4th International Conference on
Earthquake Geotechnical Engineering
June 25-28, 2007
Paper No. 1260

JOINT INVERSION ANALYSIS USING THE DISPERSION CHARACTERISTICS OF LOVE AND RAYLEIGH WAVES

Il-Wha Lee [1], and Sung-Ho Joh[2]

ABSTRACT

Rayleigh wave and Love wave are the major elastic waves in the category of surface waves. It is known that Love wave is not disturbed by P waves because of the particle movement direction. Therefore, the information that Love wave carries is more distinct and clearer than the Rayleigh wave. In this study, the dispersion characteristics of Love wave was investigated by theoretical and experimental approaches using the SASW technique. Theoretical analyses were performed for the multi-layered ground to determine the phase velocities of Love wave as well as those of Rayleigh wave. Also, SASW tests were performed at geotechnical sites to verify the results obtained by the theoretical analysis. Based on this research, the joint inversion analysis which uses both Love wave and Rayleigh wave dispersion information was proposed. Purpose of the joint inversion analysis is to improve accuracy and convergence of inversion results. The proposed technique consists of a forward modeling using modified transfer matrix, a sensitivity matrix determined to the ground system, and a DLSS (Damped Least Square Solution) as an inversion technique. The theoretical analysis and the field testing results indicated that the joint inversion analysis can be applied to better evaluate the stiffness of subsurface structures.

Keywords: Surface wave, Love wave, Rayleigh wave, SASW method, Joint inversion, Dispersion

INTRODUCTION

The Spectral-Analysis-of-Surface-Waves (SASW) test, which is used to verify earth structures and is applied in geophysics for oil exploration, was first introduced in civil engineering works by Stokoe (Stokoe et al, 1982). This test measures the dispersion characteristics of a surface wave separated from the body wave. It is based on nondestructive and nonintrusive seismic methods for the evaluation of stiffness profiles from stress wave measurement. The SASW method was established through a number of laboratory experiments and practical experiences conducted by Roesset (1991), Gucunski (1991), Al-Hunaidi (1994), Joh (1996), and others. In the SASW method, the shear-wave velocity profile is evaluated from the phase-velocity dispersion curve measured in the field by the inversion analysis. Since Nazarian and Stokoe (1984) tried to invert a shear wave velocity profile by an iterative forward modeling analysis, several researchers have taken different approaches in the inversion analysis. Foinquinos and Röesset (1991) used the least square approach for inverting a shear wave velocity profile. Rix and Leipski (1991) followed the maximum likelihood approach. Yuan and Nazarian (1993) presented an automated inversion technique using the general inverse theory. And Williams and Gukunski (1995) applied neural networks to perform the inversion procedure for SASW testing of asphalt concrete pavements.

Recently, Joint inversion method which is simultaneously used some information of the ground is attempted to increase the accuracy. Shapiro(2000) is obtained the data from the Monte Carlo inversion of the Rayleigh and Love wave group velocity dispersion curves measured using broad band

[1] Senior researcher, Track and Geotechnology Team, Korea Railroad Research Institute, Korea, Email: iwlee@krri.re.kr

[2] Associate professor Professor, [2]Department of Civil Engineering, Chung-Ang University, Korea, Email: shjoh@cau.ac.kr

seismogram data. Carlo. et al. (1998) researched about the simultaneous inversion of Rayleigh phase velocity and attenuation for near surface site characterization. They calculated the effective phase velocity about multi mode of phase velocity and attenuation based on nonlinear viscous-elastic analysis. Lai (1998) developed a method the simultaneously invert experimental dispersion and attenuation curves to obtain the shear wave velocity and shear damping ratio profiles at a site.

The inversion analysis comprises of two analysis steps, which are forward modeling and optimization. The forward modeling is to generate a theoretical dispersion curve for a given layering system, and optimization is a procedure to find the optimum solution in the given model space. Most of the aforementioned inversion analyses employed the dynamic stiffness matrix method (Kausel and Röesset, 1981) to generate a theoretical dispersion curve. Dynamic stiffness matrix method is a preferable approach, because it generates a phase velocity dispersion curve for a superposed mode of surface-wave normal modes and body waves, which is a more realistic simulation of an experimental dispersion curve determined in the field. Joh (1996) proposed “array inversion” technique to reduce non-uniqueness in the inversion analysis by employing more information on the dispersive characteristics of a given site. In the array inversion technique, the source-receiver geometry is incorporated in forward modeling, which characterizes the dispersive feature of a given site more accurately. Until now, the vertical component of the Rayleigh wave was used to evaluate the stiffness profiles at the site since this wave was more dominant than the others. However, if the layer structures are complex or existed significant stiffness differences between layers, it is difficult to calculate the stiffness profile due to the low resolution of the inversion analysis. This problem is known as the near field effect, and is caused by the higher-mode and the interference of the body waves.

