4th International Conference on
Earthquake Geotechnical Engineering
June 25-28, 2007
Paper No. 1269

NUMERICAL MODELLING OF SEISMIC RESPONSE OF CANTILEVER EARTH-RETAINING STRUCTURES

Rajeev PATHMANATHAN [1], Paolo FRANCHIN [2], Carlo LAI [3], Paolo PINTO [2]

ABSTRACT

In current engineering practice the seismic design of earth retaining structures is usually carried out using empirical methods. Dynamic earth pressures are calculated assuming seismic coefficients acting in the horizontal and vertical directions calculated with either the Mononobe-Okabe or the Wood method depending on the anticipated movement that the structure will undergo when subjected to earthquake loading.

This paper illustrates the results of a research investigation aimed to assess the appropriateness of using the Mononobe-Okabe method for determining the dynamically-induced lateral earth pressures on the stem portion of concrete, flexible cantilever retaining walls sustaining a granular backfill. A series of non-linear dynamic finite element analyses have been performed using the computer program DIANA (DIsplacement ANAlyzer). The analyses included a static-phase of stress initialization caused by placement of soil and incremental construction of the wall followed by dynamic analyses.

Soil response was simulated using an elasto-plastic, Mohr-Coulomb constitutive model. The influence of variables such as wall stiffness and strength parameters of the backfill were investigated through a parametric study. Special attention was given to the selection of seismic input in order to represent realistic ground motion scenarios corresponding to different levels of severity. Dynamic earth pressures obtained from numerical simulations were compared with those determined using pseudo-static approaches through a series of benchmark tests. Co-seismic and post-seismic displacements of the wall were also calculated using simplified pseudo-dynamic methods. Reliability of the results obtained with DIANA was assessed through a comparison with available results of analyses performed using the finite difference-based program FLAC (Fast Lagrangian Analysis of Continua).

Keywords: Retaining walls, Mononobe-Okabe, Seismic earth pressure, Elasto-plastic, Mohr-Coulomb.

INTRODUCTION

Regardless the multitude of studies that have been carried out over the years, the dynamic response of earth-retaining walls is far from being well understood. There is, in current engineering practice, a lack of conclusive information that can be used in design method. The most commonly used methods to design earth-retaining structures under seismic conditions are *force-based* equilibrium approaches like the pseudo-static analysis (e.g. Mononobe-Okabe 1926, 1929) and pseudo-dynamic techniques (Steedman and Zeng 1990), and *displacement-based* procedures such as the sliding block method (e.g. Richards and Elms 1979). In the limit-state methods of analyses in which the wall is considered to displace or deform sufficiently at the base to fully mobilize the shearing strength of the backfill.

Even under static conditions, prediction of actual retaining wall pressures and deformations constitute complicated soil-structure interaction problem. The dynamic response of even the simplest type of

[1] European School for Advanced Studies in Reduction of Seismic Risk (ROSE School), Pavia, Italy. Email: prajeev@roseschool.it
[2] Department of Structural Engineering, Università degli Studi di Roma “La Sapienza”, Italy.
[3] European centre for Training and Research in earthquake Engineering (EUCENTRE), Pavia, Italy.

retaining wall is therefore a quite complex phenomenon. It depends on the mass and stiffness of the wall, the backfill and the underlying ground, as well as the interaction among these components and the nature of the seismic input motions.

The purpose of this study was to develop a finite element numerical model to throw light into understand the dynamic behavior of cantilever earth-retaining structures, in particular to find the magnitude and distribution of dynamic lateral earth pressures, as well as the displacements induced by horizontal ground shaking. In all the analyses, the soil was assumed to behave as a homogeneous, elasto-plastic medium with a Mohr-Coulomb failure criterion. The wall was assumed to behave as linear elastic material. The numerical model for the wall and surrounding soil has been developed using DIANA, a commercially available finite element program.

The results obtained with DIANA were compared with results obtained from pseudo-static analysis using the procedure by Mononobe-Okabe and, to some extent, the results obtained with FLAC by Green and Ebeling (2003) for the same case-study. The two models, namely, the DIANA model and the FLAC model by Green and Ebeling, have the same meshing and material properties (mass density, friction angle) with the only exception of shear wave velocity profile. This latter is taken constant and equal to the weighted average along the height of the values in Green and Ebeling. As shown later all qualitative trends observed by Green and Ebeling are also found with the DIANA analyses.

