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MODELING OF CYCLIC MOBILITY – AN ENERGY APPROACH

Stanislav LENART[1]

ABSTRACT

Different researches show that dissipated energy in soil subjected to dynamic loading is related to the residual pore pressure change. Besides residual also the temporary pore pressure change occurs during undrained dynamic loading of saturated soils. It is caused by the transmission of compressive stresses onto the pore water. In this paper, temporary and residual pore pressure changes were evaluated with respect to the energy dissipated during the cyclic loading of the soil. A numerical model is presented which includes both parts of the pore pressure changes. It is possible to determine fairly exactly the pore pressure changes during cyclic loading using the concept of dissipated energy based on only two parameters and a known function of the loading. The advantages of the proposed pore pressure model have been observed mainly in the case of soils which are cyclically loaded in compression, where the pore pressure oscillates noticeably during the loading. The effect of pore pressure oscillations upon strain progression during the vertical cyclic loading of a saturated soil was found to be large, because extreme pore pressure values occur at the same time as the extreme load. Based on the presented pore pressure model 1D stress-strain relation in normal mode for cyclically loaded saturated soils was established. Special attention was focussed on short-term flow during cyclic mobility. The effect of dissipated energy upon the flow duration and soil stiffness within range of short-tem flow was determined.

Keywords: dissipated energy, cyclic mobility, liquefaction, short-term flow, pore pressure model

INTRODUCTION

Cyclic mobility is exhibited by saturated medium to dense cohesionless soils subjected to dynamic loading. It happens due to soil skeleton contraction at small shear strain and dilation at large shear strain excursions. The mechanism results in phases of soil softening known as short-term flow following by significant regain in soil stiffness and strength, which limits the magnitude of cyclic deformation. Shear strains of different range are imposed upon the soil skeleton during dynamic loading. The soil particles movement intends to cause changes in skeleton pore volume. As the soil is saturated, and water in pores has very high compressibility and is due to the dynamic loading not able to drain, the pore water pressure increases or decreases sequently during dynamic loading. Changes in pore water pressure result in changes of effective stress, which result in soil stiffness changes.

A relationship between pore pressure development and the dissipated energy during the dynamic loading has been found recently. The dissipated energy is represented by the area of the hysteric stress-strain loop and could be determined experimentally. The energy concept for the analysis of densification and liquefaction of cohesionless soils was introduced by Nemat-Nasser and Shokooh (1979). It is based on the idea that during deformation of these soils under dynamic loads part of the energy is dissipated into the soil. This dissipated energy is represented by the area of the hysteric strain-stress loop and could be determined experimentally. The accumulated dissipated energy per unit volume considers both the amplitude of shear strain and the number of cycles, combining both the

[1] Slovenian National Building and Civil Engineering Institute (ZAG), Ljubljana, Slovenia, e-mail: stanislav.lenart@zag.si

effects of stress and strain (Dief and Figueroa, 2001). The effect of random motions of an earthquake or other random dynamic load can be taken into account by using the Palmgren-Miner cumulative damage hypothesis, originaly developed for metal fatigue evaluations. Its basic premise is the assumption that the energy applied during any stress cycle has a cumulative damage effect on the material. For sinusoidal cyclic loading, the cyclic energy is directly related to the cyclic stress, which is more convenient to calculate in certain cases, but not in the case of random loading.

Experimental studies have been conducted by Law et al. (1990), Figueroa (1990), Liang et. al (1995), Hsu (1995), Davis and Berrill (2001) and Green (2001) to establish a relationship between pore-water pressure increments and dissipated energy. It has been common to all of them that only residual pore pressure changes were taken into account. This kind of situation is typical for cases when only cyclic shear stress is imposed on the soil element. In general, specialy in case of vertically cyclic loaded soil (foundation vibration, traffic load etc.), soil element is loaded also in compression. Lenart (2006a) showed that also temporary pore pressure changes happen during dynamic loading of saturated soil. Theirs source is mainly dynamic compression loading. The effect of temporary pore pressure change is large, because the extreme temporary pore pressure change occurs at the same time as the extreme load. Therefore it is necessary to take into account this kind of pore pressure changes in the modeling of dynamic loaded saturated soils.