In this paper, as an effort to further reduce non-uniqueness in the inversion analysis, a joint-inversion analysis algorithm was proposed to use both Rayleigh-wave and Love-wave dispersive characteristics. First, the joint inversion algorithm is proposed based on the modified transfer matrix method (Tzong and Penzien, 1983). Later, the synthetic phase-velocity dispersion curves were generated for Rayleigh and Love waves for the inversion analysis. And field tests were performed to verify the proposed inversion algorithm.

SURFACE WAVES IN LAYERED GROUND

In a uniform infinite medium, only P and S waves appear. However, if the medium is bounded or non-uniform, then other types of waves such as Rayleigh wave or Love wave could be appeared. A Rayleigh wave appeared as an elastic half-space. As for Love wave, theoretically, they require a superficial layer having a lower S wave velocity than the underlying half-space. Love wave consists essentially of SH wave that is trapped by multiple reflections within the superficial layer. This phenomenon is illustrated in Figure 1, which describes the case of a homogeneous superficial layer of thickness h overlying a homogeneous half-space. Love wave traveling in the x direction would involve particle displacements only in the y direction, and could be described by the following equation.

$$\upsilon(x,z,t) = V(z)e^{i(k_L x-\omega t)} \tag{1}$$

where υ is the particle displacement in the y direction, ω is the angular frequency, t is the time, $V(z)$ describes the variation of υ with depth, and k_L is the wave number of Love wave.

The important elements of the surface waves are those that propagate near the surface of the ground and travel at the regular velocity dependent on frequency. Surface waves have penetration depths that depend on each wavelength. In seismology, this phenomenon is described as "dispersion", and is used in evaluating the stiffness of the ground. Shot wavelengths represent the stiffness of shallow grounds, whereas long wavelengths represent stiffness of deep grounds. A real stiffness profile is calculated by inversion analysis base on a dynamic stiffness matrix (Kausel, 1982) or transfer matrix (Haskell, 1953). In this study, we used the dispersion characteristics of both Rayleigh wave and Love wave.

Using a Love wave is more effective than Rayleigh wave since the former wave is not affected by compression wave. Consequently, Love wave are more useful and precise.

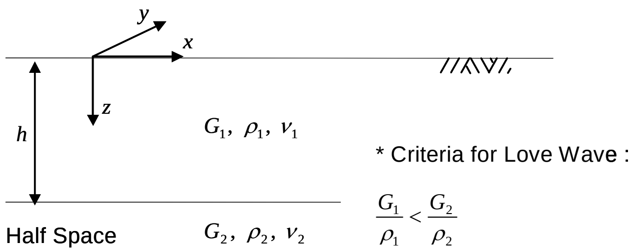


Figure 1. Simple condition for which Love wave can exist ($\frac{G_1}{\rho_1} < \frac{G_2}{\rho_2}$)

SASW TEST

The Spectral Analysis of Surface Waves (SASW) test is one of the various approaches used to evaluate stiffness profiles. The SASW test generally uses a seismic source with two receivers in line with the source. Basically, the test consists of retrieving phase velocities for a regular band of frequency. There are several methods available to calculate the phase velocities. From the measured phase velocities, the dispersion curve, which describes the relationship between the phase velocity and wavelength is obtained. The stiffness profiles can be obtained from the dispersion curves by iterative forward modeling or inversion analysis. A description used to determine stiffness profiles is shown in Figure 2.

