4th International Conference on
Earthquake Geotechnical Engineering
June 25-28, 2007
Paper No. 1259

IDENTIFICATION OF THE ESSENTIAL PARAMETERS FOR THE LATERAL IMPEDANCE OF LARGE PILE GROUP

Reza TAHERZADEH[1], Didier CLOUTEAU[2], Régis COTTEREAU[3]

ABSTRACT

The purpose of this paper is to identify the essential parameters governing the lateral impedance of pile foundations. Although the available proposed methods by (Dobry & Gazetas, 1988) and (Makris & Gazetas, 1992) can be readily applied by engineers already familiar with the use of static interaction factors of pile groups (Polous, 1970), some limitations have been noted for large piles (n>50). Thus, we present simple formulas for the impedance of pile groups and try to give additional insight on the parameters leading their behavior. Based on a numerical analysis of a wide range of foundations with varying the values for the radius, number of pile, piles bending stiffness, soil stiffness, pile length, an analysis of the major parameters is conducted. In order to check the accuracy of the proposed formulas, the seismic response of a gas reservoir is studied and compared with a more accurate solution.

Keywords: dynamic, lateral impedance, piles, identification

INTRODUCTION

For dynamic loads, the behavior of pile group is much more complicated than in the static case. Very complex wave patterns are created within the pile group. Pile-soil-pile-interaction (PSPI) results in a dynamic-stiffness thus is very strongly frequency-dependent. Until now, several different methods have been used to obtain the dynamic impedance of the pile group, including rigorous and simplified methods. The rigorous analysis of a pile group based on boundary element method, which solves the elasto-dynamic equations with complicated boundary and interface conditions, leads to an enormous computational effort (Kaynia & Kaussel, 1982), urging for the development of a simple approach with sufficient accuracy to consider pile-soil pile-interaction. A numerical scheme based on coupled boundary element (BEM) and finite element (FEM) techniques have been applied (Clouteau et Taherzadeh, 2006). This method requires also a large computer capacity for a large pile group. It is noted that numerical approach entail significant computational effort and are used primarily for research rather than as design tools. The interaction-factor method is one of the simplified methods using the rigorous impedances of the single pile and semi-analytical interaction factors between two piles at different spacings (Dobry & Gazetas, 1988, Makris & Gazetas, 1993). Clearly, in using superposition, the interaction between all piles simultaneously is approximated. In this paper, a parametric study based on numerical analysis (BEM-FEM) has been conducted to identify the important parameters of the soil and the piles for lateral dynamic stiffness and damping. Finally, we propose an alternative simple formulation for the dynamic stiffness and the damping of floating pile group foundations.

[1] PhD Student, Laboratoire MSSMat, Ecole Centrale Paris, Chatenay Malabry, France. Email: reza.taherzadeh@ecp.fr
[2] PhD, Laboratoire MSSMat, Ecole Centrale Paris, Chatenay Malabry, France. Email: regis.cottereau@ecp.fr
[3] Professor, Laboratoire MSSMat, Ecole Centrale Paris, Chatenay Malabry, France. didier.clouteau@ecp.fr

DYNAMIC STIFFNESS AND DAMPING

Dynamic stiffness and damping of piles are needed in the analysis of deep foundation and structures supported by piles for the effects of dynamic loads produces by operation machines, wind, earthquake and sea waves. In general, the impedance of a foundation is a complex matrix of a 6 vibration modes (swaying, pumping, rocking, torsion and their coupling). This function can be described in terms of complex stiffness:

$$Z = K + iC \quad (1)$$

The complex stiffness has a real part K reflects the stiffness and inertia of supporting soil and foundation; its dependence on frequency relates solely to the influence that frequency exerts on inertia and an imaginary part C reflects the two types of damping (radiation and material) generated in the system; the former due to energy carried by the waves spreading away from the foundation; and the latter due to energy dissipated din the soil through hysteretic action. This damping generally grows with frequency resembling viscous damping; it can be defined in terms of the constant of equivalent viscous damping

$$a_0 = \frac{\omega . B}{V_s} \quad (2)$$

Where a_0 is dimensionless frequency, ω is circular frequency, V_s is the soil shear wave velocity and B is the half width of the foundation. Then the complex stiffness can be normalised with the static stiffness of a surface footing with an equivalent dimension K_s (Wolf, 1994) and can be written by

$$Z = K_s(k + ia_0c) \quad (3)$$

The dynamic stiffness and damping of a pile group can not be computed by simply adding the stiffnesses of the individual piles, since each pile is affected not only by its own load, but also by the loads on its neighbors piles. The dynamic stiffness and damping of pile group can be determined experimentally or numerically. The latter approach is preferred because experiments, though very useful, are difficult to generalize. In numerical approach, calculating the forces needed to produce the vibration of pile group head having sole, unit amplitude in the prescribed direction, generates dynamic stiffness and damping.

In this study, the dynamic stiffness coefficient and damping ratio of a pile group foundation are computed. The piles and the soil between the piles are modeled by the FEM and the infinite soil at the boundary of the pile group is modeled by the BEM (Clouteau & Taherzadeh, 2006). In figure 1, the finite element model of a pile group with 9 identical piles is presented. Since the number of DOFs for a large pile group foundation with infinite soil is quite high and requires a large computational effort, a modal reduction technique (Craig & Bampton, 1967) is used to decrease the size of the problem. In this approach the displacement fields are decomposed on a basis of dynamic modes with fixed interface and unit displacement at all degrees of freedom of the interface between FEM and BEM.

NUMERICAL VALIDATION

In order to check the accuracy of the BEM-FEM solution developed in the present study, a comparison is presented with the rigorous numerical solution obtained by Kaynia and Kausel (1982), based on an integral equation formulation and the results of an analytical solution for infinite long piles derived by Makris and Gazetas (1993), where pile-soil interaction is represent through a dynamic Winkler model. For this purpose, the lateral dynamic stiffness coefficient and damping ratio of a 3 by 3-square pile group with fixed head without cap are plotted in figure 3. In the results, it is assumed that the only variable is the pile spacing ratio (s/d=2, 5 and 10) and the other parameters of the soil and the piles are Young modulus ratio $E_p/E_s=1000$, mass density ratio, $\rho_p/\rho_s=1.42$,slenderness ratio, $L/d=15$, hysteretic damping of soil, $\beta=0.05$ and Poisson's ratio $\nu_p=0.25, \nu_s=0.4$.

