with an initial arrangement of four particles per cell. The computational mesh is a grid of  $2m \times 2m$  four-node quadrilateral elements. All particles have a peak friction angle of  $50^\circ$  and a residual friction angle of  $40^\circ$ . The softening behavior is controlled by the plastic modulus  $H$  defined as the slope of the  $q$  vs.  $\varepsilon_{ef}^p$  plot;  $q$  is the effective deviatoric stress, defined by:

$$q = \left( \frac{3}{2} \mathbf{s}_{ij} \mathbf{s}_{ij} \right)^{1/2} \quad (19)$$

where  $\mathbf{s}_{ij}$  are the deviatoric stresses, given by  $\mathbf{s}_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk}$ .

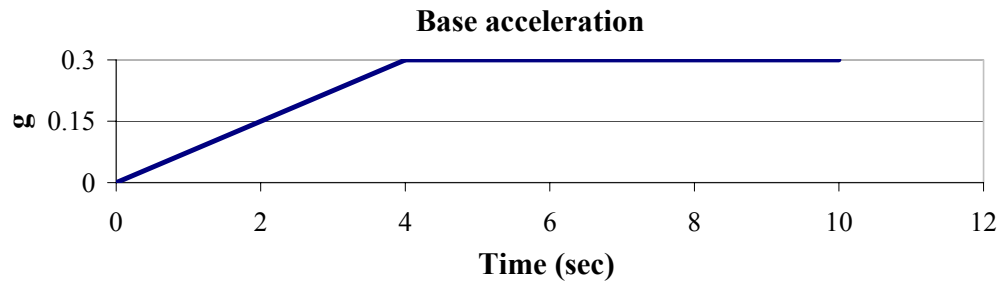
The model was analyzed introducing a mesh size dependent plastic modulus (Zienkiewicz et al., 1995) in order to prevent the pathological cell size dependence of the softening branch. Variation of the initial strength and stiffness with pressure was not considered.



**Figure 6. Concrete faced gravel dam. Particle model. (scales in meters)**

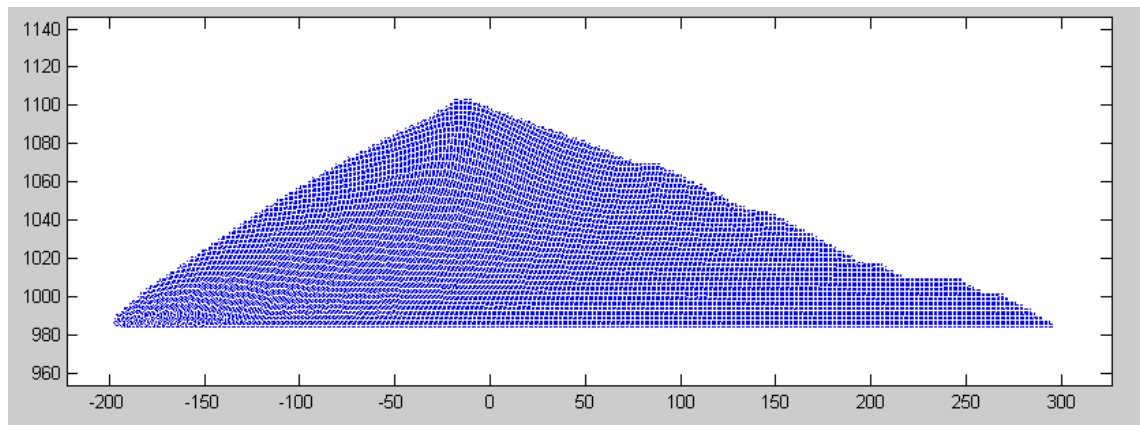
*Analysis for lateral pseudo-static base acceleration.*

Once self weight loads were applied, the model was subjected to a increasing lateral base acceleration. The lateral acceleration function is a ramp that reaches a maximum of  $0.30g$  in 4 seconds (Figure 7).

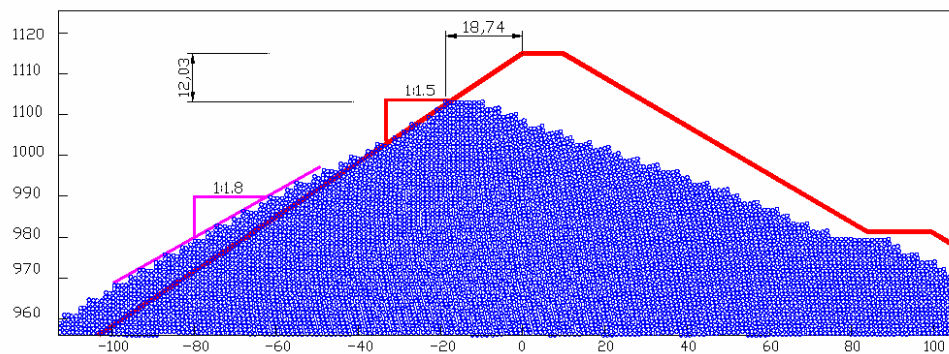


**Figure 7. Horizontal base acceleration function.**

The analyses were performed without water pressure on the upstream slope. With acceleration acting in downstream direction, the upstream slope deformed and bulged, dragging the dam crest in upstream direction and reducing the dam height, as shown in Figures 8 and 9. The large displacements and deformations obtained can be evaluated by comparison of the deformed model with the initial state at the dam crest (Figure 9). The upstream slope changes from 1:1.5 at initial configuration to approximately 1:1.8 at the end of base acceleration. Final effective plastic deformation has a diffuse pattern as shown in Figure 10. Figure 11 shows decimated particle trajectories (1 every 32).