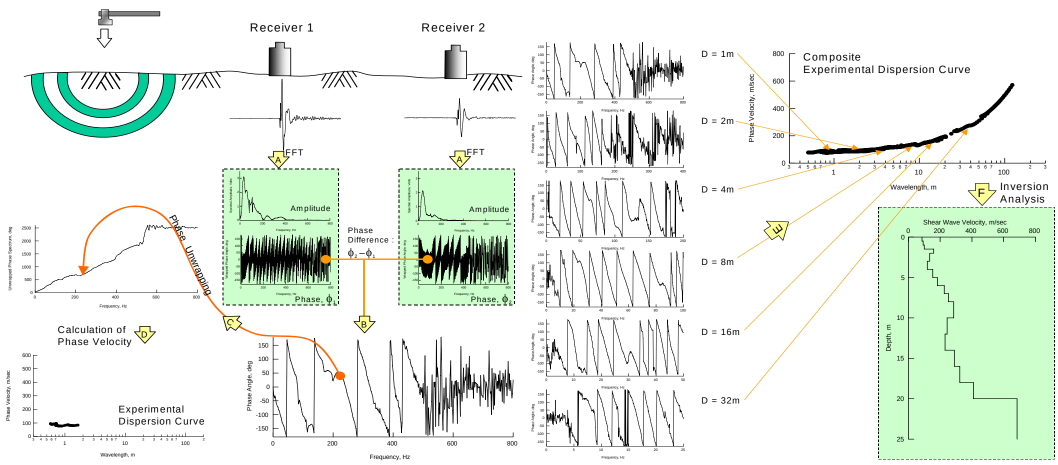


Figure 2. SASW (Spectral Analysis of Surface Waves) Method Scheme [Joh, 1996]

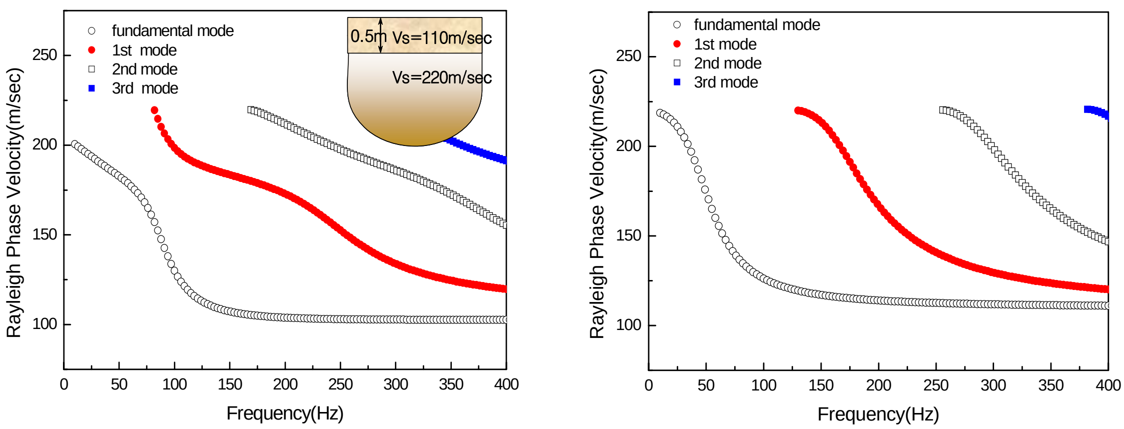
OPTIMIZATION OF THE INVERSION ANALYSIS

In the SASW method, the shear-wave velocity profile is evaluated by an inversion analysis of the phase-velocity dispersion curve measured in the field. Nazarian and Stokoe (1989) inverted shear wave velocity profiles by iterative forward modeling analyses. However, several other researchers have taken different approaches. The inversion analysis is comprised of two analysis steps, which are forward modeling and optimization. Forward modeling is performed to generate a theoretical dispersion curve for a given layering system, and optimization is a procedure to find the optimum solution in the given model space. Most of the aforementioned inversion analyses used the dynamic stiffness matrix method (Kausel, Eduardo and Peek, 1982) to generate a theoretical dispersion curve.

The dynamic stiffness matrix method is favored, because it generates a phase velocity dispersion curve for a superposed mode of surface-wave normal modes, which is a more realistic simulation of the experimental dispersion curve determined in the field. The joint inversion analysis consisted of the iterative forward modeling and the inversion analysis. In this paper, the modified transfer matrix method (Tzong and Penzien, 1983) was used as the iterative forward modeling, and the damped least square solution (DLSS) (Santamarina, 1998) for the inversion analysis.

Dispersion Characteristics of Surface Waves Using Modified Transfer Matrix

In order to investigate the dispersion characteristics of the Rayleigh and Love wave which were calculated by the modified transfer matrix, the dispersion characteristics of each wave was examined by the theoretical model shown in Figure 3(a). It was assumed that the upper layer was soft ground and the lower layer was half space where Love wave could occur. Each wave was calculated up to the 3rd mode. Figure 3 describes the phase velocity by each mode. According to the data, the phase velocity increases with the mode sequence. Therefore, higher modes were formed based on the cut-off frequency, and the phase velocity of the Love wave increases by approximately 10%. Since the dispersion characteristics of surface waves in natural grounds are the most affected by lower modes, the inversion analysis is generally done using fundamental modes only in conventional SASW test. As shown in Figure 3, the graph of the fundamental mode of the Rayleigh wave can properly express stiffness differences layer. However, its sensitivity decreases as the mode increases whereas the sensitivity of Love wave remains the same pattern. The decrease in sensitivity can be attributed by the near field effect and interference of body wave.



