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SIMPLIFIED APPROXIMATE APPROACH TO GROUP EFFECT IN PILE DYNAMICS

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ABSTRACT

One of the main objectives of the machine foundation design is to limit the amplitude of excitations to an acceptable tolerance set by the manufacture. In order to achieve this, a detailed dynamic analysis is required. Calculating stiffness and damping constants for pile groups is one of the most challenging steps in this dynamic analysis. Calculating these constants using the available tools requires a sophisticated procedures and calculations. This gives rise to the importance of developing a simple and easy method of analysis that can simplify the calculations, at least for the preliminary stage of the design, without compromising the accuracy of the results. In this paper, a simplified approximate approach that can assist in calculating the stiffness and damping constants for pile groups in the vertical and horizontal directions was presented and verified against the exact solution.

Keywords: dynamic analysis, stiffness and damping constants, interaction factors, pile groups

INTRODUCTION

The main objective in the design of machine foundations is to limit the amplitude of excitations to an acceptable tolerance set by the manufacturer. Failure to achieve this objective will result in an unsatisfactory performance of the supported machinery and will cause disturbance to people/structures in the close vicinity of the machine. Successful design of the foundation requires accurate evaluation of its dynamic characteristics, i.e., stiffness and damping. For foundations supported on piles, the evaluation of stiffness and damping involves the calculation of the stiffness and damping of single piles and the consideration of pile-soil-pile interaction (i.e. group effect). This analysis is therefore complicated and requires sophisticated procedures and calculations. Thus, there is a need to develop a simple method of analysis that can simplify the calculations without compromising the accuracy of the results.

The objective of this paper is to present a simplified approach that can assist in calculating approximate values of the stiffness and damping constants for pile groups in the vertical and horizontal directions using a handheld calculator.

IMPEDANCE FUNCTIONS OF A SINGLE PILE

Data on the single pile impedance are available in the literature for different soil profiles and different pile materials. Novak (1974), Novak and El Sharnouby (1983) and Sheta and Novak (1982) presented solutions for impedance functions of single pile. In general the impedance function can by expressed as:

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$K = K_1 + i\,K_2$ (1)

The impedance function has a real part, $K_1 = Re\ (K)$ and an imaginary part, $K_2 = Im(K)$. The real part represents the true stiffness and defines directly the stiffness constant k, while the imaginary part of the impedance function, K_2, describes the out-of-phase component and represents the damping due to energy dissipation in the soil medium. The constant of equivalent viscous damping can be expressed as:

$c = K_2 / \omega$ (2)

where ω is the excitation frequency. Using the Novak (1974) solution, the stiffness and damping constants for the vertical translation of a single pile can be evaluated as:

$$k_v = \frac{E_p\,A}{R} f_{v1}\ ,\ c_v = \frac{E_p\,A}{V_s} f_{v2} \qquad (3)$$

And for the horizontal translation,

$$k_u = \frac{E_p\,I}{R^3} f_{u1}\ ,\ c_u = \frac{E_p\,I}{R^2\,V_s} f_{u2} \qquad (4)$$

Where E_p is the elastic modulus of the pile, A and I are pile cross-sectional area and moment of inertia, respectively, R is its radius, V_s is soil shear wave velocity, f_{i1} and f_{i2} are the dimensionless stiffness and damping functions, respectively. Figures 1a and 1b show graphs of f_{i1} and f_{i2} for the vertical and horizontal directions, respectively, for different E_p/G_s ratios for fixed head piles in homogenous halfspace.

PILE GROUPS

Piles in a group are usually connected to each other through a rigid pile cap to support a superstructure. The spacing between piles has a great influence on the behaviour of the group. For larger spacing, the piles do not affect each other and the group stiffness and damping are calculated as the direct summation of the contributions from individual piles. If, however, the piles are closely spaced, they interact with each other and this pile-soil-pile interaction causes significant influence on the stiffness and damping of the group as the displacement of one pile contributes to the displacements of others.

Poulos and Davis (1980) initiated analytically based studies on the static response of pile groups. These studies showed that the group effect reduced the stiffness of the system and increased the settlement. Nogami (1980) and sheta and Novak (1982) presented analytical solutions for the dynamic response of pile groups. These studies revealed that the dynamic stiffness and damping of piles groups are frequency dependent and that the group stiffness and damping can be either reduced or increased due to pile-soil-pile interaction.

Dynamic Interaction Factors

The dynamic interaction factor is a dimensionless, frequency dependent complex value, and can be defined as follows: for any two piles, if a unit harmonic load is applied to pile 1 and the resulting displacement is calculated at the head of pile 2, then the interaction factor, α can be expressed as:

$$\alpha_m = \frac{\text{dynamic displacement of pile 2}}{\text{dynamic displacement of pile 1}} = \frac{f_{m21}}{f_m^s} \qquad (5)$$

Where the subscript m refers to the translation direction, f_{m21} is the complex dynamic deflection of pile 2 due to harmonic loading of pile 1 and f_m^s is the dynamic flexibility of a single pile. In general, the complex interaction factor can be given in the form:

$\alpha = \alpha_1 + i\,\alpha_2$ (6)