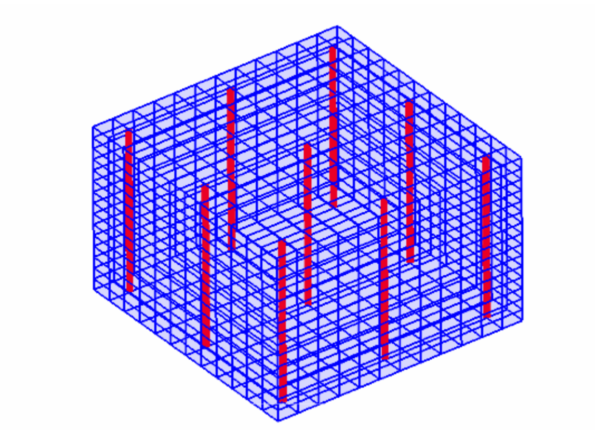


Figure1. Modeling of a 3 by 3 pile group

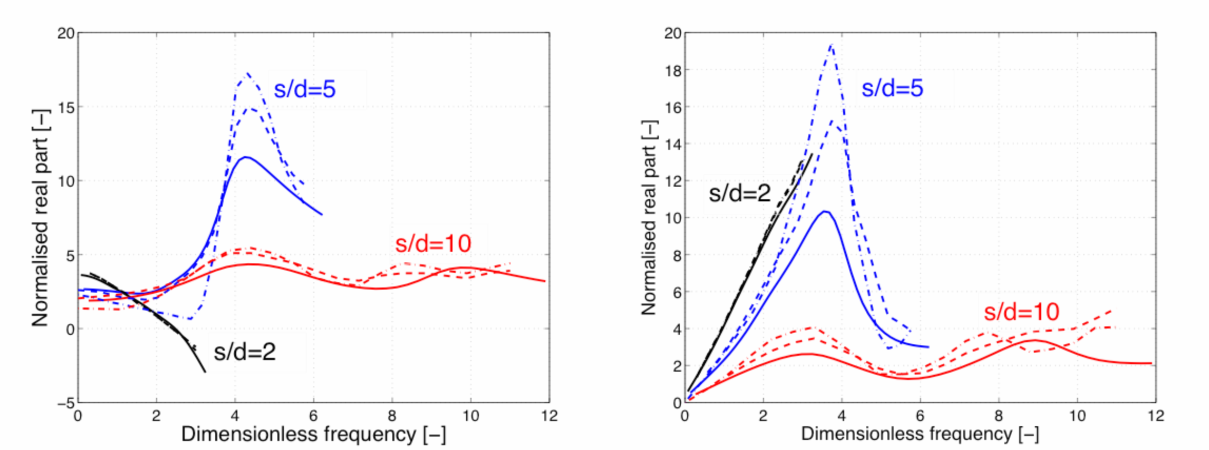


Figure 2. Lateral dynamic stiffness and damping: Comparison of FEM-BEM (Solid line) with rigorous solution (dashed line) and simplified analytical model (dashed-dotted line) for a 3 by 3 square pile group in a homogenous halfspace.

For very close spacings (*s/d=2)* the stiffness and damping group exhibit a smooth variation with frequency, with the pile group behaving very much like an isolated embedded foundation. Indeed, while the damping increase linearly with the frequency, the dynamic stiffness decrease steadily with increasing frequency. The peaks in the dynamic stiffness and damping ratio appear as a result of pile-soil-pile interaction taking place among the piles in the group. If the spacing between the piles is larger (*s/d=5 or s/d=10*) the soil between adjacent piles may vibrate out of phase, with the piles (Fig.3), which results in a large stiffness for the pile group. It may reach values greater than the sum of stiffness of the piles in the group, of the spacing between the piles equals half the wavelength of the waves in the soil (Kaynia, 1982). The good agreement in all approaches (rigorous solution, BEM/FEM and Winkler model) can be observed for the closer pile group (*s/d=2*). In another cases (*s/d=5, 10*) despite the overall satisfactory performance, the simple method over estimate the peaks and the FEM/BEM under estimate or both stiffness and damping curves.

In this section, we carry out the numerical validation for a larger pile group with a 4 by 8 piles. Figure 4 shows the numerical modeling of the group. In this model, the *130* [cm] diameter pile pass through *9.5* [m] very soft soil having an S-wave velocity *Vs=73* [m/s], mass density *1500* [kg/m^3], and Poisson's ratio $\nu_s=0.49$. The piles with the spacing *s=2.6* [m] are socketed in a stiff sand layer having *Vs=330* [m/s] into which they penetrate *6* [m]. The lateral dynamic stiffness and damping is presented in figure 5. As seen in figure 5, the results obtained by superposition method, show higher discrepancies. Since this approach take into account the interaction between all piles, simultaneously, as if intermediate piles don't have any shadow effects. It is expected that for the larger pile group (n>100), the superposition method show stronger discrepancies.