(a) Rayleigh Wave **(b) Love Wave**

Figure 3. Dispersion Characteristics each Surface Wave Mode

Inversion Analysis

Since the number of known data (d_m) and unknown (m_n) are different (m>n), in terms of the order of matrix in an inversion analysis, it is impossible to use general inverse matrix method. Therefore, non-linear inversion analysis methods based on SVD (Single Value Decomposition) are applied. This paper used DLSS (Damped Lease-Squares Solution) in order to improve the accuracy. In inversion analysis based on the SASW method, for a given frequency band are not used at same range. In other words, even though high-frequency band data can be used in various frequency ranges with high availability, the low-frequency band data (deeper ground) have relatively lower availability. One may then deduce that as the ground gets deeper, less information become available. With the DLSS, mixed matrices are calculated by minimizing that error. If it is assumed that $(G^T \cdot G + \eta^2 \cdot I)$ is nonsingular, the solution on ($m_n^{<est>}$) using the value ($d_m^{<meas>}$) shall be as follows:

$$d_m^{<est>} = (G^T \cdot G + \eta^2 \cdot I)^{-1} \cdot G^T \cdot m_n^{<meas>} \quad (2)$$

where, G is the joint sensitivity matrix, I is the unit matrix, and η is the damping coefficient.

The inversion analysis is performed with in the assumed test depth.

Constitution of the Joint Sensitivity Matrix

The sensitivity of the forward equation with respect to the model parameter is described by the sensitivity matrix (G) which is defined in equation (3). Partial derivative equation calculated by the inversion analysis evaluating the measured data (d_m) and model parameter (m_n).

$$G_{ji} = \frac{\partial V_{ph,j}}{\partial V_{s,i}} \tag{3}$$

where V_{ph} is phase velocity and V_s is shear wave velocity.

The partial derivative matrix for the joint inversion analysis is shown in equation (4).

$$[G_n^{mn}] = [\frac{\partial d}{\partial m}] = \begin{bmatrix} \begin{bmatrix} G_{11} & \cdots & G_{1m} \\ \vdots & [G]_{Love} & \vdots \\ G_{p1} & \cdots & G_{pm} \end{bmatrix} \\ \begin{bmatrix} G_{11} & \cdots & G_{1m} \\ \vdots & [G]_{Rayleigh} & \vdots \\ G_{q1} & \cdots & G_{qm} \end{bmatrix} \end{bmatrix} \tag{4}$$

where G_{ji} is the partial derivative of the j^{th} phase velocity with respect to the shear wave velocity of the i^{th} layer. And $[G]_{Love}$ is partial derivative matrix of Love wave and $[G]_{Rayleigh}$ is partial derivative matrix of Rayleigh wave.

Often the sensitivity matrix is called the Frechet derivative matrix or the data kernel.

Algorithm of the Joint Inversion Analysis

The procedure of the joint inversion analysis, which was structured using the analysis methods mentioned above set the initial shear-wave velocity profile against the given layer structure, using a representative dispersion curve. A reasonable estimation of the initial shear-wave velocity profile is essential to prevent the divergence and local minimum of the inversion solution. In order to estimate the initial shear-wave velocity profile a revision of the method created by Foinquinios (1991) was adopted. Then the initial shear-wave velocity profile was calculated based on an assumed relationship between the penetration depth and the wavelength. A revision of the model parameter (m_i), (or the change of model parameter (∂ m)) was determined based on Equation (4), using the sensitivity matrix (G) and the difference (∂ d) between the theoretical and experimental values of the dispersion curve. The dispersion characteristics of Love wave and Rayleigh wave were simultaneously considered when the joint sensitivity matrix (G) was organized. In addition, the model parameter which was calculated by inversion analysis was equally applied to Love wave and Rayleigh wave. Whether the result of the inversion analysis is reasonable or not could be decided by using the RMS (Root Mean Square) error between experimental and theoretical values. If errors are not allowed, the joint sensitivity matrix can be reorganized and the inversion analysis conducted. The calculation will be repeated until the final FMS error reaches an acceptable value, by changing the model parameter depending on the result of the inversion analysis, which can be diagramed as Figure 4.