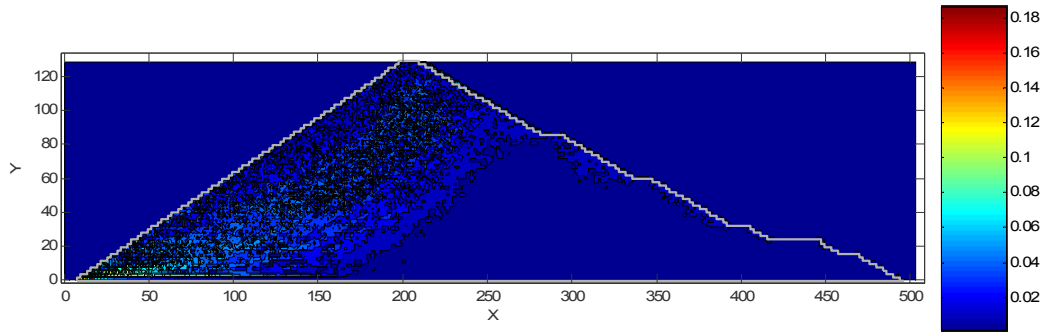


**Figure 8. Concrete faced dam. Deformed configuration.  
Non-magnified displacements. (scales in meters)**

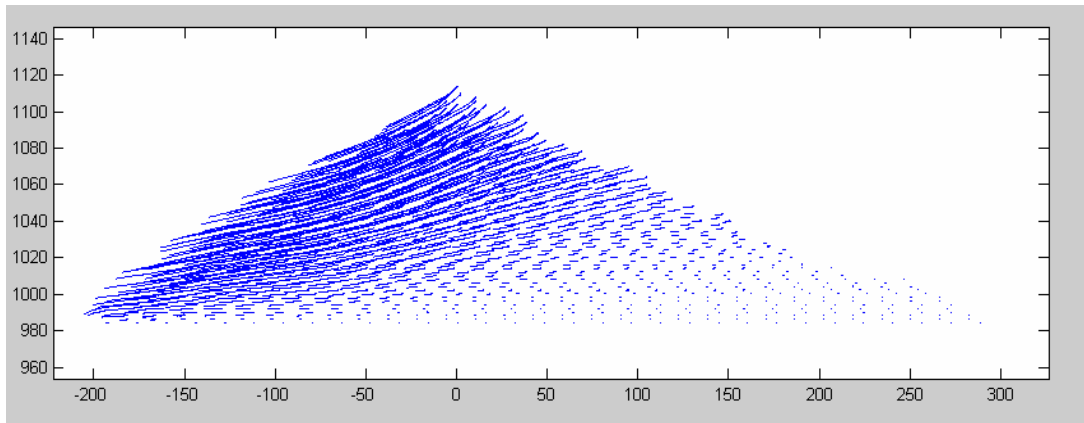


**Figure 9. Concrete faced dam. Crest displacements (m) (Non-magnified).**





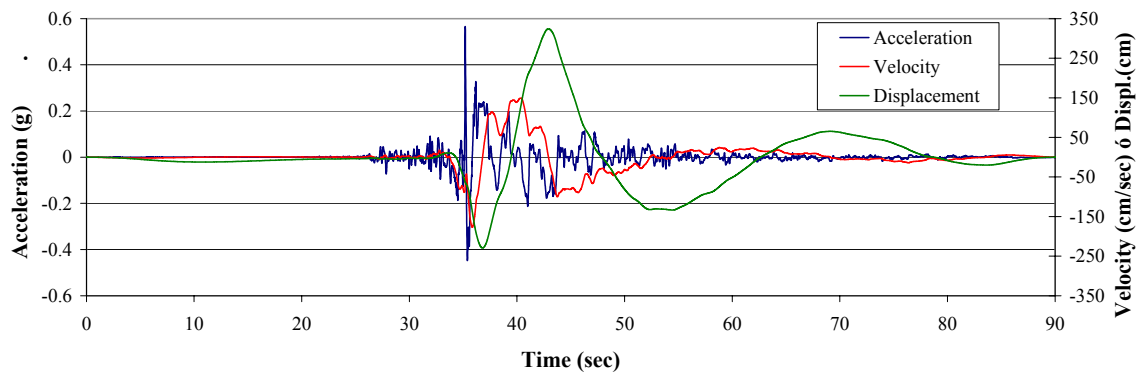
**Figure. 10: Concrete faced dam. Effective plastic strain contours.**