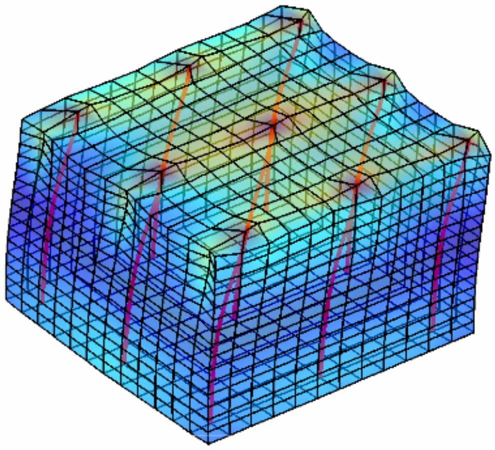


Figure 3. Wave-length of wave in the soil between the piles equal to half

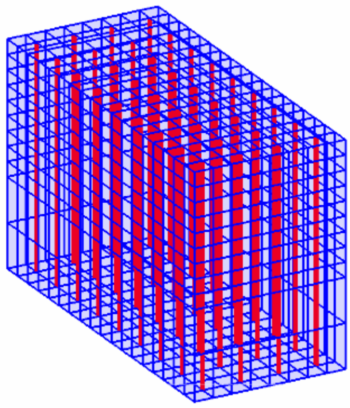


Figure 4. Modeling of a 4 by 8 pile group

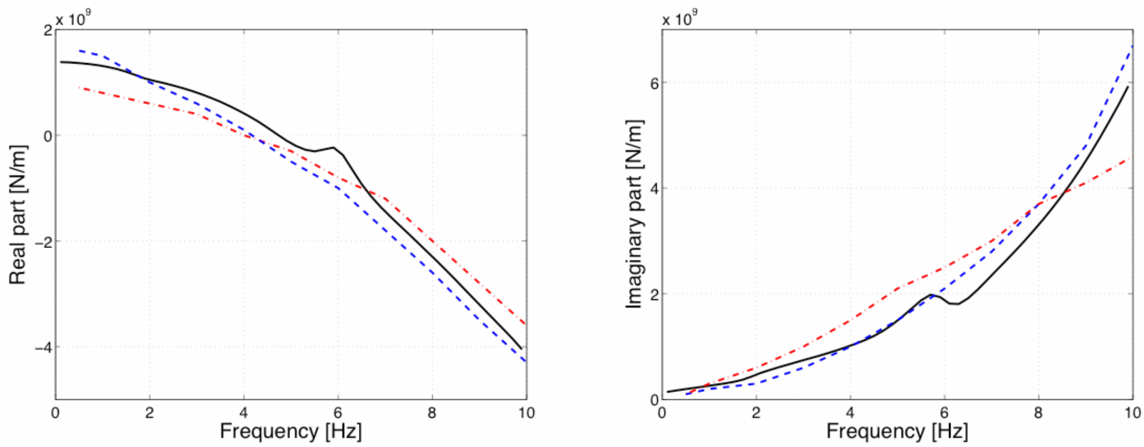


Figure 5. Modeling of a 4 by 8 pile group: FEM-BEM (solid line), BEM (dashed line), simplified analytical model (dashed-dotted line)

PARAMETRIC STUDY

A parametric study with numerical BEM/FEM solution has been conducted over varying geometrical and mechanical properties of large pile group to observe the leading parameters affecting on the lateral dynamic stiffness and damping ratio. The parametric studies are based on 18 models of large pile group with varying properties of soil and piles, including the number of piles (N), Young modulus of pile (E_p), shear wave velocity of the soil (V_s), pile diameter (d), foundation width ($2B$), pile spacing (S) and pile length (L). These models cover a wide range of the soil and pile properties as given in table 1. In all cases, square pile groups are considered.

Figures 7-11 present the dynamic stiffness and damping are normalized with the static stiffness of the equivalent surface footing with respect to the dimensionless frequency, $a_0=\omega B/Vs$. As it is observed in the range dimensionless frequency ($0<a_0<8$), the dynamic stiffness of pile group is frequency dependent and larger than the equivalent larger than that of a surface footing with equivalent plan dimension. For closer pile spacing and the soil is very weak relative to the pile, dynamic stiffness varies strongly with frequency and can become negative as the inertial effects increase with increasing frequency (Fig 7-11, dotted lines). Damping of a pile group also is frequency dependent and can be larger than that of a surface footing, with the same area. The damping increases in closer pile spacing and softer soil or stiffer piles (Fig 7-11, dotted lines). Figure 12 presents the effect of pile slenderness to dynamic stiffness and damping. Since the slender piles in horizontal direction are in flexible, the pile length and tip condition are not important parameters.

The above observations suggest that the most important factors controlling the lateral dynamic stiffness and damping are: the pile concentration in the foundation area, relating soil stiffness to pile stiffness. Based on homogenization theory of pile group as an equivalent composite foundation made of a soil matrix and piles (Postel et Aubry, 1985) we can define the equivalent longitudinal modulus E_{11} and transversal modulus G_{12} (Fig 6) of pile group as the important parameters including the major factors for the lateral dynamic stiffness and damping:

$$\frac{1}{G_{12}} = \frac{c}{G_s} + \frac{1-c}{G_p} \tag{4}$$

$$E_1 = cE_s + (1-c)E_p \tag{5}$$

$$c = \frac{S^2 - A_p}{S^2} \tag{6}$$

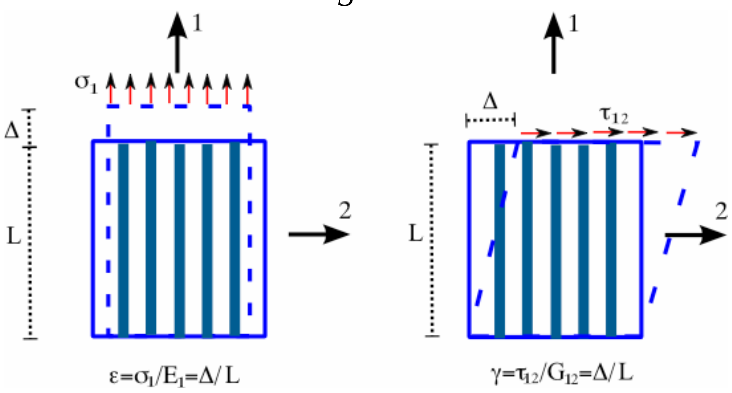


Figure 6. Longitudinal and transversal deformation of a composite foundation made of soil matrix and piles

In which c is termed as the soil concentration in foundation area and A_p is the section area of single pile. Conceptually, the longitudinal and transversal modulus is Young and shear modulus of soil ($G_{12}=Gs$ & $E_1=Es$) if there is a surface footing. On the other hand, the longitudinal and transversal modulus equal to Young and shear modulus of pile ($G_{12}=Gp$ & $E_1=Ep$) if we have a single pile.