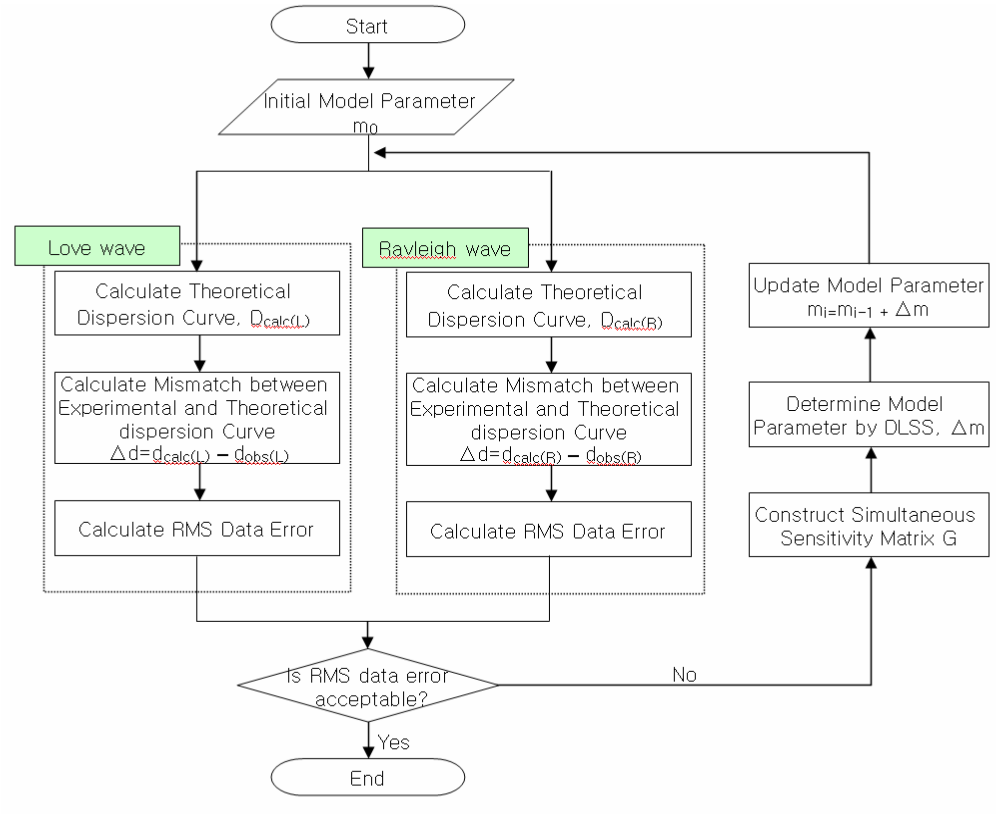


Figure 4. Flowchart of the Joint Inversion Analysis

In the joint inversion analysis, Love wave and Rayleigh wave can regarding the difference frequency range (dispersion range) but be the same in terms of the thickness of the layer. In a joint inversion, the initial layer thickness is important. Because the participation factor of each wave varies and the frequency range of field testing data is different, the range and the number of layers of frequency should be carefully decided in order to consider the characteristics of the two waves.

JOINT INVERSION ANALYSIS USING THEORETICAL MODEL

This study assessed accuracy using the theoretical model shown in Figure 5 in order to verify the joint inversion analysis method. The theoretical model was organized with two upper layers (2m thick) and a lower half space.

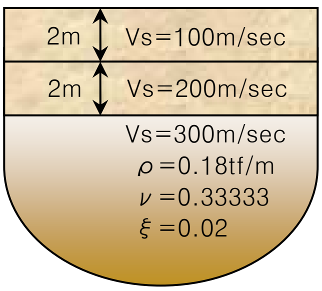
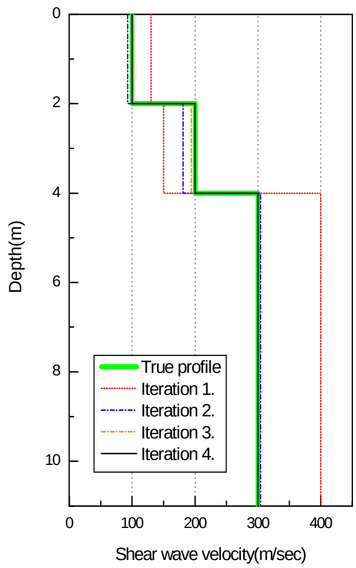