Dobry and Gazetas (1988), and Gazetas and Makris (1991) presented a set of formulas for dynamic interaction factors between two piles in a homogeneous halfspace. These interaction factors are given by:

$$\alpha_v \approx \frac{1}{\sqrt{2}}\left(\frac{S}{d}\right)^{-0.5} e^{-\beta a_o \frac{S}{d}} e^{-i a_o \frac{S}{d}} \text{ and } \alpha_u(\theta^o) \approx \alpha_u(0^o)\cos^2\theta + \alpha_u(90^o)\sin^2\theta \qquad (7)$$

where

$$\alpha_u(0^o) \approx \frac{1}{2}\left(\frac{S}{d}\right)^{-0.5} e^{-\beta\omega \frac{S}{V_{La}}} e^{-i\omega \frac{S}{V_{La}}} \quad \text{and} \quad \alpha_u(90^o) \approx 0.75\alpha_v \qquad (8)$$

where α_v and α_u are vertical and horizontal interaction factors, respectively, S/d = pile spacing to diameter ratio, a_o is the dimensionless frequency, $a_o = \omega d/V_s$, θ is the angle between the direction of load action and the plane in which piles lie, and $V_{La} =$ the so-called Lysmer's analog velocity $= \dfrac{3.4 V_s}{\pi(1-\nu)}$.

Evaluation of Group Impedances

The single pile impedances can be used in conjunction with the interaction factors to estimate the impedance functions of pile groups. In this section, the overall group stiffness and damping constants will be evaluated using the impedance function of a single pile in conjunction with the complex interaction factors.

Consider a group of n piles subjected to a harmonic load in the vertical direction. The group flexibility matrix, F_v, that relates the displacement vector at the pile heads, v, to the applied load vector, P_v, may be expressed as:

$$\{v\} = [F_V]\{P_V\} = f_v^s\,[\alpha_v]\,P_v = \frac{1}{K_V^S}\,[\alpha_v]\,P_v \qquad (9)$$

Where f_v^s, is the vertical flexibility of single pile, K_v^s is the vertical stiffness of the single pile, and α_v is the nxn vertical interaction matrix shown below.

$$\alpha_V = \begin{bmatrix} 1 & \cdots & \alpha_{v1j} & \cdots & \alpha_{v1n} \\ \vdots & \ddots & & \iddots & \vdots \\ \alpha_{vi1} & \cdots & 1 & \cdots & \alpha_{vin} \\ \vdots & \iddots & & \ddots & \vdots \\ \alpha n1 & \cdots & \alpha_{vnj} & \cdots & 1 \end{bmatrix} \qquad (10)$$

Where α_{vij}, is the vertical interaction factor that relates the displacement of pile i to the applied load on pile j. Defining the stiffness of the pile group as the load that produces a unit displacement at the pile head, connected to a rigid pile cap, Equation (9) is solved for pile loads, and the vertical group stiffness, K_v^G, is given by:

$$K_V^G = K_V^S \sum_{i=1}^{n}\sum_{j=1}^{n} \varepsilon_{ij}^V \qquad (11)$$

Where ε_{ij}^V are the elements of the inverse of the interaction matrix (α_v^{-1}).

DEVELOPMENT OF GROUP INTERACTION FACTORS

The program Dyna5 (Novak et al., 1999) was used to calculate the vertical and horizontal impedance for both single piles and pile groups with different configurations. The group impedance considering pile-soil-pile interaction obtained from Dyna5 analyses is denoted by K_i^G, where i = v or h for the vertical and horizontal directions, respectively. The impedance neglecting the pile-soil-pile interaction is given by:

$$K^G_{i\,\mathrm{without}} = nK_i^s \tag{12}$$

Whereas, the group interaction factor, α, is given by:

$$\alpha_i = \frac{nK_i^s}{K_i^G} \tag{13}$$

Pile groups of 2x2, 3x3 and 4x4 were considered for L/d ratio of 20 and a hysteretic material damping, $\beta_s = 0.05$, for pile rigidity ratios between 300 and 1000, and spacing to diameter ratios, s/d = 2, 5 and 10. For other values of S/d or E_p/G_s interpolation can be used. Figure 2 shows the group interaction factors for 2x2 piles group configuration, while, Figure 3 shows the group interaction factors for the 3x3 piles group configuration. The group interaction factors for the 4x4 piles group configuration is shown in Figure 4.

THE PROPOSED SIMPLIFIED APPROACH

The effect of the pile-soil-pile interaction decreases as the spacing between the piles in the pile group increases (El-Marsafawi et al., 1992). Accordingly, all piles that are spaced at a distance s/d ≥ 20 will have insignificant influence on the reference pile (see Fig. 5). For example, a pile group of 8x16 with s/d=5 shown in Figure 6, assuming that the reference pile is in the second row, the piles after the 5th row of piles have an s/d ratio to the reference pile of at least 20. Therefore, if a mapped pile group of 4x4 is placed at the location of the reference pile of the 8x16 group the same interaction relation is expected as far piles will not affect the interaction relation. Thus, to evaluate the stiffness and damping for a given pile group, the following procedure can be followed:

1. Obtain single pile stiffness and damping by using Equations 3 and 4 along with Figures 1a and 1b.
2. Obtain the equivalent group interaction factors from the nearest square group to the width of the considered piles group. For example, for a 3x9 pile group use the interaction factors of the 3x3 group given by Figure 3, whereas, for the 8x16 group shown in Figure 5 use the interaction factors of the 4x4 group given by Figure 4.
3. Calculate the overall group stiffness and damping using Equation (13).