EARTH-RETAINING STRUCTURE-SOIL SYSTEM

Figure 1 shows the soil-wall system that has been studies in this paper. The height of the flexible wall is 6.1m. The backfill and foundation soil is assumed to be medium-dense, cohesion-less, compacted fill. Its most important geotechnical properties are as follows: mass density: γ_s = 19.6 kN/m^3; effective angle of internal friction: φ' = 40˚. The water table is assumed located well below the foundation of the wall and thus the analyses are performed assuming dry soil.

The properties of the concrete and of the reinforcing steel used for designing the wall are as follows: concrete unit weight : γ_c = 23.6 kN/m^3; concrete compressive strength: f'_c = 27.6 MPa; steel yield strength: f'_y = 413.4 MPa.

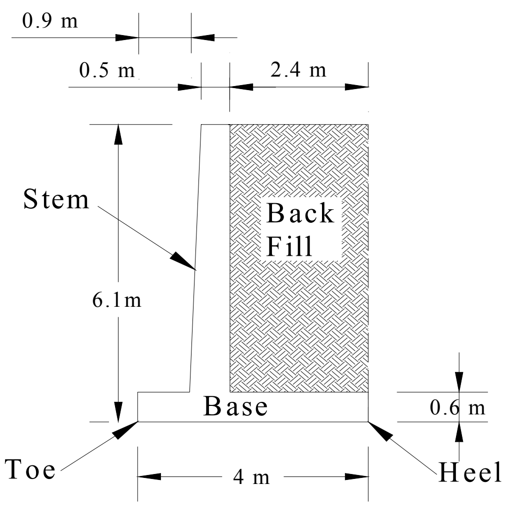

Figure 1. Dimensions of the wall-soil system studied in this work

NUMERICAL MODEL

Overview of FEM model developed using DIANA

The finite element model set up in DIANA consists of the upper 9.1 m of the wall-soil system and it contains the wall, the backfill and 3m of the underlying natural soil below the foundation of the wall.

The model extends laterally for approximately 26.0 m to include 9.35 m of existing soil in front of the wall and about 16.65 m of the backfill/existing soil behind the wall (see Figure.2).

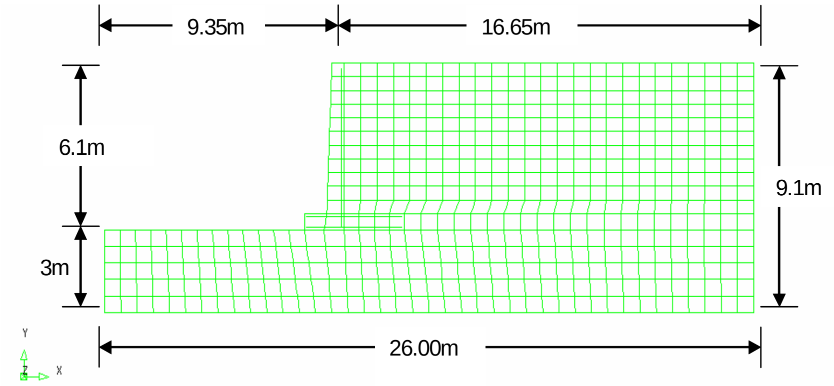

Figure 2. Mesh of FEM model developed using DIANA

The soil and the wall are modeled using eight-nodes quadrilateral isoparametric plane-strain elements. The size of the elements varies from 0.50 m to 0.60 m. A total of 688 elements are used in the model. The employed elements (see Figure 3) are based on quadratic interpolation of displacements and Gauss integration. Each node has two translation degrees of freedom along the X and Y directions. The polynomial for the displacements u_ξ and u_η can be expressed as:

$$u_i = a_0 + a_1\xi + a_2\eta + a_3\xi\eta + a_4\xi^2 + a_5\eta^2 + a_6\xi^2\eta + a_7\xi\eta^2 \quad (1)$$

This polynomial yields a strain $\varepsilon_{\xi\xi}$ which varies linearly in ξ direction and quadratically in η direction. The strain $\varepsilon_{\eta\eta}$ varies linearly in η direction and quadratically in ξ direction. The shear strain $\gamma_{\xi\eta}$ varies quadratically in both directions (see DIANA User's manual 9.0).