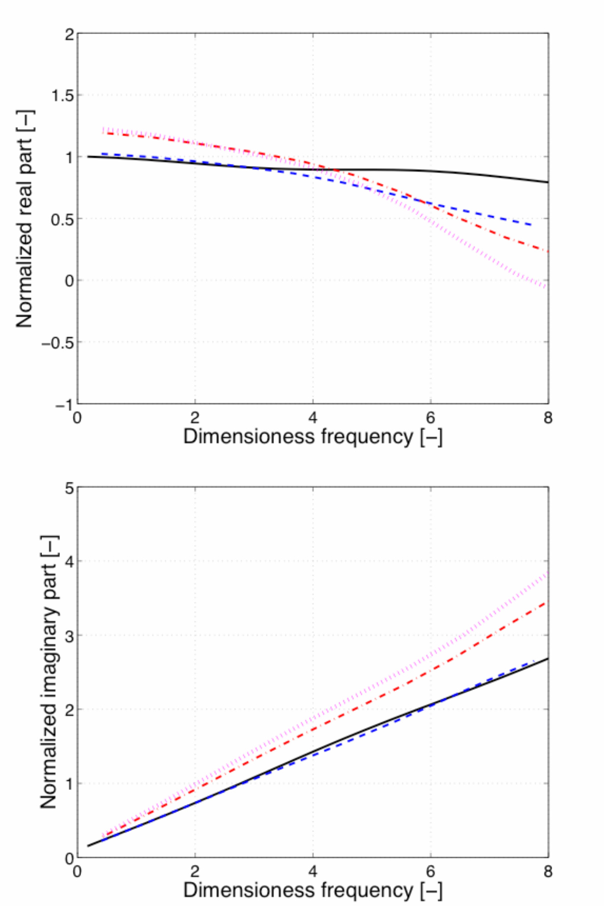


Figure 7. Effect of pile numbers on dynamic stiffness and damping: surface footing with an equivalent dimension (solid line), PH1 (blue dashed line), PH2 (red dashed-dotted line), PH3 (magenta dotted line)

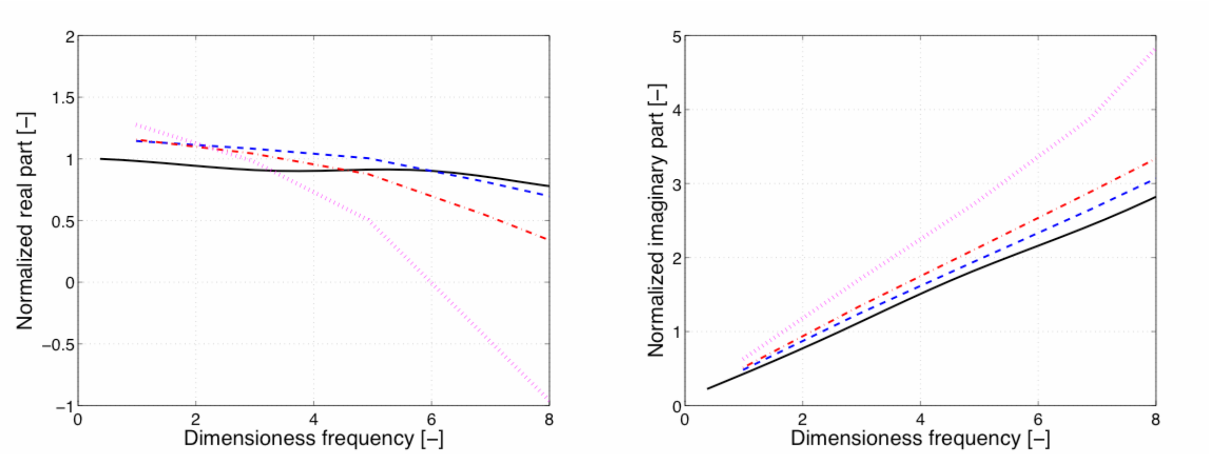


Figure 8. Effect of pile diameter on dynamic stiffness and damping: surface footing with an equivalent dimension (solid line), PH9 (blue dashed line), PH8 (red dashed-dotted line), PH7 (magenta dotted line)

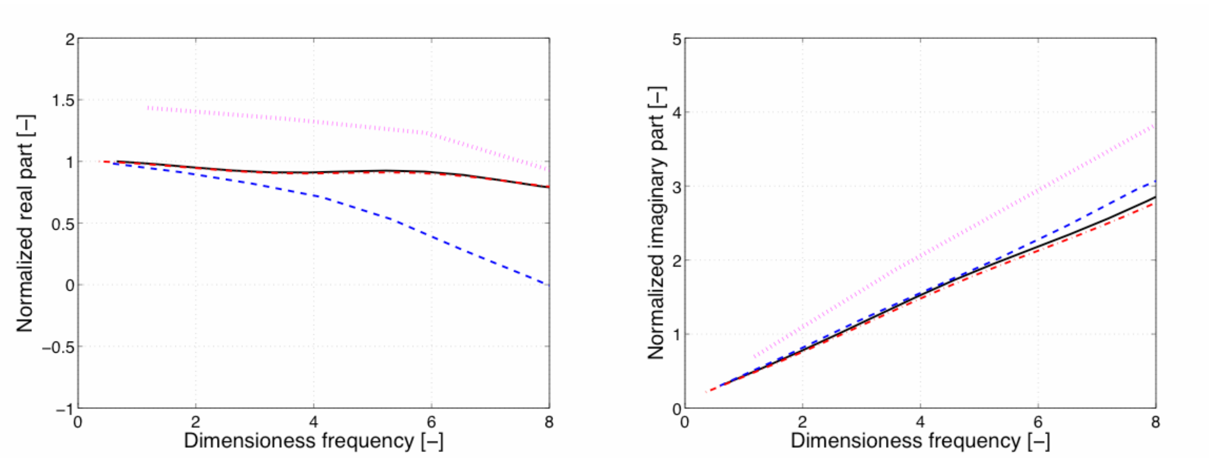


Figure 9. Effect of foundation width on dynamic stiffness and damping: surface footing with an equivalent dimension with PH13 (solid line), PH13 (blue dashed line), surface footing with an equivalent dimension with PH15 (red dashed-dotted line), PH15 (magenta dotted line)