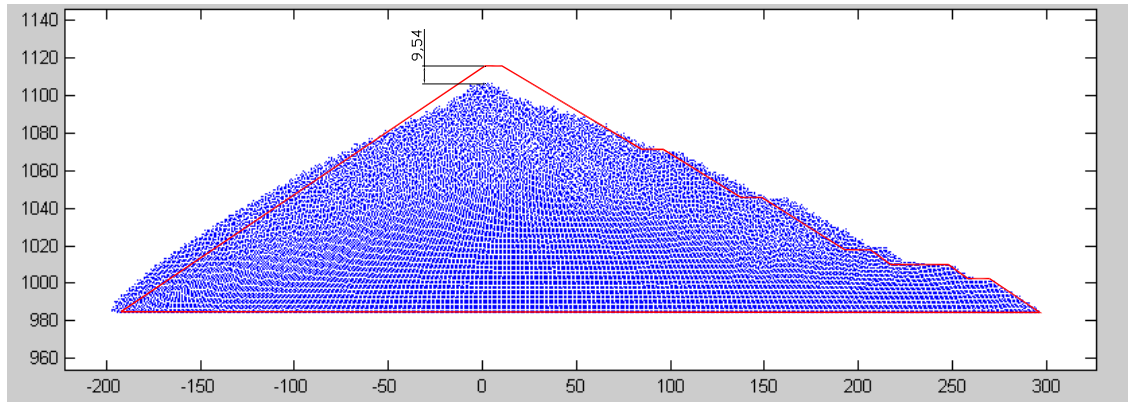
**Figure. 11: Concrete faced dam. Particle trajectories.  
Non-magnified displacements. (scales in meters)**

*Analysis for Chi- Chi (9/20/1999) earthquake TCU068W acceleration register.*

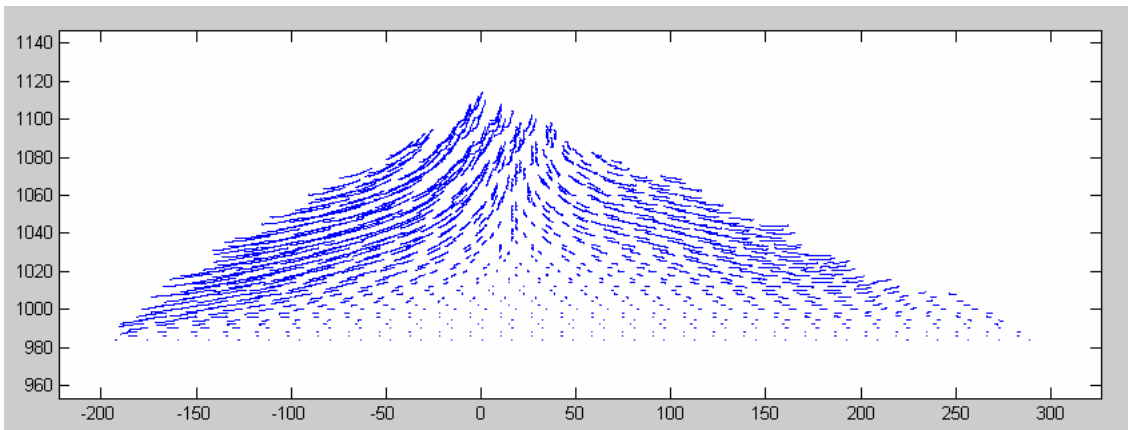
The model response to a horizontal base acceleration time history is obtained in this analysis. The input acceleration is given by the W-E register of Chi–Chi 20/9/1999 earthquake recorded at TCU068 station (PEER, 1999) (Figure 12). The final configuration and crest displacements are shown in Figure 13. The deformation pattern shows that sliding of both upstream and downstream slopes occurred during the earthquake (Figure 14), as a result of the action of bidirectional inertia forces. Deformation leading to the final resulting geometry of the dam involves large displacements, which are captured by the model. The calculated freeboard loss is approximately 9 m.



**Figure. 12: Concrete faced dam. Acceleration time history. Chi Chi 09/20/99, TCU068W.**



**Figure. 13: Concrete faced dam. Deformed configuration for TCU068W acceleration register. Non-magnified displacements. (scales in meters)**



**Figure. 14: Concrete faced dam. Particle trajectories for TCU068W acceleration register. Non-magnified displacements. (scales in meters)**

## CONCLUSIONS AND FINAL COMMENTS

The Material Point Method can be applied to seismic analysis of geotechnical structures at a relative low cost of software development because it has many common features with the Finite Element Method. The first example shows that the MPM is able to adequately model strain localization and the generation of collapse mechanisms under horizontal pseudo-static loading at small strains. The second is a different type of example in which the MPM appears to simulate reasonably well large deformation and displacements with diffuse shear strain distribution. Extensive verification effort must be done in order to use the MPM in practice as a design tool.

A MPM based geotechnical analysis program called GEOPART is currently under development at the University of San Juan (Argentina) and at the Polytechnic University of Catalonia (Spain). In order to enhance program capabilities more complex models as a modified Pastor- Zienkiewicz generalized plasticity critical state model (Zabala & Oldecop, 2000) are under implementation. Also pore pressure computation using an adapted u-p formulation is being tested.



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