Based on a realistic pore pressure model, which describes well all kind of pore pressure changes happening in the dynamic loaded soil element, it is possible to establish a complete stress-strain relation for cyclically loaded saturated soils. Many plasticity models were developed already trying to capture the characteristics of cyclic mobility (Finn et. al., 1977; Prevost, 1985; Pastor and Zienkiewicz, 1986; Cubrinovski and Ishihara, 1998; Li and Dafalis, 2000). Currently (Elgamal et. al. 2003), reliable computational modeling of cyclic-mobility deformation initiation still remains a major challenge. As the shear stress approaches the failure envelope (phase transformation envelope) significant strain may develop without appreciable change in stress. To solve the numerical versatility this highly yielded segment was defined (Elgamal et. al. 2003) as a distinct phase. Shear deformation is observed to accumulate in cyclic mobility process (Yang et. al. 2003). Much of the shear strain accumulation occurs rapidly during the transition from contraction to dilation near the phase transformation surface. Lenart and Logar (2006) observed that the highly yielded segment is effected by the dissipated energy. They have found for a type of a soil beeing tested that energy dissipated during short-term flow phase is related to residual pore pressure ratio.

EXISTING ENERGY-BASED PORE PRESSURE MODELS

The application of cyclic loading to cohesionless soil results in a permanent or temporary decrease in volume, even in the case of dense sand, which may dilate alternately. For a saturated soil, if drainage is unable to occur during the time span of the loading sequence, the tendency for volume reduction results in the increase of pore water pressure. During soil particles movements, which lead to the soil skeleton contraction, a part of an energy, which is introduced to the soil through cyclic loading, is dissipated by the viscous and frictional mechanism. Whitman and Dobry (1993) stated that at large strain amplitudes, the frictional mechanism is expected to dominate. Therefore, at large strain range, which is also the case of cyclic mobility, the total amount of dissipated energy is approximated by the energy dissipated through the frictional mechanism. Laboratory studies have shown that for a given amplitude load, the quantity of energy dissipated by the frictional mechanism is independent of the frequency of the applied loading (Hardin, 1965). For this reason the use of dissipated energy is independent of frequency and loading rate, and that is one of its main advantages. The expression for accumulated dissipated energy per unit volume normalized by initial mean effective pressure, *w* is given by (Desai, 2000; Green, 2001)

$$w(t) = \frac{1}{p'_0} \int_0^t \sigma^T d\varepsilon^p \ , \qquad (1)$$

where t = time, σ = stress vector, ε^p = irreversible or plastic strain vector and p'_0 =initial mean effective pressure. The dissipated energy defined by equation (1) represents the summation of areas under the hysteresis loops of the stress-strain curve for cyclic loading divided by initial mean effective pressure. In this paper the dissipated energy defined by equation (1) is named as dissipated energy ratio (because of its dimensionless).

The first suggestion of a possible relationship between dissipated energy and pore pressure increase in soils was by Nemmat-Nasser and Shokooh (1979). They established governing differential equations relating energy dissipation to the densification of dry soil and to the generation of excess pore pressures in saturated soil. Besides to the dissipated energy, they related the pore pressure increase also to the initial void ratio of soil, e_0. The same characteristic appears in a much simpler form of equation derived from the liquefaction evaluation procedure (Mostaghel and Habibaghi, 1979):

$$r_u = \frac{1}{e_0} w, \qquad (2)$$

where $r_u = \Delta u / p'_0$ and Δu = the pore pressure in excess of hydrostatic conditions when the applied cyclic stress is zero.

The idea of a relationship between dissipated energy and pore pressure increase was further developed in the context of liquefaction risk analysis (Davis and Berrill, 1982; Berrill and Davis, 1985; Yamazaki et al., 1985; Law et al., 1990; Figueroa et al., 1994; Liang et al., 1995; Hsu, 1995; Green et al., 2000). More or less all the proposed models are empirical curve fit equations of laboratory data with different numbers of parameters. A typical general form derived from the proposed equations could be written as

$$r_u = a \cdot w^b, \qquad (3)$$

where a and b are functions of soil type, relative density of soil, stress conditions, initial soil state parameters, etc.

