

## SEISMIC DESIGN OF RETAINING WALLS CONSIDERING VERTICAL GROUND ACCELERATION

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### ABSTRACT

Repeated observations from recent earthquakes show that the vertical acceleration can be amplified significantly and even become greater than the horizontal acceleration in certain situations. Concern has therefore arisen over the possible effect of vertical ground motion on the performance of geotechnical structures. In this paper, this emerging issue is addressed for retaining walls using the well-accepted pseudostatic approach and Newmark sliding block theory. Earth pressures, factors of safety against sliding and overturning, and wall displacements are calculated and compared for different loading conditions to identify the influence of vertical acceleration.

Keywords: retaining walls; earth pressures; seismic effects

### INTRODUCTION

Ground motion induced by an earthquake is characterized by a vector with its components along vertical and two horizontal directions. Current engineering practice has tended to focus mainly on the effect of horizontal motion and simply disregard the vertical component. This practice is due partly to the consideration that engineering structures have adequate resistance to dynamic forces induced by the vertical acceleration that is generally much smaller in magnitude and richer in high frequencies than its horizontal counterparts. While the effect of vertical motion on earth structures have been discussed in some studies (Iai, 2001; Ling and Leshchinsky, 1998), it still remains a debatable issue.

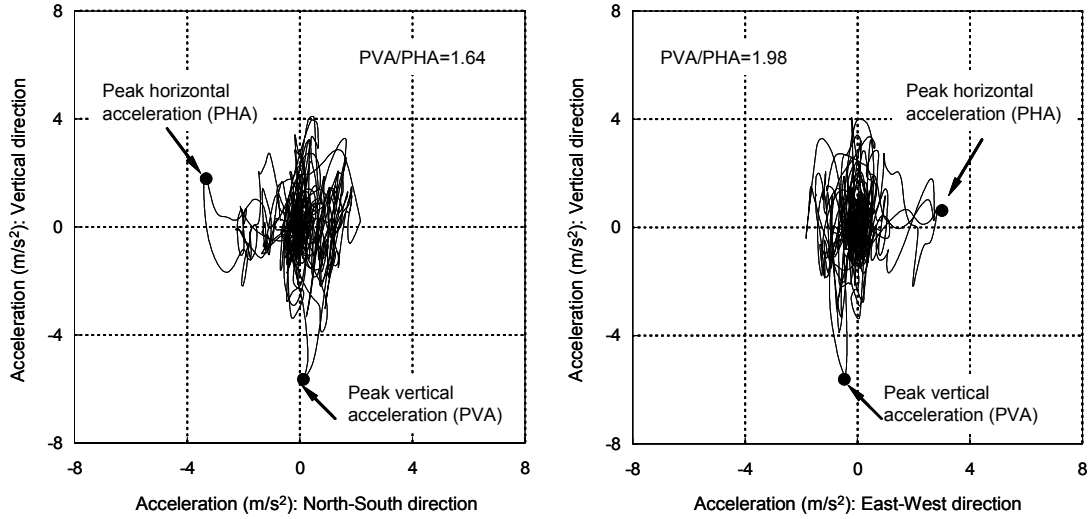
In dealing with this issue some observations from recent earthquakes are worthwhile noting. First, large vertical accelerations have repeatedly been recorded in the near field of moderate and large earthquakes, which indicate that the rule-of-thumb ratio, 1/2, between peak vertical and horizontal ground acceleration may not be a good descriptor. For example, the peak vertical acceleration at the surface of a reclaimed site during the 1995 Kobe earthquake was found to be twice as high as the peak horizontal acceleration (Yang and Sato, 2000), as shown in Figure 1. Second, the ground motion amplification in vertical direction can significantly be affected by groundwater conditions (Yang and Sato, 2000; 2001). These observations suggest that various possible combinations of the groundwater and ground motion conditions need to be examined to find out a particularly severe state of hazard for earth structures.