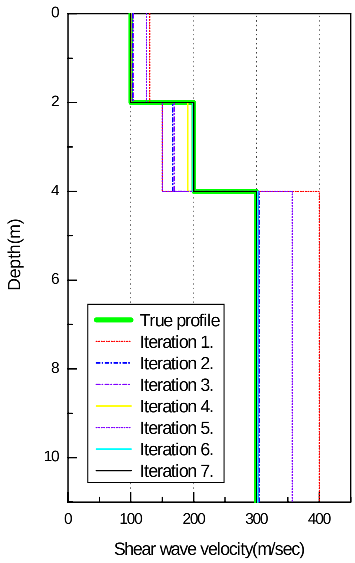
Figure 5. Theoretical Ground Model

The characteristics of the layers used the same value except for the shear-wave velocity. The frequency band was 5 - 23Hz with a perturbed coefficient (Joh, 1996) of 1×10^{-5} to calculate the sensitivity matrix. In the inversion analysis, only the fundamental mode was used.

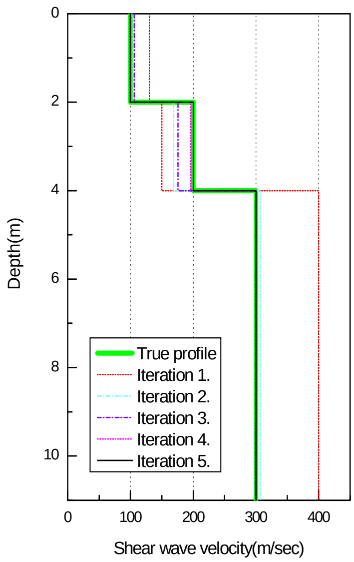
The inversion method was subdivided into three parts as a input data ; Love wave, Rayleigh wave and joint inversion which uses both waves. The initial assumption value for shear wave velocity was 130, 150, and 400m/sec as shown in Figure 6, and the layer thickness was the same with the theoretical model. When Love wave was used, the RMS error converged to 0 after 4 iterations and the analysis was very fast because the equation of motion and characteristics were considered just in a single direction in case of the forward modeling. In case of the Rayleigh wave, the layer thickness and the initial model parameter were the same with Love wave. However, the routine which decided the dispersion characteristics at forward modeling was different. Regarding the analysis of the Rayleigh wave, the RMS error converged to 0 after 7 iterations. The same layer thickness and initial model parameter were used in the joint inversion analysis as well. In the inversion analysis, the joint sensitivity matrix (G) which included dispersion information of both Love wave and Rayleigh wave was used. In order to calculate the parameter (m_n) of the new model through the inversion analysis, the differences ($\triangle$d) of phase velocity were equally applied to both waves for the formation of a sensitivity matrix. In the joint inversion analysis, the RMS error converged to 0 after 5 iterations. During the iterations, the decimal fractions of the shear-wave velocity value were ignored.



(a) Love wave



(b) Rayleigh wave



(c) Joint Inversion

Figure 6. Comparison the Shear-wave Velocity Profile each Inversion Method

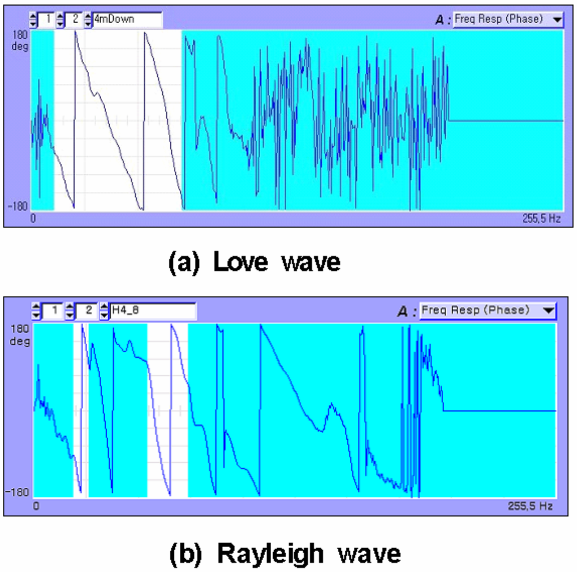
JOINT INVERSION ANALYSIS OF THE FIELD TEST RESULTS

Field Test

In order to investigate the applicability of the joint inversion analysis, field tests were conducted. The results were compared under three different inversion method; only the Rayleigh wave, Love wave, and both. The field tests were conducted in a hill area which is about 100m long flat. The ground consists of a colluvial layer that goes up to 0.5m deep from the surface, and a weathered soil of 5m deep. Below the 5m, weathering layers (fracture zone) are formed. Because the boring site was far from the test location, the result of the boring test was used just to analyze the general trend of the ground structure.