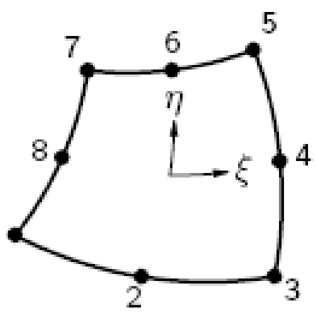

Figure 3. CQ16E 8 node 2-D plane strain element used in this study

An elasto-plastic constitutive model, with Mohr-Coulomb failure criterion, is used to model soil response under both static and dynamic loading conditions. Plane-strain elements are also used to model the concrete retaining wall with linear elastic material.

To simulate realistic earth pressures developed at the back of the wall as it deforms during construction, the wall-backfill system is "numerically constructed" in DIANA similarly to the way an actual earth-retaining structure would be constructed in reality. The backfill is placed in 0.50 m lifts, for a total of 12 lifts with the model being brought to static equilibrium after each increment.

To properly simulate the Sommerfeld radiation condition of an unbounded medium, transmitting boundaries are attached along the base of the model. For the vertical boundaries, horizontal active-type soil pressures needs to be applied at the corresponding nodes. This is done manually by first keeping track of the reaction forces acting in the model with fixed boundaries and then applying them to the model with horizontal rollers used for dynamic analyses. The corresponding horizontal displacements after application of self weight are very small in the order of 10^{-4} m.

Selection and specification of seismic input

Three acceleration time-histories were used for the seismic analyses of the earth-retaining structure shown in Figure 1 and they include the 1940 Imperial Valley earthquake (California), the 1999 Chi-Chi earthquake (Taiwan) , and the 1995 Hyogoken-Nambu (Japan), corresponding to low, medium and high Peak Ground Acceleration (PGA), respectively. Table 1 shows the values of PGA and duration characterizing the selected accelerograms.

Table 1. Ground motion selected in this study for the seismic analyses of the soil-wall system

Earthquake	Station	PGA (g)	Significant duration (s)
Imperial Valley (1940)	117 El Centro Array	0.20	20.34
Chi-Chi (1999)	CHY 006N Chi-Chi	0.35	26.03
Hyogoken-Nambu (1995)	KJMA Kobe	0.80	8.36

The small-strain fundamental frequency of the retaining wall-soil system in the DIANA model is estimated to be approximately 9 Hz. This is expected to decrease at larger strains. Since frequencies above 15Hz are not significant for typical soil-structure systems. Later value is taken as cutoff frequency. Low-pass filtering was carried out using the computer program SEISMOSIGNAL. The incident motion at the base of the model was obtained by simply dividing by a factor of two the outcrop ground motion. The motion computed at the free-surface with DIANA compared satisfactorily with the outcropping motion.

Geotechnical parameters assumed for foundation soil and backfill

As mentioned above, an elasto-plastic constitutive model with Mohr-Coulomb failure criterion is used to model the mechanical response of both the soil and the backfill. The parameters required by this model are: the effective internal friction angle (φ'), the effective cohesion (c'), the angle of dilation (ψ) and the mass density (ρ).

As stated previously, φ' for the existing soil and the backfill is assumed to be equal to 40˚. This value refer to a medium-dense, cohesion-less, competed fill. Parameters c', ψ, and ρ are taken equal to 0, 0, and 2000 kg/m^3 respectively.

Additional parameters are required for the plane strain finite element: the small-strain Young's modulus (E) and the Poisson's ratio (v). By using theory of elasticity:

$$E = 2(1+\nu)G \tag{2}$$

Several correlations exist that relate G that is the shear modulus of the soil to other soil parameters. However, the most direct relation is between G and shear wave velocity (V_s):

$$G = \rho V_s^2 \tag{3}$$

By assuming V_s is equal to 180 m/s, the small-strain shear modulus was calculated as 64.8 MPa.

The Poisson ratio v may be estimated from the at-rest coefficient of earth pressure K_o using the standard relation by Jaky (1944) $K_0 = 1 - \sin\phi'$ and the following well-know expression:

$$\nu = \frac{K_0}{1+K_0} \tag{4}$$

Using Equation (4) above , v was determined to be equal for to 0.26 both for existing soil and backfill. Table 2 illustrates the values of the geotechnical parameters used in this study for the foundation soil and backfill.