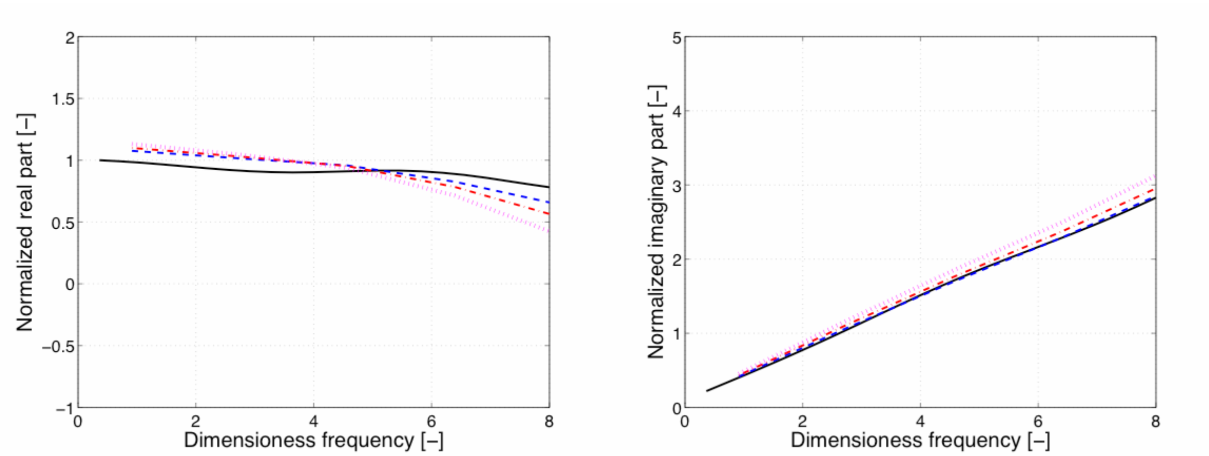


Figure 10. Effect of pile young modulus on dynamic stiffness and damping: surface footing with an equivalent dimension (solid line), PH6 (blue dashed line), PH5 (red dashed-dotted line), PH4 (magenta dotted line)

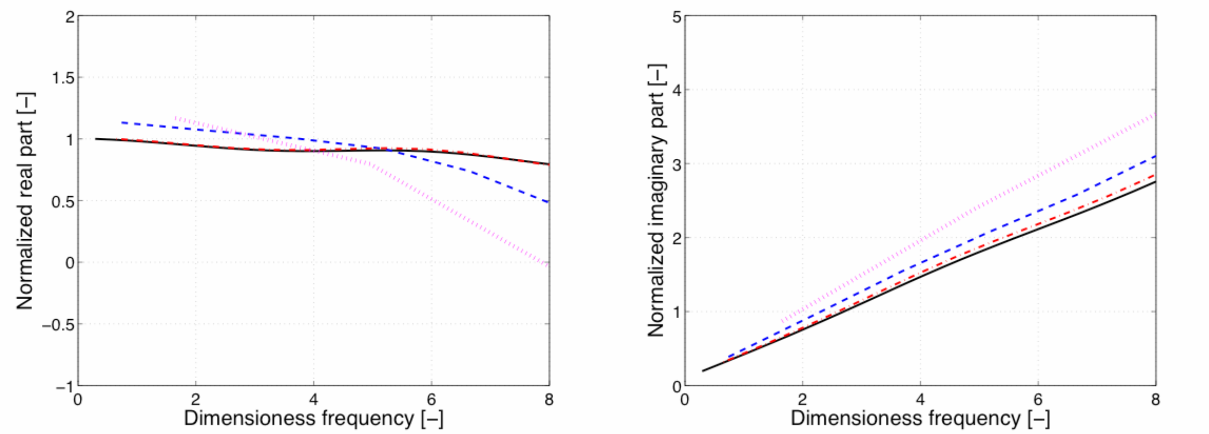


Figure 11. Effect of soil shear wave velocity on dynamic stiffness and damping: surface footing with an equivalent dimension with PH11 (solid line), PH11 (blue dashed line), surface footing with an equivalent dimension with PH12 (red dashed-dotted line), PH12 (magenta dotted line)

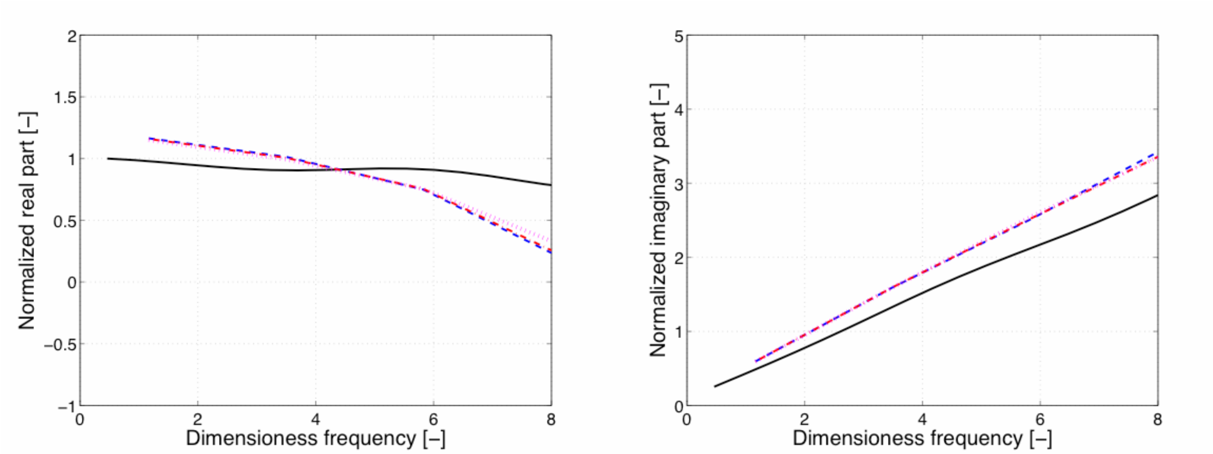


Figure 12. Effect of pile length on dynamic stiffness and damping: surface footing with an equivalent dimension (solid line), PH16 (blue dashed line), PH17 (red dashed-dotted line), PH18 (magenta dotted line)