Verification of the proposed approach

In order to verify the applicability of the proposed approach, a parameter study was conducted to determine the limitations of the approach. The normalized stiffness and damping group parameters F_{v1}^G, F_{v2}^G and F_{u1}^G, F_{u2}^G for the vertical and horizontal directions, respectively, were determined with respect to the dimensionless frequency a_o using the proposed simplified method and the dynamic analysis software Dyna5 (Novak et al., 1999) to investigate the effect of the size of the pile group on the predictability of the simplified (approximate) approach. For the 3-pile wide pile groups, 3x6, 3x9 and 3x12 pile groups configurations were studied. While for the 4-pile wide pile groups, 4x8, 4x12 and 4x16 pile groups were considered.

Figure 6a shows the normalized group vertical stiffness F_{v1}^G versus the dimensionless frequency a_o for the 3-pile wide groups with s/d=5, the solid lines represent Dyna5 solutions while the doted lines represent the proposed approximate solutions. It could be seen from Figure 6a that the approximate solution gave nearly identical results to that of the exact solution for the 3x6 pile group. As the size of the group increases, the agreement between the approximate solution and the exact solution slightly decreases. Figure 6b shows the normalized group vertical damping F_{v2}^G versus the dimensionless frequency a_o for the 3 piles wide groups with s/d=5. The same trend holds for the damping. Figures 6c and 6d show F_{u1}^G and F_{u2}^G, respectively. These graphs show that the accuracy of the approach in the horizontal direction is less, for both stiffness and damping. The maximum error in the vertical group stiffness for all of the 3 piles wide groups with s/d=5 cases was in the range of 20% over the range of a_o=0.45 ~ 0.55 and a_o= 1.75 ~ 1.85. For the vertical group damping, the error varied from 14% to 23% over the range of a_o=0.27 ~ 0.46. Outside this frequency range, the difference was under 5%. In the horizontal direction, the difference between the simplified approach and the exact solution was less than 5% for both the group stiffness and damping, except over a_o=0.72 ~ 0.88 where the error jumped to almost 30% in the stiffness, and 40% in the damping in the range of a_o=1.00 ~ 1.20.

Figures 7a to 7d show the normalized group stiffness and damping for the 3 piles wide groups with s/d=10. As it is noted from these figures, the accuracy was enhanced significantly as the spacing increased. In the vertical direction, the difference between the simplified approach and the exact solution was in the range of 2% to 5%, a maximum error of 10% over the range of a_o=0.92 ~ 1.05 for the group stiffness. In the horizontal direction, the maximum error in the group stiffness was 30% within the range a_o=0.55 ~ 0.70, and 5% outside this frequency range. For the horizontal damping, the difference was less than 5%.

For 4-pile wide groups with s/d=5, the agreement between the simplified approach and the exact solution is much better as shown in Figures 8a through 8d. The maximum difference in vertical group stiffness between the simplified and exact solutions was 20% over the narrow range of a_o=0.28 ~ 0.37, but was less than 5%, outside this range. The vertical group damping is predicted with good accuracy except within the ranges of a_o=0.72 ~ 0.88 and a_o=1.20 ~ 1.30, where the error is up to 30%. The accuracy for the 4-pile wide groups with s/d=10 was even better as seen from Figures 9a to 9d.

SUMMARY AND CONCLUSIONS

The stiffness and damping constants of pile groups were theoretically investigated utilizing the dynamic interaction factors method. A simplified approach for calculating the stiffness and damping constants was proposed and verified against the exact solution. For vertically loaded pile groups, the stiffness predictions using the proposed approach compared well with the exact solution, while the damping agreed to a lesser degree, but within the acceptable limits. For horizontally loaded pile groups, the accuracy of the simplified approach is good for S/d ≥4.

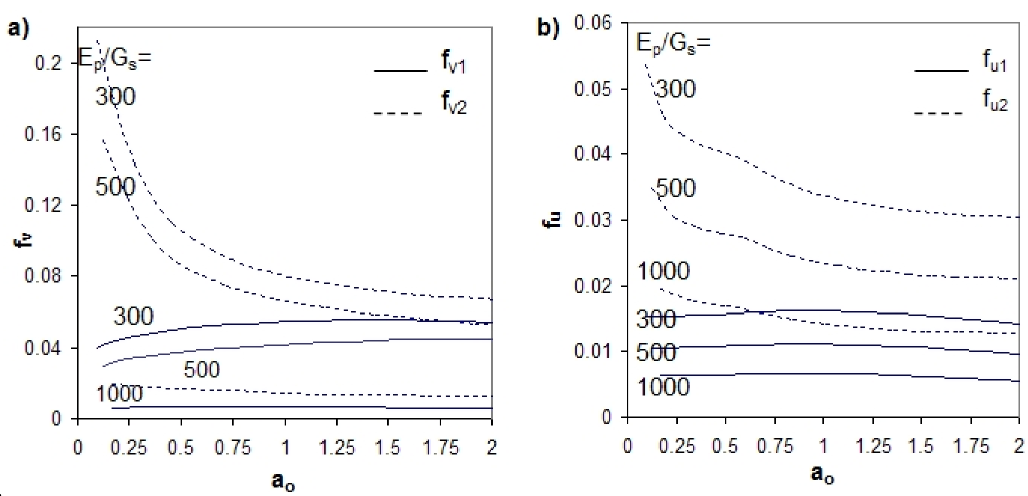

Figure 1: Stiffness & damping parameters for fixed head piles in homogenous halfspace.
a) Vertical direction, b) Horizontal direction