THE EFFECT OF PORE PRESSURE OSCILATIONS

The described existing energy-based pore pressure models relate the dissipated energy and permanent pore pressure change which happen due to the soil particles movements. The permanent pore pressure change is the pore pressure in excess of hydrostatic conditions when the applied cyclic stress is zero. Besides this kind of pore pressure changes also the temporary changes occur. They are caused by transmission of compressive stresses onto the pore water. The existing pore pressure models for cyclic loaded saturated soils neglect the temporary oscillations of pore pressure. The models of this type, being incorporated into nonlinear cyclic models, mainly concern the soil deformed in simple shear conditions (Osinov, 2003). Simple shear conditions are quite commonly used to describe deformation shape of soil deposit under seismic load. This type of models is usually simpler than tensorial ones.

Purely one dimensional stress conditions are very rare in reality. Besides shear stresses, normally normal stresses appear, too. One of their major effects is observed in oscillations of pore pressure in case of saturated soil. While pore water has no shear stiffness and can not be affected by shear stresses, the situation in case of normal stresses is completely different. Normal stresses transmit from soil skeleton onto the pore water. Changes of normal stresses from compression to extension and back is observed as oscillations of pore water pressure. The effect of pore pressure oscillations upon strain progression during the vertical cyclic loading of a saturated soil is demonstrated on Figure 1. The use of pore pressure models with and without the oscillation effect taken into account, were compared in numerical modeling. The pore pressure oscillations effect was found to be large, because extreme pore pressure values occur at the same time as the extreme load. This means that the material stiffness is the lowest at the moment when the load is the greatest.

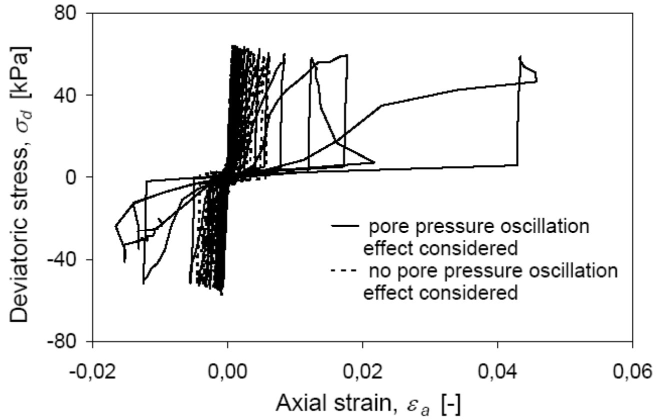

Figure 1. Progression of strain during cyclic loading with pore pressure oscillation effect taken into account and without it

NEW PORE PRESSURE MODEL

Figure 2 schematically presents the computation of dissipated energy per unit volume of material tested in cyclic triaxial test. The dissipated energy is defined as the area bounded by deviator stress – axial strain hysteresis loops and can be evaluated by equation (4). It is a kind of damage parameter, which takes into account both stress and strain histories of material, as well as material properties. Dissipated energy is a very suitable parameter to describe the pore pressure changes of material due to any kind of loading history.

$$W = \int \sigma_d \, d\varepsilon_a \tag{4}$$

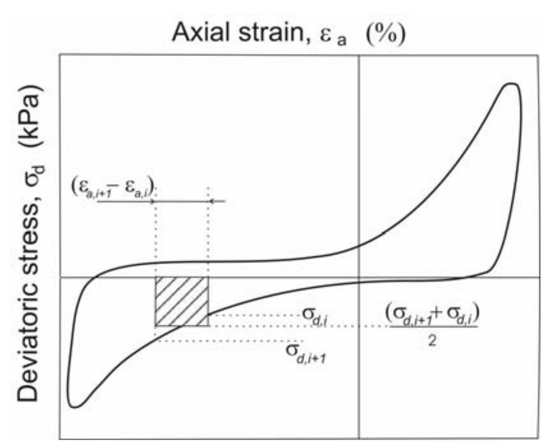

Figure 2. The dissipated energy per unit volume for a soil sample in cyclic triaxial loading

The basic idea of the used pore pressure model is in dividing of pore water pressure into two parts: the temporary pore pressure change and the residual pore pressure change. The idea was presented by Lenart (2006a) and tested on two different kinds of soils. The amplitude of temporary pore pressure change Δu_0 in every load cycle and residual pore pressure change u_0 were divided from total pore pressure changes Δu measured during triaxial tests. The amplitude of temporary pore pressure change and residual pore pressure change are discrete single values unique to the load cycle. Residual pore pressure u_0 is a pore pressure value in the mean point of the hysteresis loop; that is a pore pressure at the point where the applied cyclic stress is zero. This value was defined already by other researchers (Davis and Berrill, 1982; Figueroa et al., 1994). The amplitude of temporary pore pressure change Δu_0 is an extreme pore pressure change in a loading cycle. It was observed that this value is almost the same for loading in tension or loading in compression during the same cycle of uniform cyclic loading if the amount of dissipated energy during this half cycle is small when compared to sum of dissipated energy during the cyclic loading (Lenart, 2006a). The procedure of defining the values u_0 and Δu_0 during cyclic loading is described by author in other place (Lenart, 2007).