In view of the above observations, this study aims to explore the possible effect of vertical acceleration with regard to the seismic design of retaining structures, by taking into account a wide range of magnitudes of the vertical and horizontal accelerations and a varying water table. Effort is made to show how significant the effect could be on the earth pressure, factor of safety, yield acceleration and, particularly, permanent wall displacement.

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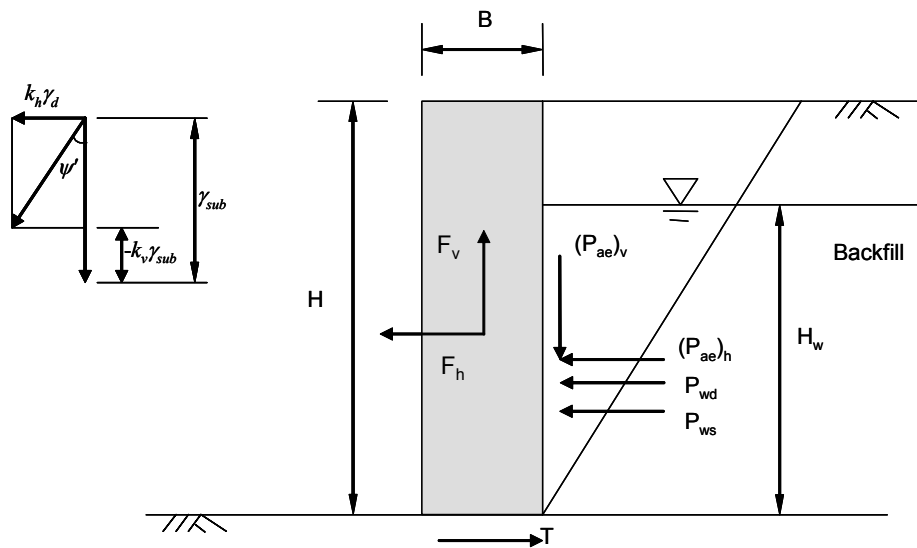
**Figure 1. Acceleration trajectory recorded at a reclaimed site during the Kobe earthquake**

### METHOD OF ANALYSIS

The pseudostatic approach is widely used in seismic design of earth structures. In this approach, the effects of earthquake are represented by pseudostatic forces,  $F_h$  and  $F_v$ , in the following form:

$$F_h = \frac{a_h W}{g} = k_h W \quad F_v = \frac{a_v W}{g} = k_v W \quad (1)$$

where  $k_h$  and  $k_v$  are known as horizontal and vertical pseudostatic seismic coefficients, and  $W$  is the weight of sliding wedge or sliding wall. The vertical and horizontal inertial forces can act upward or downward and toward or away from the retaining wall during earthquakes. In this study the seismic coefficients are assumed positive when they point upward and away from the wall.



**Figure 2. An idealized retaining wall model**

The earth pressures on a retaining wall due to dry backfill are usually estimated using the Mononobe-Okabe equation (Kramer, 1996), which was derived by modifying Coulomb's classical earth pressure theory to account for inertial forces. For a vertical wall retaining a horizontal backfill (Figure 2), the total active earth thrust  $P_{ae}$  is given by:

$$P_{ae} = \frac{1}{2} \gamma_d H^2 (1 - k_v) K_{ae} \quad (2)$$

where

$$K_{ae} = \frac{\cos^2(\phi - \psi)}{\cos \psi \cos(\delta_{wall} + \psi) \left[ 1 + \sqrt{\frac{\sin(\delta_{wall} + \phi) \sin(\phi - \psi)}{\cos(\delta_{wall} + \psi)}} \right]^2} \quad (3)$$

$\gamma_d$  and  $\phi$  are the unit weight and internal friction angle of the dry backfill,  $\delta_{wall}$  is the angle of friction between the backfill and the wall, and  $\psi$  is defined as  $\psi = \tan^{-1}[k_h/(1 - k_v)]$ . The dynamic active earth pressure  $\Delta P_{ae}$  is assumed to act at 0.6H above the base of the wall (Seed and Whitman, 1970). Combining the static ( $P_a$ ) earth pressure, the total active thrust therefore acts at a height:

$$h = \frac{P_a (H/3) + \Delta P_{ae} (0.6H)}{P_{ae}} \quad (4)$$

Recent experiments show that the point of application of the dynamic active earth pressure  $\Delta P_{ae}$  is dependent on the mode of wall movement, and is somewhere between 0.45H and 0.55H above the wall base. Nevertheless, the suggestion by Seed and Whitman (1970) always yields a conservative design of the wall.