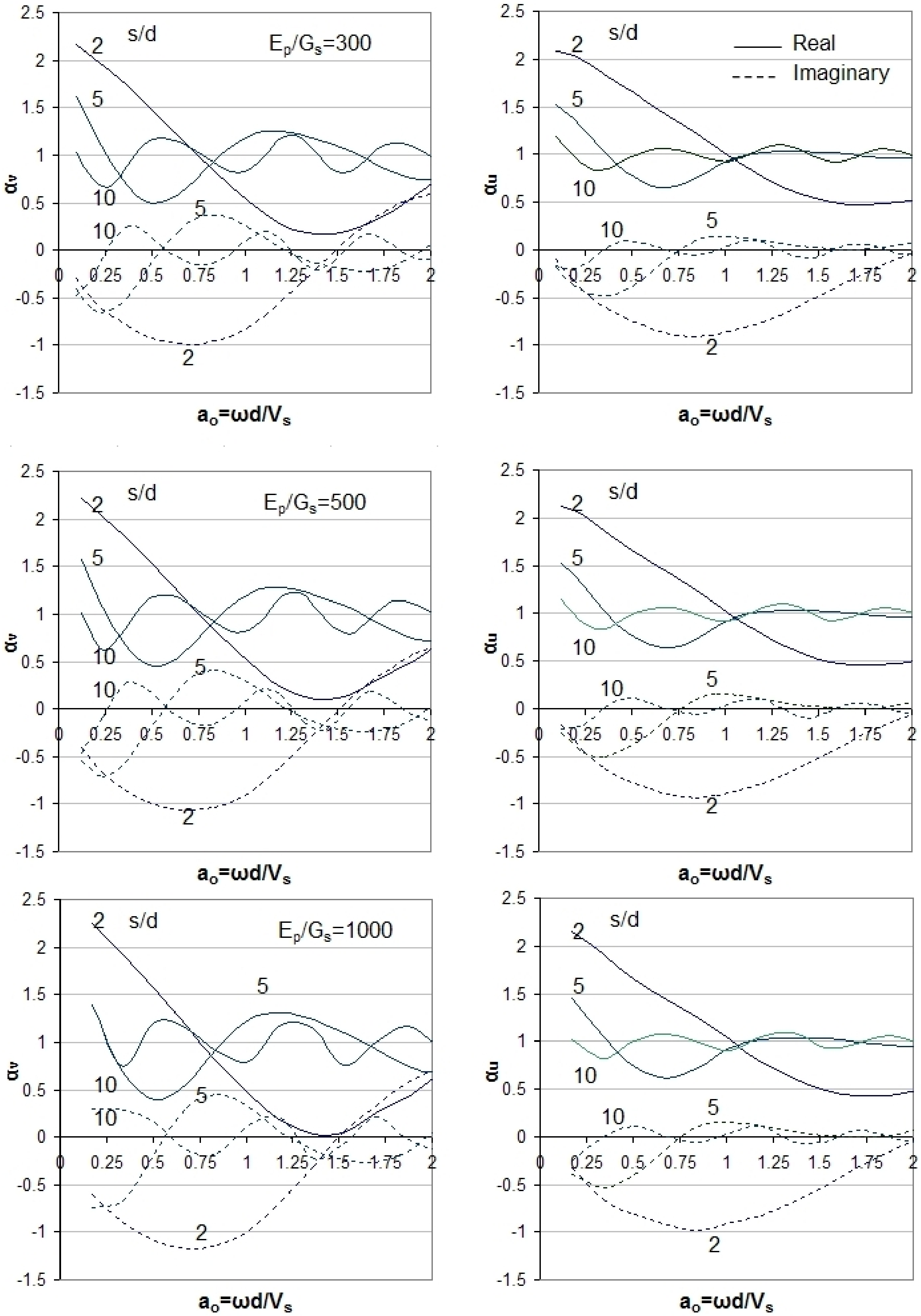

Figure 2: Vertical and horizontal interaction factors for 2x2 pile group in homogenous halfspace with length to diameter ratio L/d = 20, damping ratio, β_s = 0.05, and Poisson's ratio, ν = 0.4.