Pore pressure change is presented by a pore pressure ratio, r_u which consist (5) from its residual $r_{u,r}$ and temporary $r_{u,t}$ part. Equation (6) defines each of them. Dissipated energy per unit volume divided by initial mean effective pressure results in dimensionless dissipated energy ratio, w (7), which was defined for general case already in equation (1).

$$r_u = \frac{\Delta u}{p'_0} = r_{u,r} + r_{u,t} \tag{5}$$

$$r_{u,r} = \frac{u_0}{p'_0} \quad \text{and} \quad r_{u,t} = F(t) \cdot \frac{\Delta u_0}{p'_0} \tag{6}$$

$$w = \frac{W}{p'_0} \tag{7}$$

$F(t)$ in equation (6) is a time function of compressive stresses acting upon the soil element. It was observed that the amplitude of temporary pore pressure change and the residual pore pressure change are related to the dissipated energy per unit volume. Using a proper function of energy and normalized by initial mean effective pore pressure and time function respectively, relations can be transformed into a linear one (Figure 3). Their slopes give parameters k_r and k_t, where r and t mean residual and temporary changes, respectively. Some typical results of both parameters and observed relations between parameters, material void ratio e and cyclic stress ratio CSR for tested materials are presented in other places (Lenart, 2007).

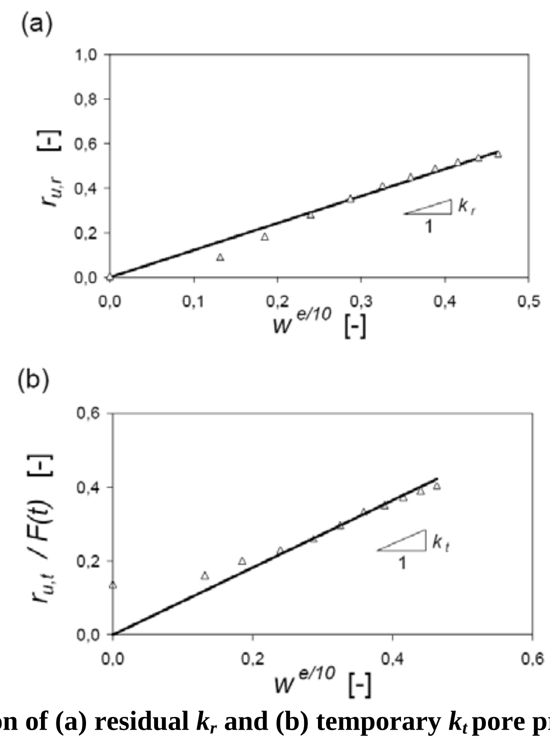

Figure 3. Evaluation of (a) residual k_r and (b) temporary k_t pore pressure parameters; Lacustrine carbonate silt, p'_0=200 kPa, e=0.40, CSR=0.195

Knowing the time function $F(t)$ of dynamic loading to which soil is subjected, one can write the equation for evaluating the pore pressure ratio, r_u (8). The equation includes the temporary as well as residual pore pressure changes. As it is based on dissipated energy ratio, it is independent of loading frequency or rate impacts. The equation was used in constitutive relation defined in the sequel.

$$r_u = (w)^{e/10} \cdot [k_r + k_t F(t)] \tag{8}$$

SOFTENING AND HARDENING DURING CYCLIC MOBILITY

There is another phenomenon of soil behavior, besides pore pressure changes, which makes a perceivable impact on strain progression in soil during cyclic loading. A short-term flow developed during cyclic mobility of soil can be seen from the stress-strain relationship observed during cyclic triaxial tests. After few cycles of loading soil starts to exhibit very low stiffness at the beginning of a load cycle and it strengthen latter. Figure 4 shows this kind of behavior. It is obviously that strain developed during this short-term flow presents the main share of a total strain developed during cyclic loading and it leads therefore the deformation behavior of a soil. The duration of this phenomenon and stiffness of soil during this phase is in the question.