Table 2. Geotechnical parameters used in the FEM model for foundation soil and backfill

Parameters	Value
Poisson's ratio (-)	0.26
At-rest pressure coefficient (-)	0.36
Small-strain Young's modulus (MPa)	163.13
Effective friction angle (deg)	40º
Density (kg/m^3)	2000
Cohesion (MPa)	0.00
Dilation angle (deg)	0º

Material parameters assumed for concrete-wall

The concrete wall is modelled as a linearly elastic material. The required parameters are the mass density (ρ), the Young's modulus (E_c) and the Poisson's ratio (v). Parameter E_c is estimated using the following relation:

$$E_c = 5000\sqrt{f_c^{'}} \tag{5}$$

where f'_c is compressive strength of concrete. Values of the other parameters are reported in Table 3.

Table. 3 Material parameters used in the FEM model for concrete wall

Parameters	Value
Elastic modulus of concrete (MPa)	30,000
Yield strength of steel (MPa)	413.400
Young's modulus of steel (GPa)	200
Unit weight of concrete (kN/m^3)	23.6

Wall-soil interface elements

No special interface elements are used between the soil and the wall. The horizontal displacements of the nodes of wall and the soil elements are tied together (i.e. glued interface) whereas the vertical displacements are assumed to be independent of each other.

Dimensions of finite element mesh

Proper dimensioning of the finite element mesh is required to avoid numerical distortion of propagating ground motions in addition to accurate computation of model response. Kuhlemeyer and Lysmer (1973) recommended that the length of the element Δl be smaller than one-tenth to one-fifth of the wavelength λ_{min} associated with the highest frequency f_{max} component of the input motion, i.e.:

$$\Delta l \leq \frac{\lambda_{\min}}{10} \tag{6}$$

The wavelength λ is related to the shear wave velocity of soil ($V_s=\sqrt{G/\rho}$) and the frequency (f) of the propagating wave by the relation:

$$\lambda = \frac{V_s}{f} \tag{7}$$

Substituting equation (7) into equation (6) gives:

$$\Delta l \leq \frac{V_s}{10 f_{\max}} \quad or \quad f_{\max} \leq \frac{V_s}{10 \Delta l} \tag{8}$$

Since the shear wave velocity of the soils is taken to be equal to 180 m/s, and the seismic input of Table 1 are low-pass filtered with cut-off frequency at 15 Hz, the minimum wave length λ is 12m. Hence the maximum allowable element size is 1.2m. Equation (8) shows that by using $\Delta l=0.50m$, the finite element mesh used in DIANA should adequately propagate shear waves having frequencies up to approximately 36 Hz. This value is well above the 15 Hz cutoff frequency used in the dynamic analyses and well above the estimated fundamental frequency of the retaining wall-soil system being modeled (i.e. ≈ 9 Hz).

Damping

Inherent to the elasto-plastic Mohr-Coulomb constitutive model is the characteristic that once the induced dynamic shear stresses exceed the shear strength of the soil, the plastic deformation developing in the soil introduces a considerable amount of hysteretic damping. However, for dynamic stresses lower than the shear strength, the soil behaves elastically (i.e. without damping). DIANA allows Rayleigh damping to be specified with an approximately constant value of damping ratio over a restricted range of frequencies. A lower bound damping ratio of 1% is set at the first natural frequency of the system and at the predominant frequency of the excitation.

DISCUSSION

Dynamic analyses were performed using the acceleration time-histories described in Table 1. The results obtained from DIANA were compared with those determined using pseudo-static method (i.e. following the approach by Mononobe-Okabe).

Dynamic earth pressures

Following Green et al.(2003) the dynamically-induced lateral earth pressures acting on the stem of the wall and the section along the heel (see Figure 1) were computed by assuming constant stresses within the element. The corresponding lateral earth pressure coefficient ($K_{j,DIANA}$) could then be back-calculated at time increment j from DIANA results using the following expression:

$$K_{DIANA} = \frac{2 \cdot P_{DIANA}}{\gamma_t \cdot H^2 \cdot (1 - k_v)} \tag{9}$$

where P_{DIANA} is the resultant of stresses computed by DIANA and acting on the stem or the section along the heel of the wall, γ_t is the total unit weight of the backfill, H is the height of the wall, and k_v is the vertical inertial coefficient (assumed in this study equal to zero). Equation (9) is used to compute K_{DIANA} values at times corresponding to the peaks in the time-history of the horizontal inertial coefficient (k_h) acting towards and away from the backfill. A plot of the computed K_{DIANA} values versus k_h is shown in Figure 4. Also shown in this figure are the lateral dynamic earth pressure coefficients (active: K_{AE}; Passive: K_{PE}) computed using the Mononobe-Okabe expressions for the wall-soil system (Okabe 1924; Mononobe & Matsuo 1929) and Wood (1973) solution for rigid wall.