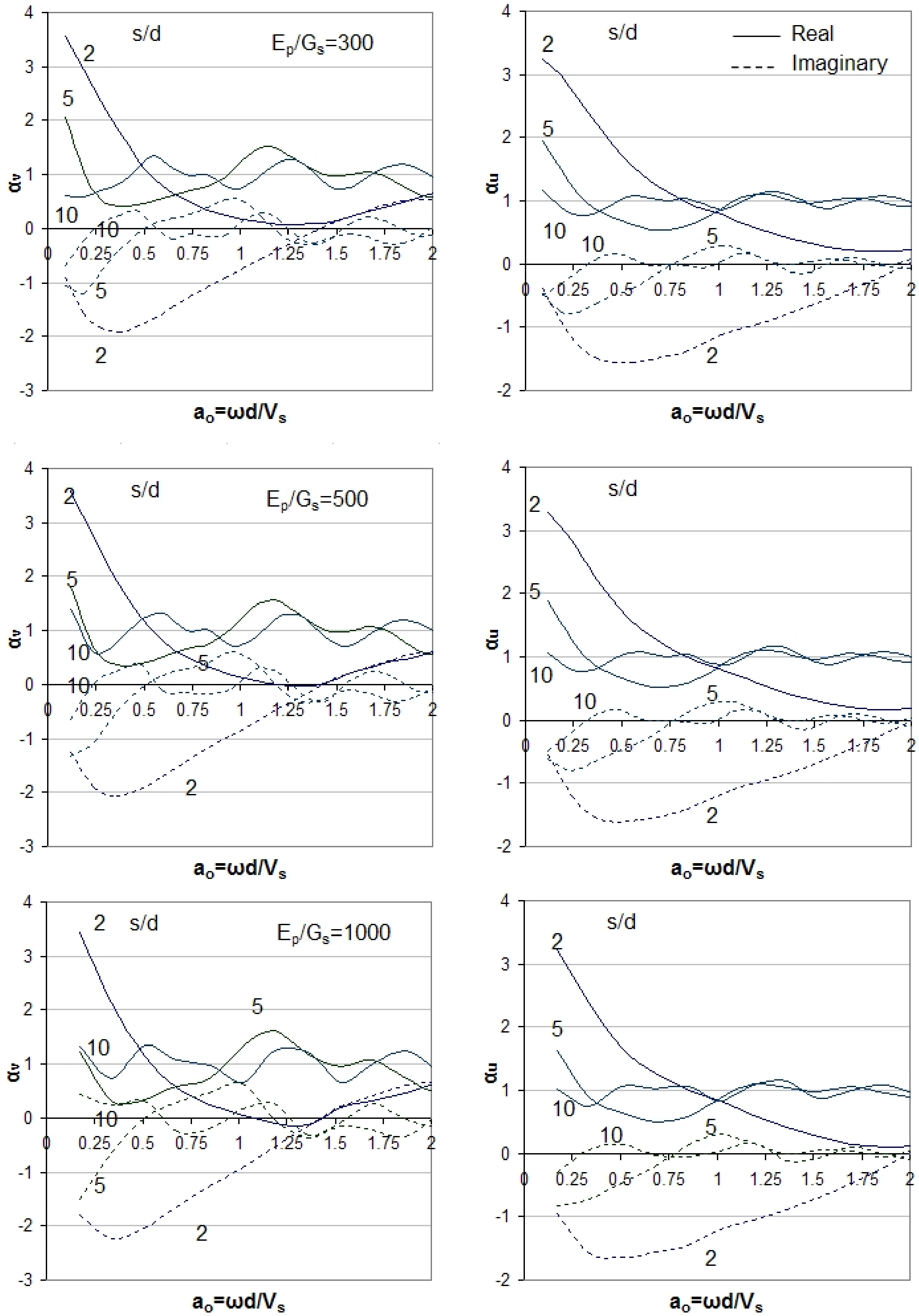

Figure 3: Vertical and horizontal interaction factors for 3x3 pile group in homogenous halfspace with length to diameter ratio L/d = 20, damping ratio, β_s = 0.05, and Poisson's ratio, ν = 0.4.