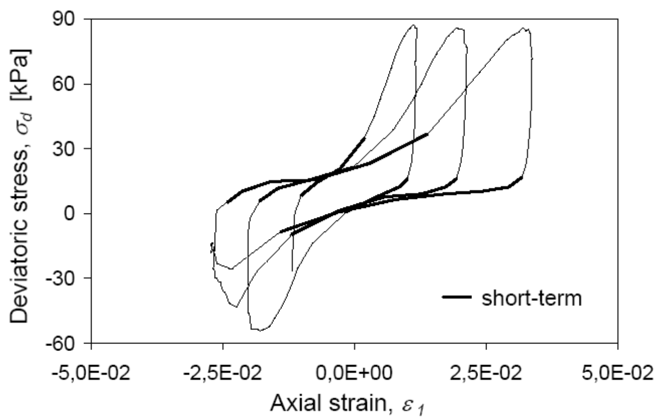

Figure 4. Typical stress-strain behavior of cyclic loaded soil with short-term flow phase

It has been found (Lenart, 2006a) that energy dissipated in short-term flow phase, duration of this phase, as well as shear modulus of soil during the same phase are related to residual pore pressure ratio. Typical relations are shown on figure 5 and 6. The beginning of short-term flow during cyclic loading and the length of this phase can be easily obtained for typical test results from figure 5. During cyclic loading the pore pressure in saturated soil increase gradually if drainage is prevented. The short-term flow occurs after few cycles, usually, when some residual pore pressure exists already. The value of this threshold residual pore pressure can be identified from figure 5 easily, if we presume that no energy was dissipated during short-term flow before that moment. The length of a short-term flow phase can be defined in every moment from the linear relation between the energy dissipated during flowing phase and the residual pore pressure (figure 5).

Stiffness of soil during flowing phase is decreasing as the residual pore pressure increases. Figure 6 presents the typical values of shear modulus of soil during short-term flow phase normalized by initial shear modulus and the impact of residual pore pressure increase upon it. It is obvious that the value of shear modulus during the flow is very small as compared to the modulus without the flowing phase. As seen on the figure 6, typical values of the shear modulus during flowing phase are lower than 1% of the initial shear modulus. As the residual pore pressure increases during cyclic loading, the shear modulus of soil in short-term flow phase decreases.

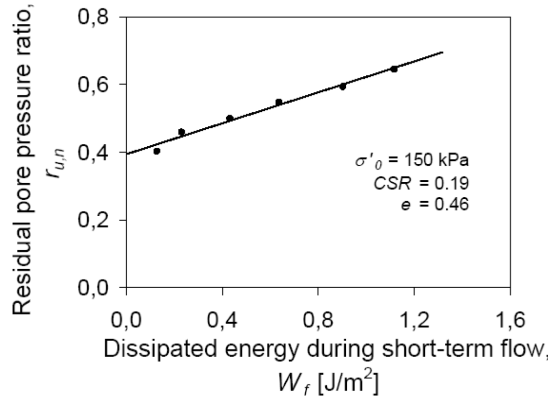

Figure 5. Dissipated energy and residual pore pressure ratio during short-term flow phase are linearly related

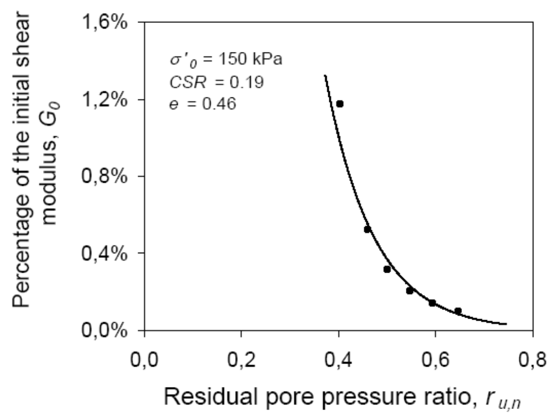

Figure 6. Typical relation between the shear modulus and residual pore pressure ratio during short-term flow phase in compression

MODELING OF CYCLIC TRIAXIAL TESTS

After the short-term flow phase is completed, soil starts to strengthen again. The shear modulus increases again up to the value from before of the flowing phase. The relation between dissipated energy and soil stiffness has been defined for a case of reconstituted samples of lacustrine carbonate silt (Lenart, 2006b). The shape of a curve obtained in this way (figure 7) is very similar to the well known shape of the relation between strain and stiffness. The advantage of the energy-stiffness curve is that it is less sensitive to the strain changes and it includes both stress and strain histories of a material relating them to the stiffness properties.