Furthermore from Figure 4, one can make the following additional consideration:
1. $K_{heel} > K_{stem}$, when k_h is directed toward the backfill.
2. $K_{stem} > K_{heel}$, when k_h is directed away from the backfill.
3. The largest K_{stem} occurs, when k_h is directed away from the backfill.
4. The largest K_{heel} occurs, when k_h is directed toward the backfill.
5. The computed K values show a general scatter around the curve obtained using the Mononobe-Okabe dynamic active earth pressure curve.

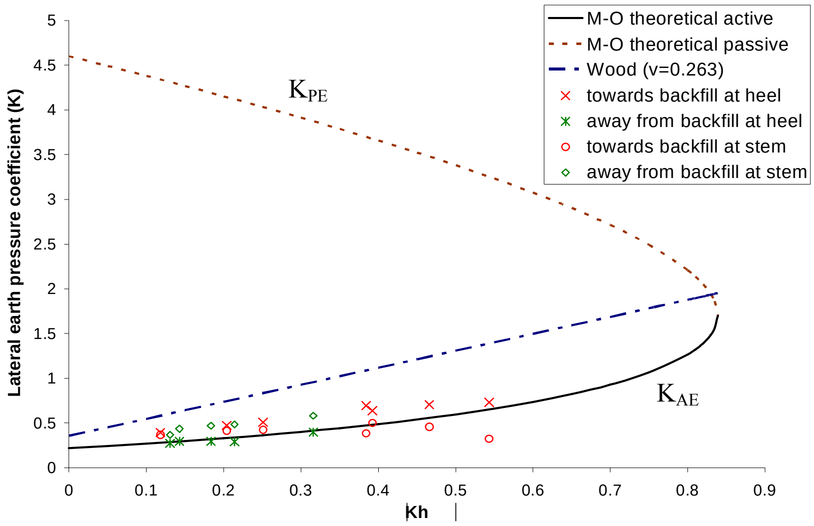

Figure 4. Comparison of DIANA and Mononobe-Okabe dynamic lateral earth pressure coefficients

The stress distribution along the stem of the wall before the dynamic analysis was very close to the theoretical active earth pressure distribution calculated using Rankine theory (see Figure 5). Figure 5 shows that at the end of the dynamic analysis and for all three seismic inputs of Table 1 the active pressure distribution is close to the values associated with the at-rest soil pressures.

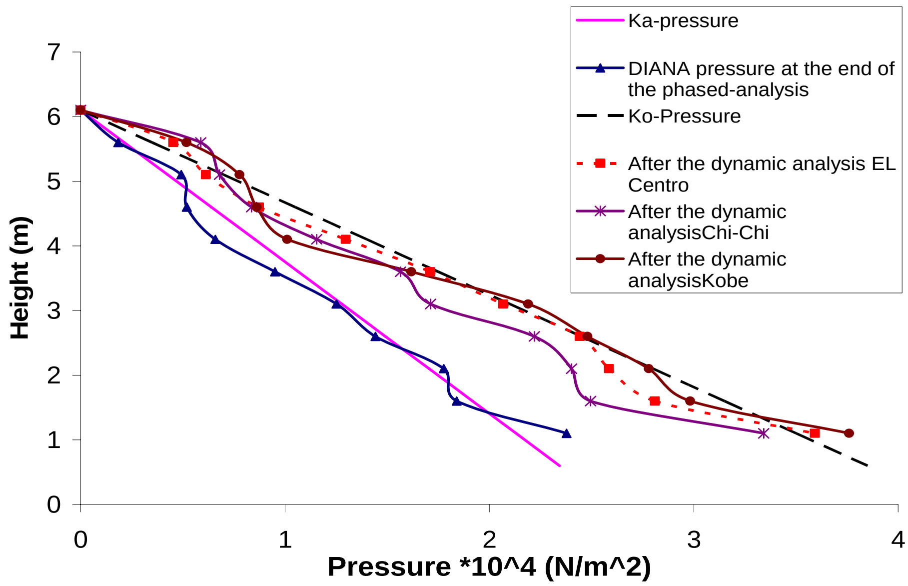

Figure 5. Comparison of pressure distributions along the stem of the wall before and after the dynamic analysis

In general, the values of K_{DIANA} values are somewhat higher than the Mononobe-Okabe active lateral earth pressure coefficient. This phenomenon is discussed in detail by Green et al.(2003) where a similar soil-wall system is analyzed with the computer program FLAC. It is due to the failure wedge in the backfill that is composed of several failure wedges rather than a single wedge as inherently assumed in the Mononobe-Okabe theory. Figure 6 offers a schematic illustration of this.