### Submerged backfill

When Mononobe-Okabe method is applied to submerged backfill soil, two modifications are needed. For fully submerged backfill, the vertical inertial force  $(1 - k_v)\gamma_d$  should be replaced by  $(1 - k_v)\gamma_{sub}$  in Mononobe-Okabe equations. The angle of seismic coefficient  $\psi$  should be replaced by the apparent seismic coefficient by taking into account the vertical and horizontal inertial forces in submerged conditions. The apparent seismic coefficient depends on the soil properties.

For highly permeable backfill soil such as gravelly and coarse sandy backfills, pore water can move freely in the voids. During earthquakes, the pore water remains stationary while the backfill soil is moving. Therefore, the horizontal inertial force is proportional to the dry unit weight  $\gamma_d$  of the soil. Therefore, the angle of seismic coefficient is given by (Figure 2)

$$\tan \psi' = \frac{\gamma_d}{\gamma_{sub}} \frac{k_h}{1 - k_v} = \frac{G_s}{G_s - 1} \tan \psi \quad (5)$$

where  $G_s$  is the specific gravity of soil particles. When using the above equation, both static and dynamic water pressure should be considered. The approximate solution for dynamic water pressure of free stand water along vertical wall caused by a horizontal earthquake motion (Westergaard, 1933) is used here:

$$P_{wd} = \frac{7}{12} k_h \gamma_w H_w^2 \quad (6)$$

where  $\gamma_w$  is the unit weight of water and  $H_w$  is the depth of water.

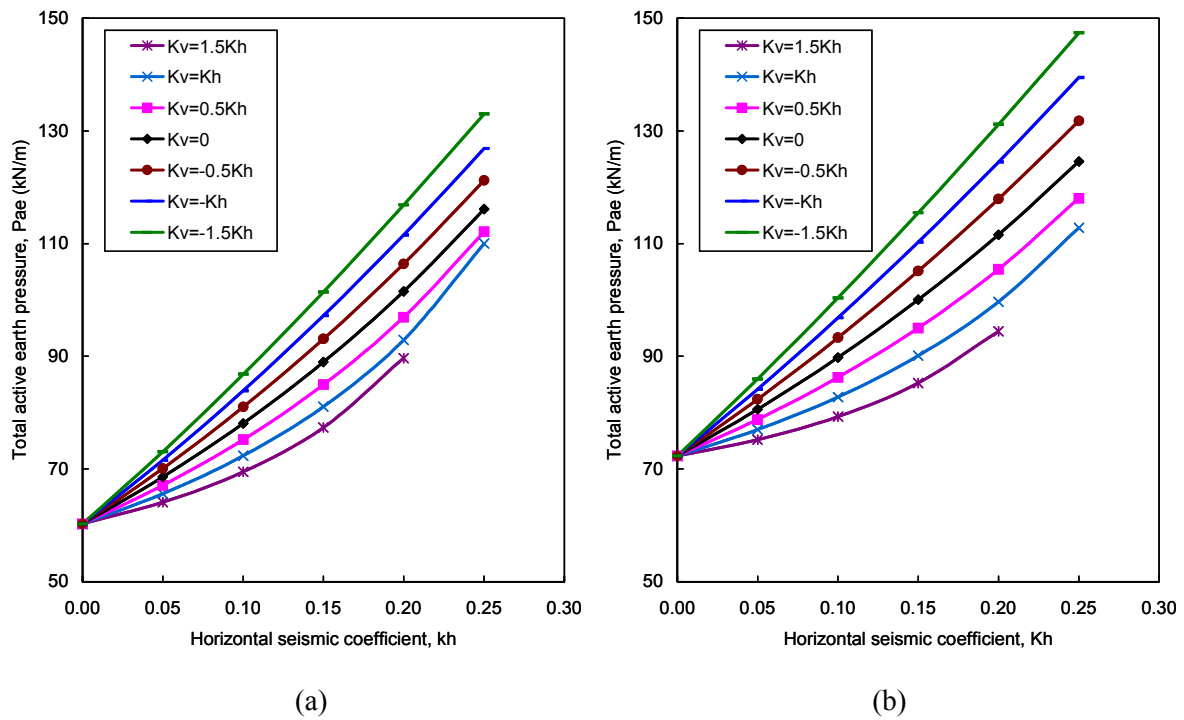
In the conventional stability design of retaining walls, the factors of safety of the wall against sliding and overturning need to be checked. In the performance based design which limits the displacement of the wall after earthquakes, the permanent displacement needs to be estimated. The Newmark sliding block theory (Newmark, 1965) is one of the most common methods to predict permanent displacements of retaining walls. The Newmark analysis involves determination of the yield acceleration, selection of an appropriate earthquake record and double integration of the parts in the acceleration time-history that exceed the yield acceleration. The advantage of the performance based design is that for large earthquake, it would be more economical to allow for an acceptable level of displacement rather than restricting any displacement using stability design.