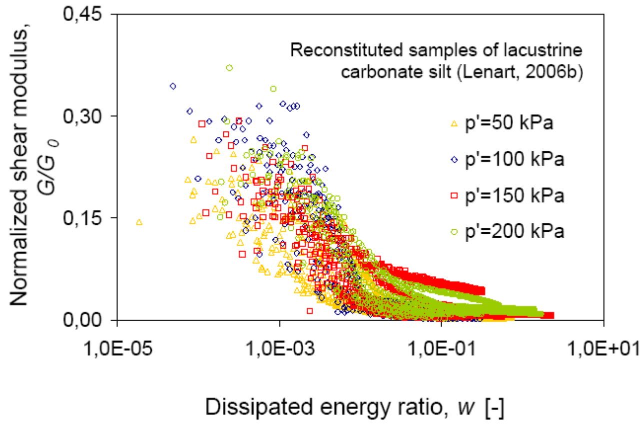

Figure 7. The relation between normalized shear modulus and the ratio of dissipated energy

A simple numerical model of a vertically cyclically loaded soil body was used to model a series of cyclic triaxial tests (Lenart and Logar, 2006). The dynamic response of a soil sample loaded vertically by cyclic load was proposed to be one-dimensional. A concentrated mass connected by a nonlinear spring and a viscous damper to a base was vertically loaded by a known loading function of time, *F(t)*. The differential equation of motion is well known differential equation (9), where *m* means the mass of the oscillated body, *k* is the stiffness of a spring and *c* presents the damping of a viscous damper.

$$m\ddot{y} + c\dot{y} + ky = F(t) \tag{9}$$

The soil stiffness defined above (figure 7) was used in the model for a loading history before short-term flow appearance. During the short-term flow phase the stiffness of a soil was defined by relation presented in figure 6. Soil starts to strengthen when short-term flow is finished. It has been assumed in the model that the stiffness of soil during strengthening and unloading is equal to the average stiffness from the last loading cycle. The soil stiffness during the cyclic loading defined above reflects hysteretic behavior of soil. The viscous damping of a system used in numerical model was calculated as 25% of critical damping C_{crit} (10). G_0 means initial shear modulus of soil and ρ is a density of soil. Viscous damping was used only during the unloading phase, while viscous damping during loading was neglected. The whole process during cyclic loading, soil transformation from normal state into the flowing phase and back, is governed by the pore pressure changes, specially the residual pore pressure changes, as defined in the first part of this paper.

$$C_{crit} = 2\sqrt{G_0 \cdot \rho} \tag{10}$$

A simple numerical model described above was verified on results obtained by cyclic triaxial tests. Above defined pore pressure model and relations obtained by studying of a short-term flow during cyclic mobility were used in a model. Their contributions seem the most important to the obtained results. Comparison of calculated and measured values of pore pressure, stress states, displacements, strains and stresses are shown for one typical test in figure 8.