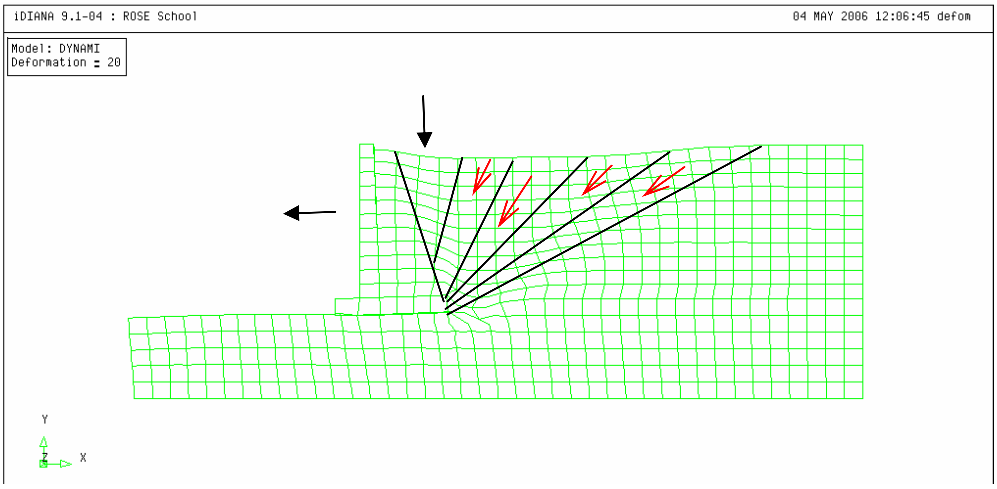

Figure 6. Deformed mesh obtained from the dynamic analysis of the wall-soil system using the 1995 Hyogoken-Nambu acceleration

Wall horizontal displacement

The permanent horizontal displacement of wall-soil system was directly calculated by DIANA. The horizontal displacement time-history at the base of the wall for the 1940 Imperial Valley accelerogram is shown in Figure 7. The permanent horizontal displacement at the base of the wall found at the end of the dynamic analysis for all three records as shown in Table 4.

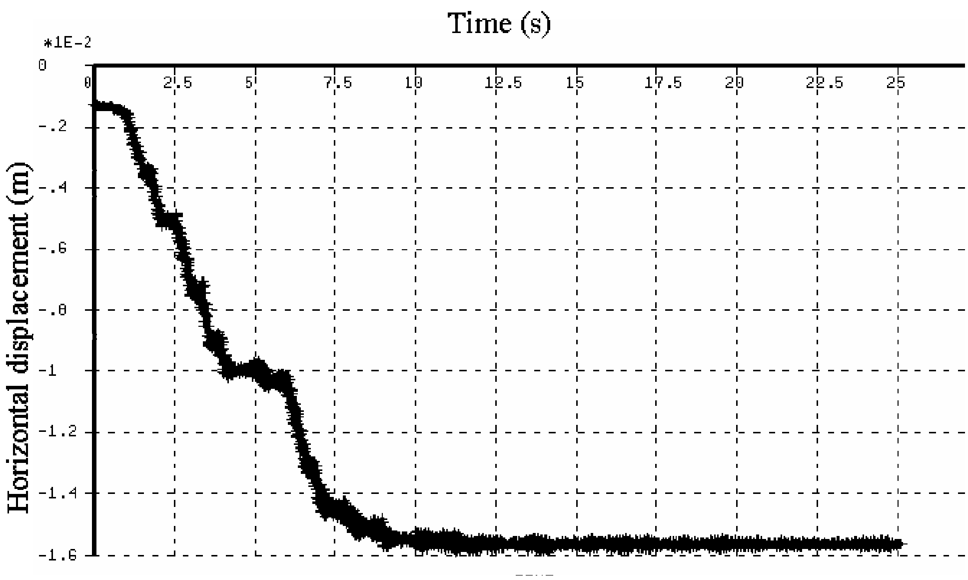

Figure 7. Relative displacement time-history at the base of the wall using the 1940 Imperial Valley (California) accelerogram