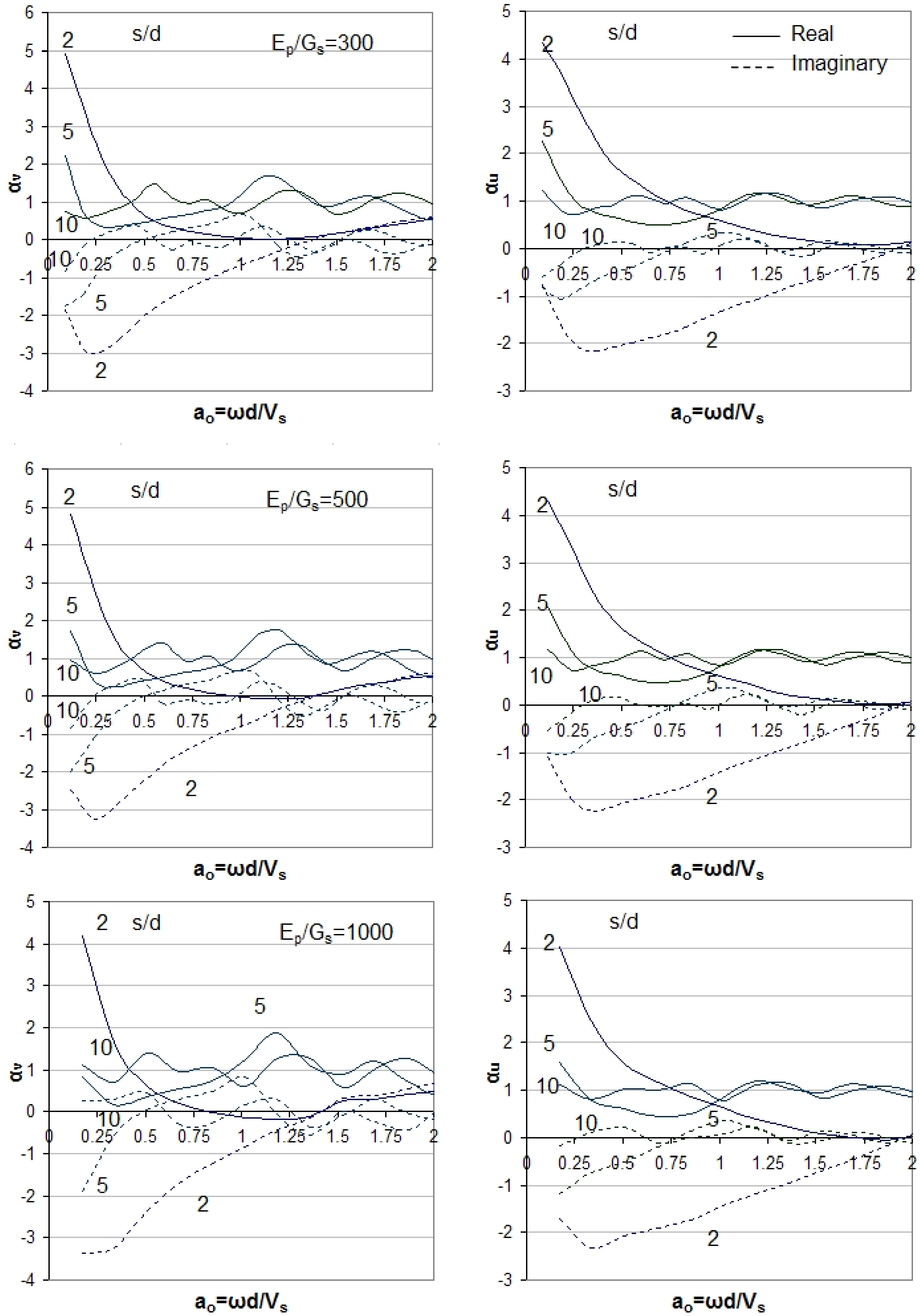

Figure 4: Vertical and horizontal interaction factors for 4x4 pile group in homogenous halfspace with length to diameter ratio L/d = 20, damping ratio, β_s = 0.05, and Poisson's ratio, ν = 0.4.

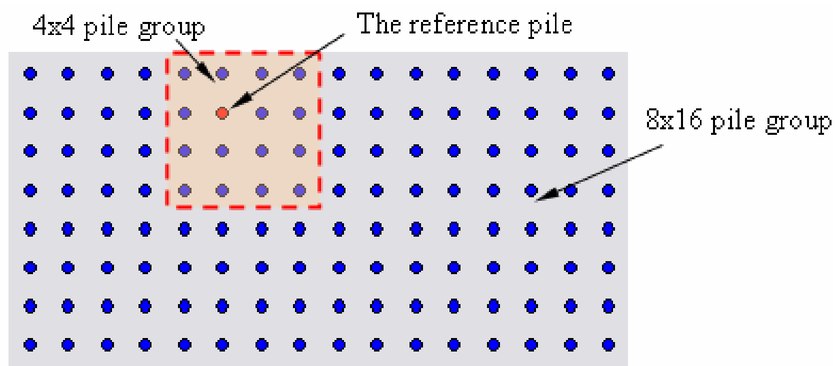

Figure 5: illustration of the mapping method.

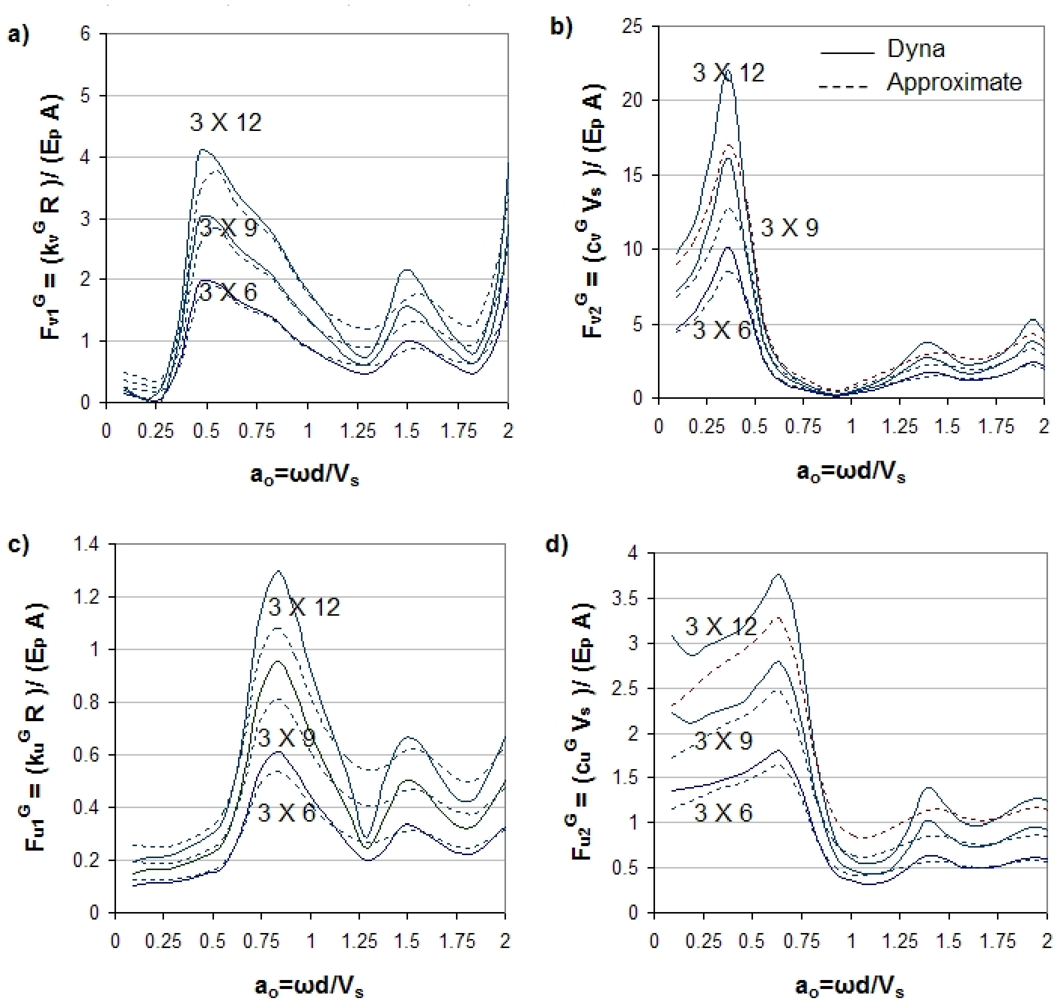

Figure 6: The normalized stiffness and damping group parameters for the 3 piles wide groups with s/d=5. a) Vertical stiffness, b) Vertical damping, c) Horizontal stiffness and d) Horizontal damping.