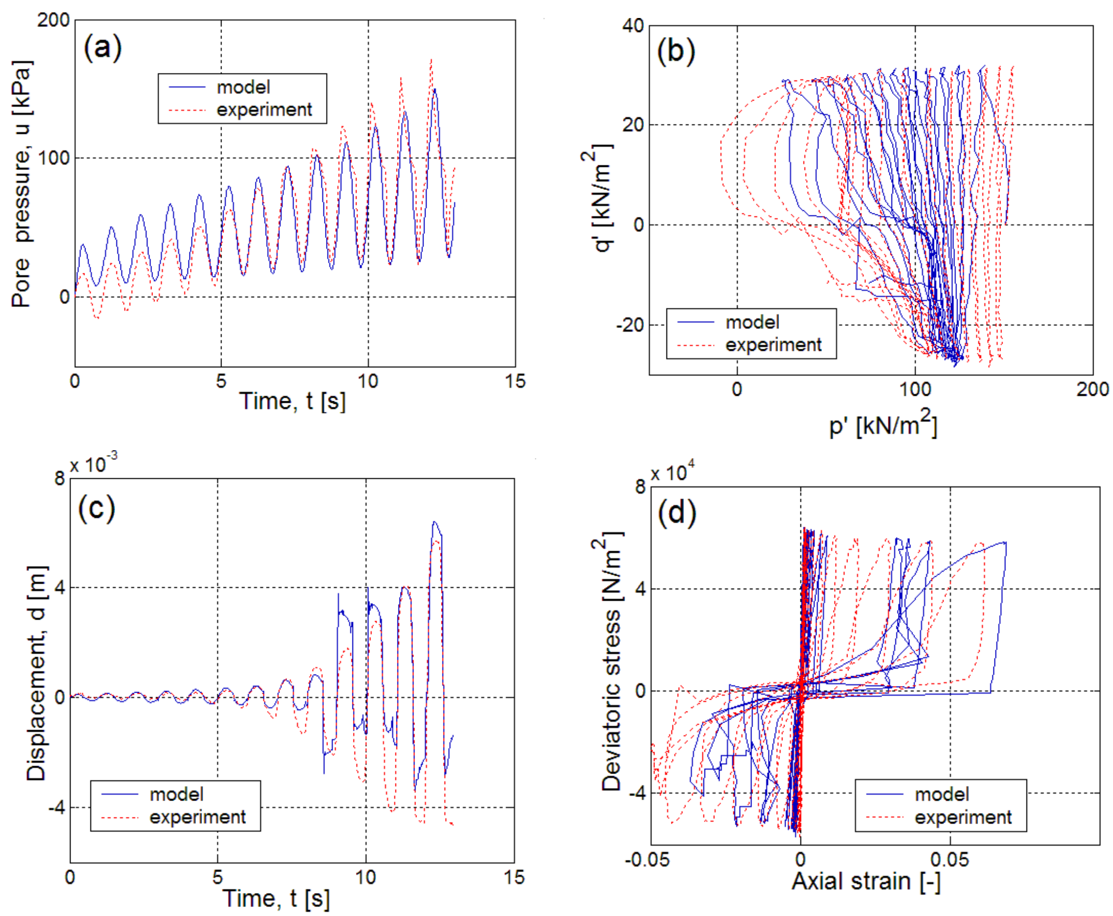

Figure 8. Typical simulation of a pore pressure (a), stress states (b), displacements (c) and stress-strain relation (d) for a cyclic triaxial test of a reconstituted sample of lacustrine carbonate silt (Lenart and Logar, 2006)

CONCLUSIONS

A simple constitutive model proper for a cyclic mobility is presented in the paper based on an energy approach. Its main part present a new pore pressure generation model and findings related to short-term flow observed during cyclic mobility. The energy approach was chosen because of its independence of the rate and frequency of the loading in the large strain range, which is characteristic for cyclic mobility phenomena.

The pore pressure change is divided into temporary and residual pore pressure change. Both two parts are described as a function of a dissipated energy. The model takes into account the effect of pore pressure oscillations caused by compression cyclic loadings of soil. The pore pressure oscillations effect upon the strain progression was found to be large.

The phase of very low stiffness at the beginning of a load cycle was named as a short-term flow. It has been proved that the dissipated energy as well as the stiffness of soil during this phase are related to the residual pore pressure.

The presented model has two main constants: residual k_r and temporary k_t pore pressure parameters, which can be determined by undrained cyclic triaxial test. Besides these two constants also an initial stiffness of tested soil is needed. Softening and hardening process during cyclic mobility can be properly modeled by using the relation between dissipated energy and residual pore pressure, and between soil stiffness and residual pore pressure during short-term flow phase. Some typical relations are presented in the paper. More researches in this direction are needed.

The model is set to be 1D in a normal stress-strain mode. Cyclic mobility and problems related to dynamically loaded saturated soils are usually based on earthquake loading and may escalate to the liquefaction of soils. Therefore the proposed model is transformed into the shear stress-strain form to compare it with other similar models and theirs parameters. Nevertheless, some more work should be done to enable the model to be readily used for the seismic response of liquefiable level ground.

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