Joint Inversion Analysis

The same layer thickness and initial assumption value were used in the joint inversion analysis. Besides, the sensitivity matrix of Love wave and Rayleigh wave were used simultaneously. The new model parameter (m_n) was equally applied to both the Love wave and the Rayleigh wave for the forward modeling. Figure 7 compares the phase spectrum at each receiver spacing. Receiver spacing is 4m, the frequency band is 20 - 75Hz. Even though the whole band range was available in case of the Love wave, however the band range was partially available in case of the Rayleigh wave. Also as illustrated in the Gabor spectrum shown in Figure 8, the energy was concentrated in the some band in case of Love wave. The Gabor spectrogram, which represents the original signal as a linear combination of time and frequency shifted Gaussian functions, is a good tool in the investigation of wave groups. This discrepancy may be explained by a difference in the participation factor of displacement and stress depending on the frequency or location of the layer boundary. Depending on the ground conditions, the results of the Rayleigh wave tests can be improved. In other words, the range of available frequency band decided depending on the type of wave, ground conditions, and receiver spacing. This can be used in calculating a more accurate shear-wave profile by joint inversion analysis.



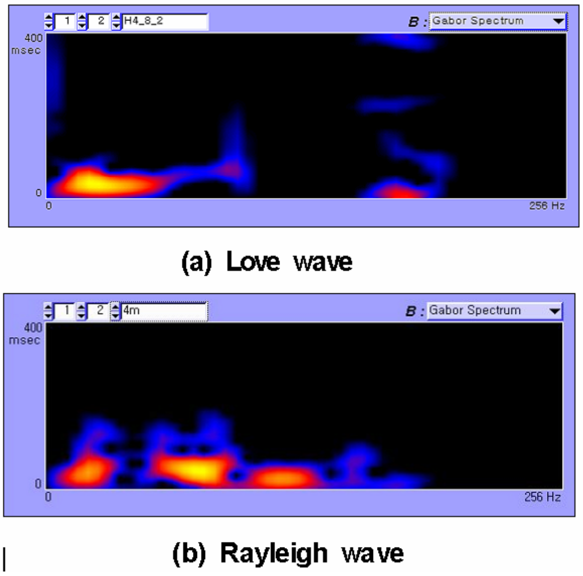


Figure 7. Phase Spectrum (4m spaing) **Figure 8. Gabor Spectrum (4m spacing)**

In order to equalize the data ranges of Love wave and Rayleigh, 20 - 130Hz was used as the frequency band for the joint inversion analysis. Figure 9 compares the results which were calculated by Love wave, Rayleigh wave and the joint inversion analysis. It was turned out that the result of the joint inversion analysis was more subordinate to Love wave than Rayleigh wave. Besides, the RMS error decreased compared to the inversion analysis using Rayleigh wave only. Although the three inversion data turned out similar up to 7m, beyond that level they varied according to the inversion method.