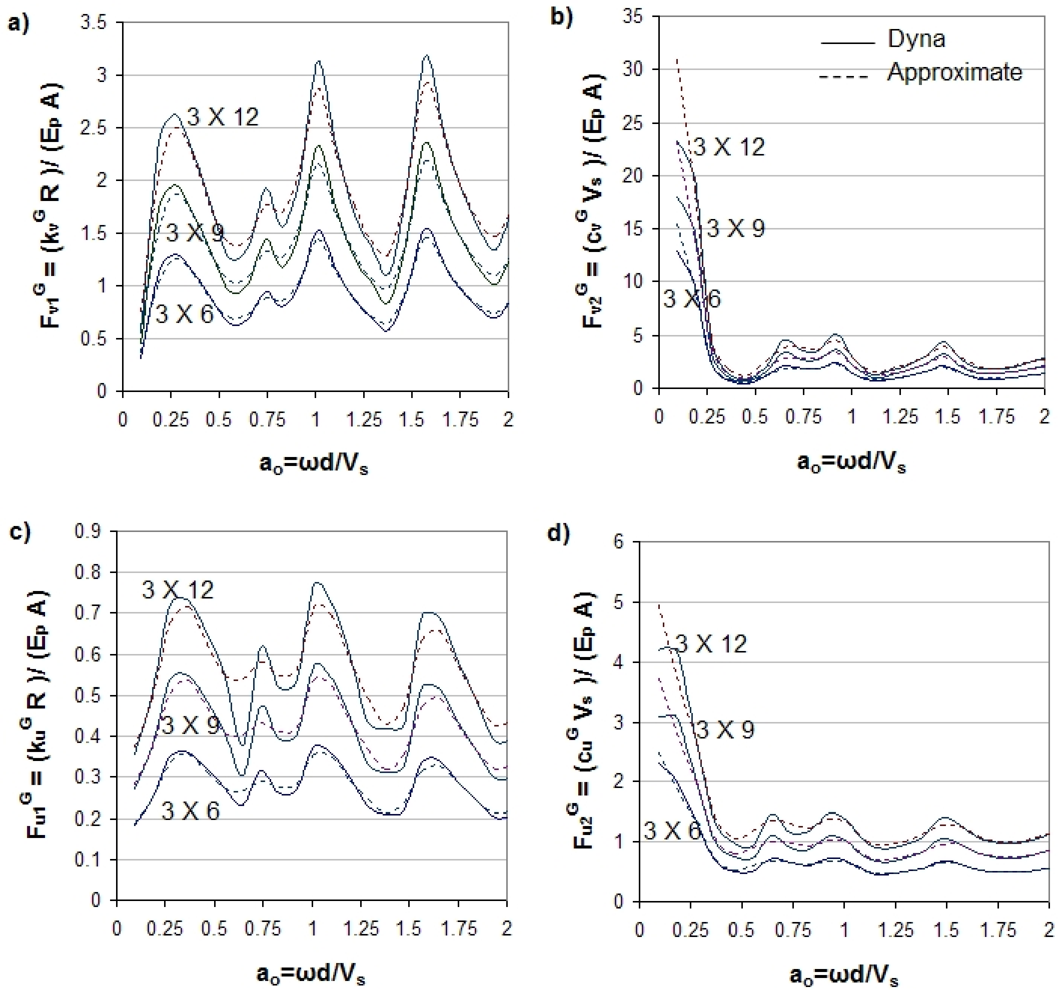

Figure 7: The normalized stiffness and damping group parameters for the 3 piles wide groups with s/d=10. a) Vertical stiffness, b) Vertical damping, c) Horizontal stiffness and d) Horizontal damping.