Table 1. Different samples of large pile group foundation

	Pile number	Pile Young modulus Ep [Pa]	Pile dia. d [m]	shear wave Velocity V_s [m/s]	Foundation Width $2B_f$ [m]	Pile Length L [m]	Pile Spacing ratio S/d	Shear Modulus ratio G_{12}/G_s	Young Modulus ratio E_1/E_s	Slenderness ratio L/d
PH1	49	2e10	1	200	30	20	3.75	1.05	6.5	15
PH2	121	2e10	1	200	30	20	2.5	1.14	13.4	15
PH3	225	2e10	1	200	30	20	1.9	1.28	23.1	15
PH4	169	2e10	0.8	130	40	16	3.6	1.06	16.3	20
PH5	169	1e10	0.8	130	40	16	3.6	1.06	8.6	20
PH6	169	5e9	0.8	130	40	16	3.6	1.06	4.8	20
PH7	289	1e10	1	90	30	16	1.7	1.39	71.4	16
PH8	289	1e10	0.6	90	30	16	2.8	1.11	26.3	26
PH9	289	1e10	0.4	90	30	16	4.2	1.04	12.3	40
PH10	361	2e10	1	320	50	20	2.5	1.14	5.9	20
PH11	361	2e10	1	200	50	20	2.5	1.14	13.4	18
PH12	361	2e10	1	90	50	20	2.5	1.14	63.7	16
PH13	361	2e10	1.2	200	40	24	1.7	1.38	29	20
PH14	361	2e10	1.2	200	60	24	2.5	1.14	13.4	20
PH15	361	2e10	1.2	200	80	24	3.3	1.07	8	20
PH16	324	2e10	1	130	50	12	2.6	1.12	29.2	12
PH17	324	2e10	1	130	50	16	2.6	1.12	29.2	16
PH18	324	2e10	1	130	50	20	2.6	1.12	29.2	20

The results in parametric study, on the large pile group foundation provided an important basis to develop a discrete model including mass-spring-dashpots as an impedance model for a pile group. Since the equivalent longitudinal and transversal modulus for pile group is a significant factor for the lateral dynamic stiffness and damping ratio, in the following section, we reveal a consistent relationship between the discrete model (mass-spring-dashpot) to the equivalent parameters.

PROPOSED FORMULAS

There is a need for simple engineering procedures such as the code provisions developed for the seismic design of structures on spread footings. As it is observed in the parametric study results, the parabolic variation of the dynamic stiffness and increasing linearly the simplified system, $Z=k_\Gamma-a_0^2m_\Gamma+ia_0c_\Gamma$ (Fig.13) can be introduced for the lateral dynamic stiffness and damping for pile group in a homogenous half-space and the range of frequency between $0<a_0<8$. To identify each parameter (k,

c and *m*) the *KCM* approach is applied (Cottereau et al. 2006). This approach is able to identify, Mass, rigidity and mass matrices, to construct the impedance function for the complicated cases including the internal poles corresponding the resonance of soil between piles and the modal vibration (rocking, pumping and tensional motion), but in this study, we apply this approach for lower frequency (no pole) and only for the lateral motion.
A regression analysis is then used to determine a simple relationship between the representative parameters for pile group and a coefficient of simple spring-dashpot-mass model. This model is able to predict the lateral impedance in the case of floating pile group in a homogenous half-space. The samples chosen for this statistical is the 18 models, as shown in table 1.

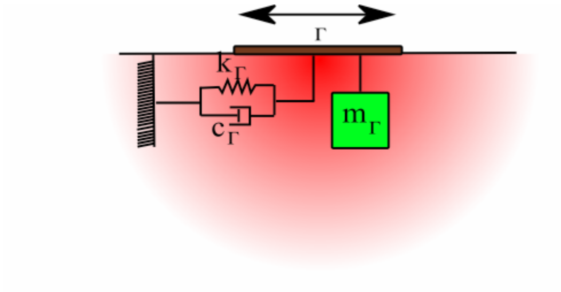


Figure 13. Simple spring-mass-dashpot model for lateral impedance function

The multiple regression analysis carried out to correlate the mass and dashpot to the soil and pile properties gives the following results with a regression coefficient (R) greater than *90%* for static stiffness and damping and *83%* for the equivalent mass:

$$\frac{k_\Gamma}{k_s} = \left(\frac{G_{12}}{G_s}\right)^{0.28} \left(\frac{E_1}{E_s}\right)^{0.036} \quad \text{(R=93\%)} \tag{8}$$

$$\frac{m_\Gamma V_s^2}{k_s B^2} = 0.013 \left(\frac{G_{12}}{G_s}\right)^{2.5} \left(\frac{E_1}{E_s}\right)^{0.35} \quad \text{(R=83\%)} \tag{9}$$

$$\frac{c_\Gamma V_s}{k_s B} = 0.5 \left(\frac{G_{12}}{G_s}\right)^{0.38} \left(\frac{E_1}{E_s}\right)^{0.16} \quad \text{(R=94\%)} \tag{10}$$

In these expressions as the equivalent longitudinal E_1 and transversal G_{12} increase the discrete model elements (mass-spring-dashpot) increase. Since the pile length don't play an important role for the lateral dynamic stiffness and the damping, the pile length is not observed in these expressions. Although the sample models chosen to obtain the proposed formulas just including large pile group, comparison with the exist formulas for single pile ($G_{12}=Gp$ & $E_1=Ep$) (Gazetas et al, 1984) or surface foundation ($G_{12}=Gs$ & $E_1=Es$) (Wolf, 1994) illustrates that it can be used for a comprehensive cases. Since numerical analysis of large pile groups is extremely costly, computer wise, the proposed expressions (8), (9) and (10) can be directly used to estimate the dynamic stiffness and damping of this type of foundation.

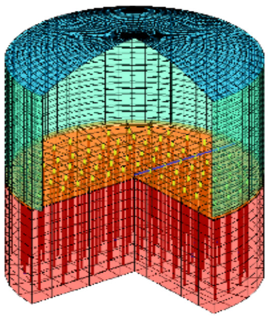


Figure 14. Numerical modeling of a reservoir resting on a pile group with 112 piles

ILLUSTRATIVE EXAMPLES AND COMPARISON

As an example, we consider the case of an array of 112 identical piles connected by a rigid cap (fig. 14). Figure (15) shows the dynamic stiffness and damping versus frequency and compares the results derived from the formulas (7), (8) and (9), from the numerical FEM/BEM. This figure illustrates that the performance of the proposed formula to predict the lateral stiffness and in particular damping with a very good approximation.