Table 4. Relative permanent displacement of the base of the cantilever wall

Earthquake	Relative permanent displacement of the base of the wall (cm)
Imperial Valley (1940)	1.59
Chi-Chi (1999)	0.70
Hyogoken-Nambu (1995)	4.12

Bending moments along the wall

The bending moments along the wall were computed using the pressures due to the dynamic forces on the wall. Figure 8 shows the maximum bending moments' envelopes along the wall for all three records, together with the moment distribution at the end of the excavation phase.

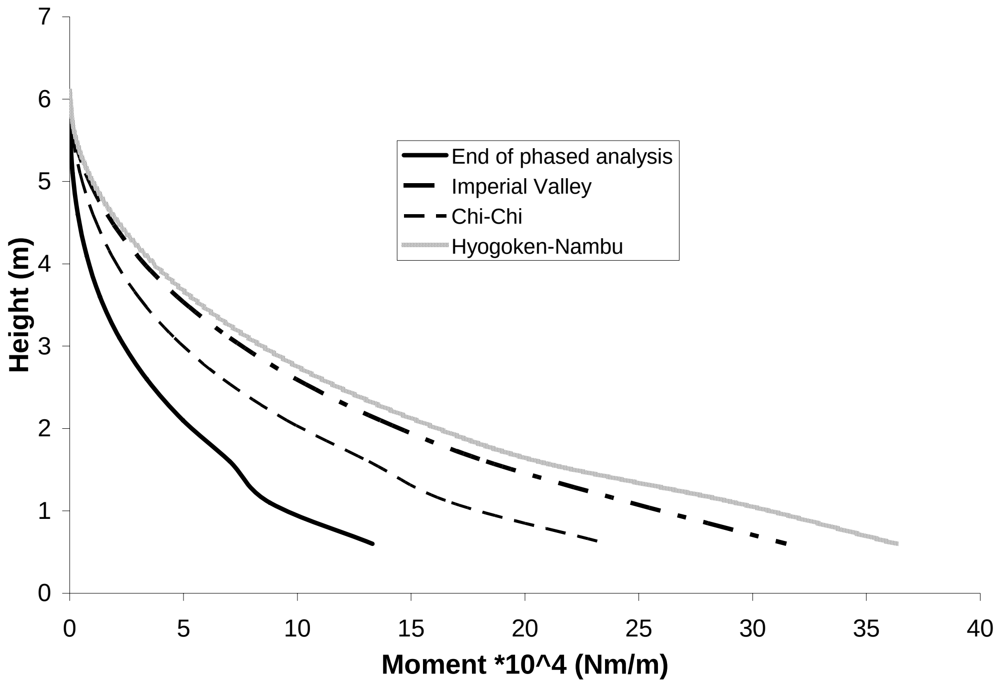

Figure 8. Maximum bending moments' envelopes

SUMMARY AND CONCLUSIONS

This paper illustrates a preliminary investigation onto the seismic behavior of flexible cantilever RC retaining wall, with horizontal dry granular backfill. A finite element model of the system is built using DIANA finite element program. For the foundation soil and backfill elasto-plastic constitutive model with Mohr-Coulomb failure criterion was used.

The results from this case study show that the pressures induced on the wall stem are larger than to those predicted by the Mononobe-Okabe method. The reason for this deviation may be attributed to a) the relative flexibility of the structural wedge and b) to the non-monolithicity of motion within the driving soil wedge. Both situations are neglected in the Mononobe-Okabe method. The dynamic response of the wall-backfill system is such that there is an incremental increase from active to at-rest earth pressure conditions in the residual stresses imposed on the stem of the retaining wall.

The conclusions drawn from this study may not apply to retaining wall system of differing geometry and/or material properties. Further research is required in order to draw more general conclusions regarding the appropriateness of the Mononobe- Okabe method to evaluate the dynamic pressure induced under seismic conditions on the cantilever walls.

REFERENCES

DIANA finite element analysis User's manual Release 9 TNO DIANA BV.