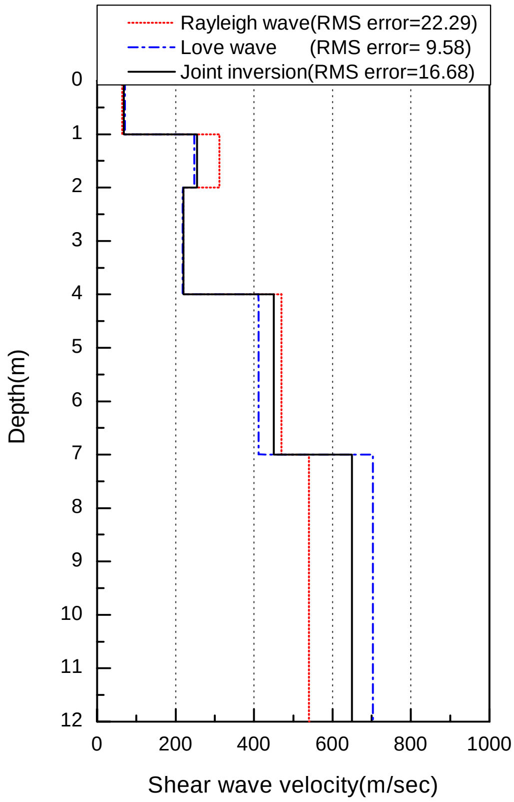


Figure 9. Comparison the Shear-wave Velocity Profile each Inversion Method

CONCLUSIONS

This study suggested a joint inversion analysis method which uses phase velocity dispersion information of Love wave and Rayleigh wave based on the SASW technique. The purpose of the joint dispersion analysis is to improve the accuracy and the convergence of the inversion result, using the each wave contribution. The method of analysis includes the organization of the modified transfer matrix in order to analyze the forward modeling of Love wave and Rayleigh wave, the sensitivity matrix and the DLSS (Damped Least Square Solution). The joint inversion analysis used dispersion information of both when organizing the sensitivity matrix. Then, the shear-wave profile of the ground was repeatedly calculated through the inversion analysis. In a theoretical study which was to verify the suggested method, a numerical analysis of the joint inversion analysis was conducted. Finally, the results were compared to field tests. According to this comparison, the joint inversion analysis was better in preventing excessive divergence and defining the change in stiffness by layer. It can be concluded that the joint inversion analysis method can improve the accuracy and the convergence speed for a solution.

REFERENCES

Al-Hunaidi, M. O., "Analysis of dispersed multi-mode signals of the SASW method using multiple filter/cross-correlation technique," Soil Dynamics and Earthquake Engineering, 13/24, 13-24p., 1994

Carlo, G. L., and Rix, G. J., "Simultaneous inversion of Rayleigh phase velocity and attenuation for near-surface site characterization", national Science Foundation and U.S.Geological Survey, Georgia Institute of Technology., 1998

Ewing, A. G., Jardetzky, W. S., and Press, F., "Elastic Waves in Layered Media," McGraw-Hill, New York, NY, USA., 1957

Foinquinos, R., Analytical study and inversion for the spectral analysis of surface waves method. B. S, C.E. The University Texas at Austin., 1991

Gucunski, N., and Woods, R. D., "Inversion of Rayleigh Wave Dispersion Curve for SASW Test," Proceeding of the 5th International Conference on Soil Dynamics and Earthquake Engineering, 127-138p., 1991

Haskell, N. A., "The distribution of surface waves on multi-layered media," Bull. Seismol. Soc. Am. 43, 17-34p., 1953

Heisey, J. S., Stokoe, K. H., II, Hudson, W. R., and Meyer, A. H., "Determination of in situ shear wave velocites from Spectral-Analysis-of-Surface –Waves", Research Report No.256-2 Center for Transportation Research, The University of Texas at Austin.,1982

Joh, S.-H., "Advancement in the Data Interpretation and Analysis in the Spectral-Analysis- of-Surface-Waves Method", Ph.D. Dissertation, The University of Texas at Austin., 1996

Jones, R. B., "Surface wave technique for measuring the elastic properties and thickness of roads", theoretical development, Journal of Applied Physics, 13, 21-29p., 1962

Kausel, Eduardo, and Peek, "Dynamic Loads in the Interior of a Layered Stratum: An Explicit Solution",Bull. Seismol. Soc. Am. 75/5, 1459-1508p., 1982

Lai, C. G.," Simultaneous inversion of Rayleigh phase velocity and attenuation for near surface site characterization", Ph.D, dissertation, Georgia Institute of Technology., 1998

Rix G. J., and Leipski, A. E., "Accuracy and resolution of surface wave inversion." *Recent advances in instrumentation, data acquisition and testing in soil dynamics*, Geotechnical special publication, No. 29, N.Y., ASCE, 17-23., 1991

Roësset, J. M., Chang, D. W., Stokoe, K. H., "Comparison of 2-D and 3-D models for analysis of surface wave tests." *Prooc. 5th Int. Conf. on Soil Dyn. and Earthq. Eng.*, Kalsruhe, 1, 111-126., 1991

Santamarina, J. C. and Fratta, D., "Introduction to discrete signals and inverse problems in civil engineering", ASCE Press., 1998

Shapiro, N. M., Gorbatov, A. V., Gordeev, E., and Domnguez, J., "Averge shear-wave structure of the Kamchaka peninsula from the dispersion of surface wave", Earth Planets Space, 52,573-577, 2000

Stoke, K. H. II., G. J. Rix, and S. Nazarian., "In situ seismic testing with surface waves," Proceedings, 12th International Conference on Soil Mechanics and foundation Engineering, Rio de Janeiro, 13-18 Aug., pp.31-334., 1989

Tokimatsu, K., Tamura, S., and Kojima, H., "Effects of multiple modes on Rayleigh wave dispersion." *J. Geotech. Engrg.*, ASCE, 118(10), 1529-1543., 1992

Tzong, T. J., and Penzien, J., "Hybrid modelling of soil-structure interaction in layered media", Rep. No. UCB/EERC-83/22, Earthquake Engineering Research Center, Univ. of California, Berkeley, Calif., 1983