Table 3. Soil and pile properties for an example model

Pile number	*Pile Young modulus [Pa]*	*Pile dia. [m]*	*Soil shear wave Velocity [m/s]*	*Cap Diameter [m]*	*Pile Length [m]*
112	*2e10*	*1*	*200*	*40*	*20*

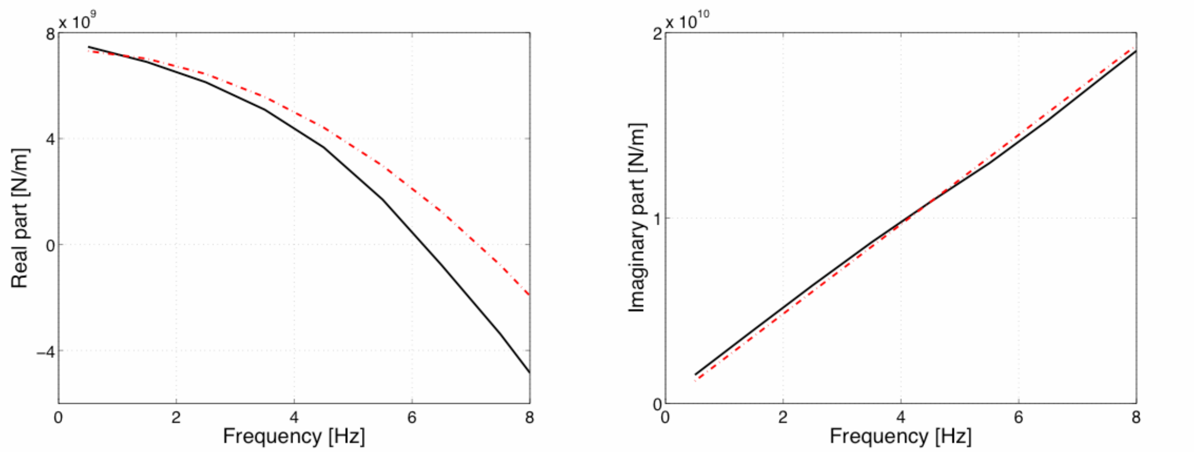


Figure 15. Comparison the proposed formulas with numerical solution: Complete solution (blue solid line), Proposed formulas (red dashed dotted line)

To examine the accuracy of the proposed expression on the response of the superstructure, a gas reservoir is chosen. This reservoir is 35m high, has a diameter 20m and is 50cm thick on an 80cm-thick cap foundation and the above pile group. To check the proposed formulas for the response of the structure two cases are shown:

a) Using the lateral dynamic stiffness, damping ratio computed from numerical FEM-BEM.

b) Using lateral dynamic stiffness and damping computed with the proposed formulas.

The harmonic transfer function at the top of the reservoir give two peaks at 2.8 Hz and 10.4 Hz corresponding the resonance of the coupled soil-structure system (Fig. 16). The time history responses are then presented, using an accelegeram (shown in Fig. 17a) recorded during the 1976 Friuli earthquake. It has a peak acceleration (PGA) is about 0.23g. The *2%* damped response spectra is also illustrated in Fig17b. This clearly shows a dominance of low period (0.2-0.4s) in the signal. As a result, the proposed formulas (7, 8 and 9) provide a very acceptable engineering estimate both frequency and time domain response (Fig. 18).

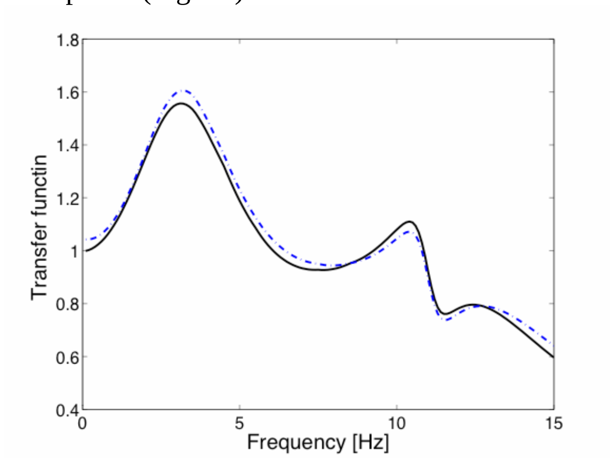
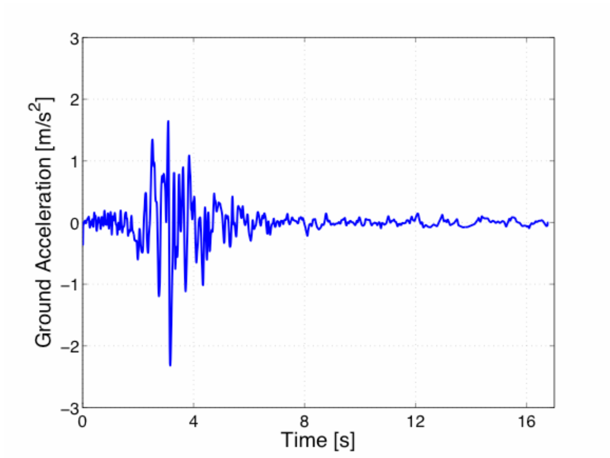
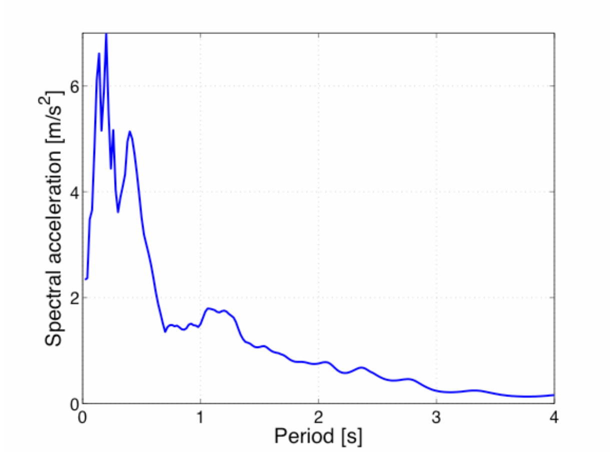