Gazetas, G., Psarropoulos, P.N., Anastasopoulos, I., and Gerolymos, N. "Seismic behaviour of flexible retaining systems subjected to short-duration moderately strong excitation" J.Soil dynamic and earthquake engineering vol.24, 537-550, 2004.

Green, R.A., and Ebeling, R.M. “Seismic analysis of cantilever retaining walls, Phase I, ERDC/ITL TR-02-3, Information technology laboratory, US army corps of engineers, Engineer research and development center, Vicksburg, MS, 2002.
Green, R.A., and Ebeling, R.M. “Modeling the dynamic response of cantilever earth-retaining walls using FLAC” Proceedings of the 3rd International Symposium on FLAC and FLAC3D: Numerical modeling in geomechanics, Sudbury, ON, Canada, 2003.
Green, R.A., Olgun, C.G., Ebeling, R.M., and Cameron, W.I.“Seismically induced lateral earth pressures on a cantilever retaining wall” P.The sixth US conference and workshop on lifeline earthquake engineering (TCLEE2003), ASCE, 2003.
Ishihara.K. “Soil behaviour in earthquake geotechnics” Oxford science publications, 1995.
Kapila, I. P.“Earthquake resistance design of retaining walls’’ 2nd earthquake symposium, Univ. of Roorkee, India, 1962.
Kramer, S.L. Geotechnical earthquake engineering. Prentice-Hall, New Jersey, pp 466-505, 1996.
Lysmer, J., and Kuhlemeyer, R. L. “Finite dynamic model for infinite media” J.Engineering mechanics division, 95 (EM4), 1969.
Madhav, M.R., and Kameswara Rao, N.S.V. “Earth pressure under seismic conditions” J.Soils and foundations vol.9 No.4, 33-47, 1969.
Nadim, F., and Whitman, R, V. “Seismically induced movement of retaining walls” J.Geotech Engrg, ASCE, 109(7), 915-913, 1983.
Peck, B., Hanson, W., and Thornburn, T. “Foundation Engineering” Willey, New York, 1974.
Prakash, S. “ Analysis of rigid retaining walls during earthquakes” Proc., Int. Conf. on Recent Adv. in Geotech. Earthquake Engrg., and Soil Dyn., Univ. of Missouri, Rolla, Mo., vol III, 1-28, 1981.
Prakash, S., and Basavanna, B. M. “Earth pressure distribution behind retaining wall during earthquake” Proc., 4th world Conf. on Earthquake Engrg, Satiago, Chile, 1969.
Psarropoulos, P.N., Klonaris.G., and Gazetas.G. “Seismic earth pressures on rigid and flexible retaining walls” J.Soil dynamics and earthquake engineering vol.25, 795-809, 2005.
Seed, H.B., and Whitman, R.V. “Design of earth retaining structures for dynamic loads”, ASCE Spec.Conf. Lateral Stresses in the ground and design of retaining structures,Cornell,pp 103-147, 1970.
SEISMOSIGNAL http://www.seismosoft.com/
Steedman, R.S., and Zeng, X. “The influence of phase on the calculation of pseudo-static earth pressure on a retaining wall” Geotechnique, 40 (1), 103-112, 1990.
Steedman, R.S. “Seismic design of retaining walls” Proc. Instn. Civ. Engrs. Geotech. Engng, 131 12-22, 1998.
Steedman, R.S. “Seismic soil-structure interaction of rigid and flexible retaining walls” Earthquake geotechnical engineering , 1999 Balkema, Rotterdam, ISBN 90 5809 1163, 1999.
Veletsos, A. S., and Younan, A.H. “Dynamic modeling and response of soil-wall systems” J.Geotech. Engrg.,ASCE, 120 (12) 2155-2179, 1994.
Veletsos, A. S., and Younan, A.H. “Dynamic soil pressures on vertical walls” Proc., 3rd Int. Conf. on Recent Adv. in Geotech. Earthquake Engrg., and Soil Dyn., Univ. of Missouri, Rolla, Mo.,1589-1604, 1995.
Veletsos, A. S., and Younan, A.H. “Dynamic response of cantilever retaining walls” J.Geotech Engrg., ASCE, 123 (2) 1090-0241, 1996.
Wood, J. H. “Earthquake-induced soil pressures on structures” Rep. EERL 73-05, Earthquake Engineering Research Laboratory, California Inst. of Technol., Pasadena, Calif, 1973.