## RESULTS OF ANALYSIS

The properties of the backfill soil and the wall used in the analysis are given in Table 1. Figure 3(a) shows the active earth pressure as a function of  $k_h$  for a partially submerged backfill ( $H_w=4$  m). The value of  $k_v/k_h$  is varied from (-1.5) to 1.5 to examine the influence of vertical acceleration. The results for a dry backfill under otherwise identical conditions are shown in Figure 3(b).

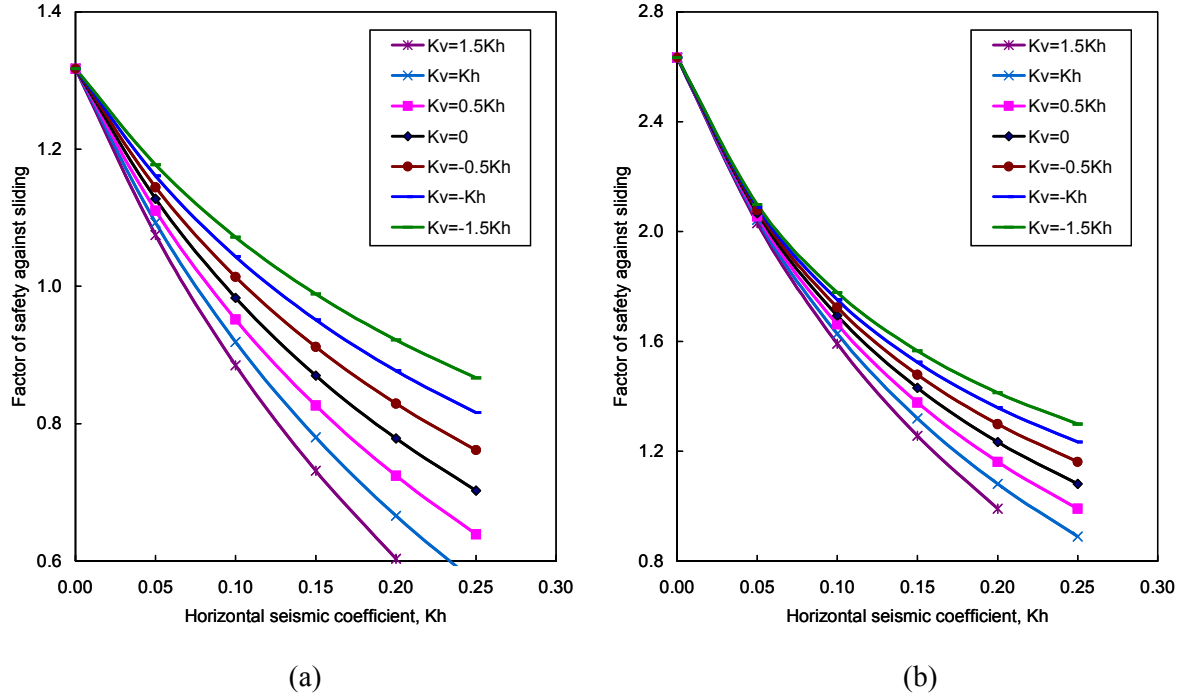
**Table 1. Parameters used in analysis**