Figure 16. Harmonic transfer function at the top of the reservoir: complete BEM/FEM (solid line), proposed formulas (dashed-dotted line)

CONCLUSION

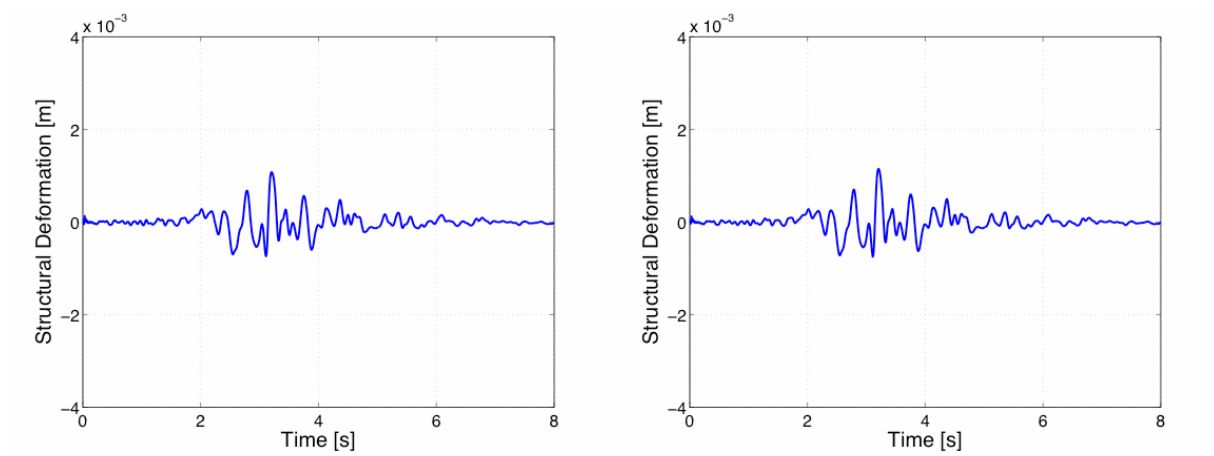
A BEM/FEM coupling technique is applied for the dynamic analysis of pile groups. Results are compared with others existing methods in the literature. A parametric study is carried out on large pile group foundations to study the influence of soil and pile properties on the lateral stiffness and damping. The dynamic large pile group behavior is highly frequency dependent. The less pile spacing or the more pile/soil rigidity, the more the inertial effect and radiation damping. Based on homogenization theory the important factors was presented in the form of longitudinal and transversal equivalent modulus of the soil matrix and the piles. The proposed formulas for static and dynamic stiffness and damping ratio was proposed a reference to further enhance the understanding the lateral dynamic behaviour of large pile group and as rather good engineering estimates. A gas reservoir is studied in different cases complete solution with SSI and application of proposed formulas. Good agreement is found between the complete SSI and the application of the proposed formulas. These formulas can be applied for the pile group in a homogenous halfspace with neglecting the peak, which occur at higher frequency and very soft soil.





a) Input acceleration at bedrock b) Response spectra of the ground motion

Figure 17. Friuli earthquake, M=6.5 (1976, Italy)



a) Relative displacement using FEM-BEM b) Relative displacement using proposed formulas

Figure 18. Time-history relative displacement of the reservoir under Friuli earthquake (1976)

REFERENCES

Aubry, D. and Clouteau, D. (1992) "A subdomain approach to dynamic soil-structure interaction," Recent advances in earthquake engineering and structural dynamics, pp. 251-272, Nantes: Ouest Edition/AFPS

Clouteau, D. and Taherzadeh, R. "Soil-Pile-Superstructure Interaction Under Seismic Loading, " First European conference on Earthquake Engineering and Seismology, Geneva, Switzerland (2006).

Cottereau, R., Clouteau, D. and Soize, C. (2007) "Construction of a probabilistic model for impedance matrices" Comput. Methods Appl. Mech. Eng. (2007), doi: 10.1016/j.cma.2006.12.001.

Craig R.J. and Bampton, M., "Coupling of Sub-structures for Dynamic Analysis.", AIAA Journal, Vol. 6, no. 7, pp.1313-1319, 1968.

Dobry, R. and Gazetas, G. (1988) "Simple Method for Dynamic Stiffness and Damping of Floating Pile Groups, " Geotechnique, Vol.38 (40), pp. 557-574.

Gazetas, G., Hess, P., Zinn, R., Mylonakis, G. and Nikolaou, A. (1998) "Seismic Response of a Large Pile Group, "11th European Conf. On Earthquake Engrg., Paris, France.

Gazetas, G., Dobry, D. (1984) " Horizontal response of piles in layered soils " Journal of Geotechnical Engineering, Vol. 110, 20-39

Aubry, D. and Postel, M. (1985) " Dynamic response of a large piles by homogenization" Proc. 2nd. Int. Conf. Soil Dynamics and earthquake engineering.

Kaynia, A.M. and Kausel, E. (1982) "Dynamic Stiffness and Seismic Response of Pile Groups" Research Report, Dept. of Civil Engineering., MIT, Cambridge Mass.

Makris, N. and Gazetas, G. (1992) " Dynamic Pile-Soil-Pile Interaction. Part II: Lateral and Seismic Response," Earthquake Engineering and Structural Dynamics, Vol. 21 pp.145-162

Mylonakis G., Nikolaou S. & Gazetas, G. (2006) "Footing under seismic loading: Analysis and design issues with emphasis on bridge foundations," Soil Dynamic and Earthquake Engineering, 26, 824-853

Mylonakis, G. and Gazetas, G. (1999) "Lateral Vibration and Internal Forces of Grouped Piles In Layered Soil," Journal of Geotechnical & Earthquake Engineering, Vol. 125 pp.16-25

Novak, M. (1991) "Pile Under Dynamic Loads: State of the Art." Proceeding 2nd Int. Conf. On Rec. Adv. In Getech. Earthquake Engrg. And Soil Dyn., Vol. 3, 2433-2456.

Roesset, J.M. (1984). "Dynamic Stiffness of Pile Groups." Pile Foundation, Special Publication No. 10, ASCE, Reston, Va., 263-286

Wolf, J.P. (1994). "Foundation vibration analysis using simple physical model." PTR Prentice Hall