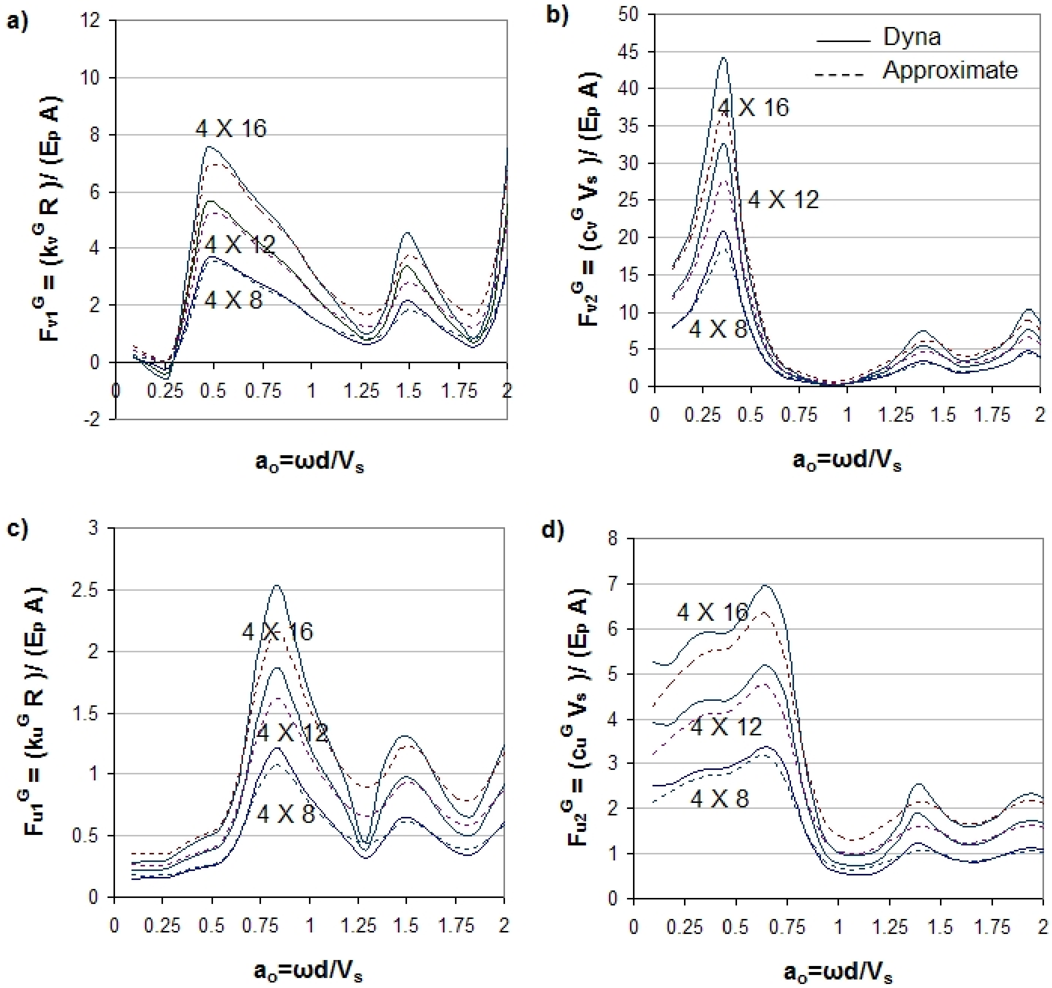

Figure 8: The normalized stiffness and damping group parameters for the 4 piles wide groups with s/d=5. a) Vertical stiffness, b) Vertical damping, c) Horizontal stiffness and d) Horizontal damping.

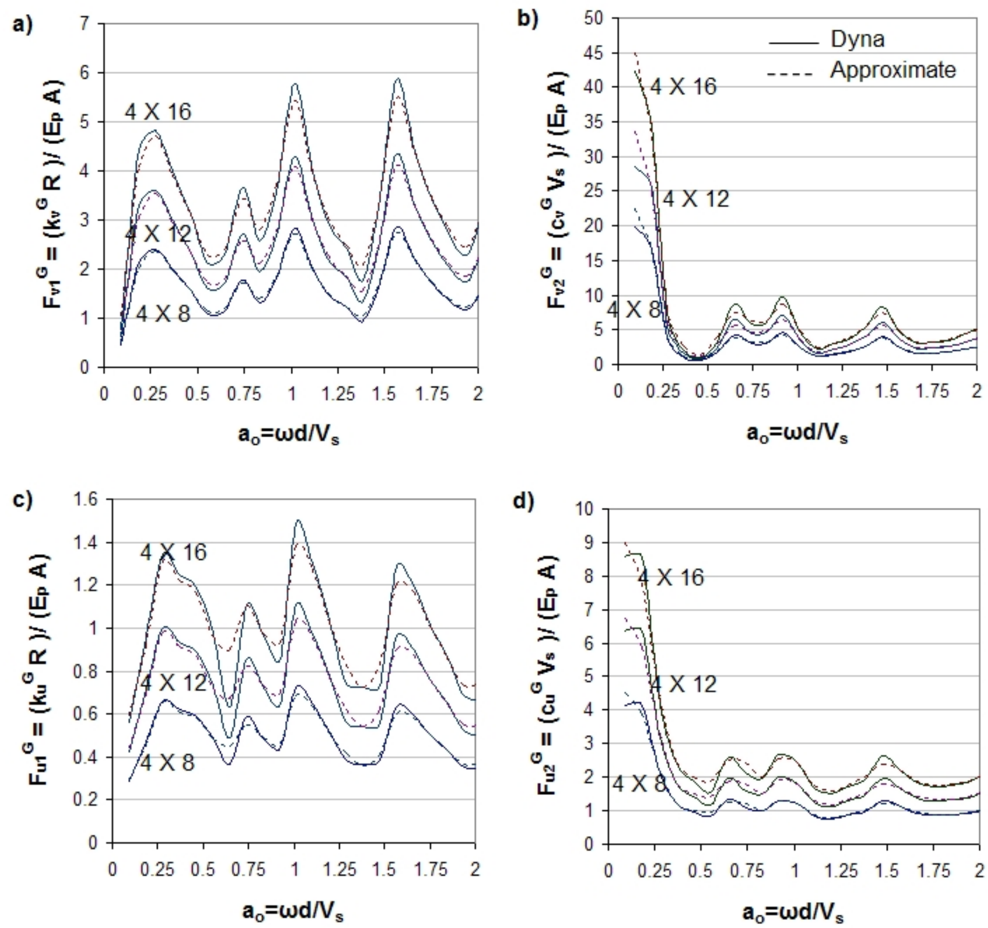

Figure 9: The normalized stiffness and damping group parameters for the 4 piles wide groups with s/d=10. a) Vertical stiffness, b) Vertical damping, c) Horizontal stiffness and d) Horizontal damping.

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