Backfill			Wall			Wall-soil interface	
$\phi$	$\gamma_{\text{sat}}$	$G_s$	$H$	$B$	$\gamma_{\text{wall}}$	$\delta_{\text{wall}}$	$\delta_{\text{base}}$
$35^\circ$	$20 \text{ kN/m}^3$	2.66	6 m	1.65 m	$24 \text{ kN/m}^3$	$17.5^\circ$	$35^\circ$

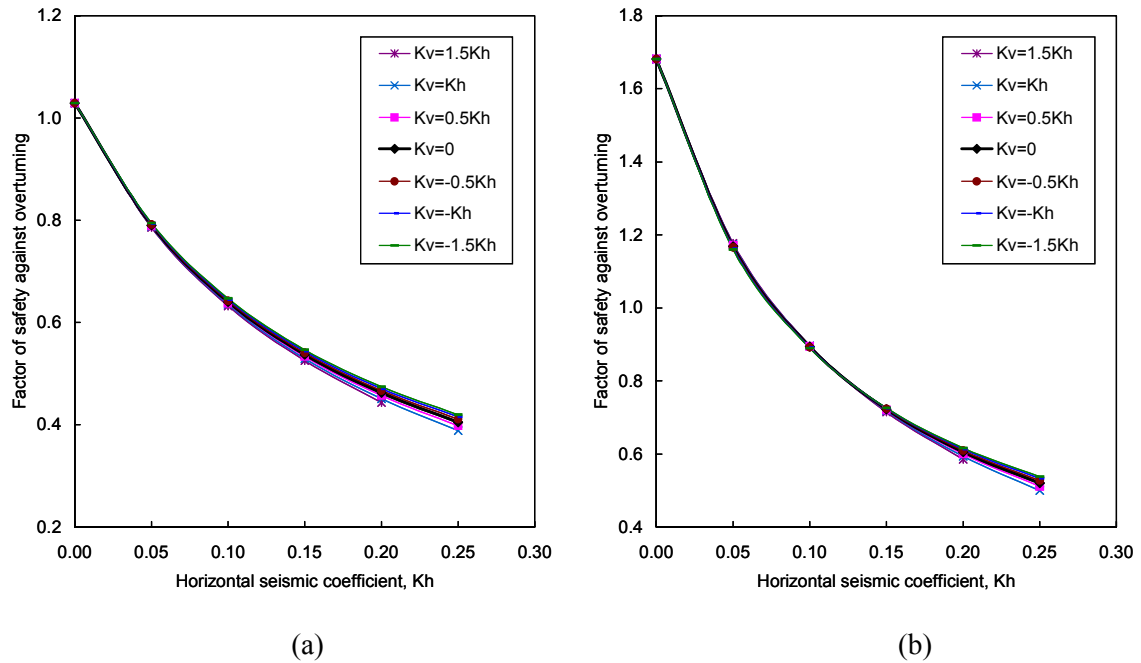


**Figure 3. Effect of vertical inertial force on active earth pressure: (a) partially submerged backfill; (b) dry backfill**

It is noted in Figure 3 that the effect of vertical inertial force on the active earth pressure is generally small when the horizontal seismic coefficient is less than 0.1. The influence tends to be significant at large values of  $k_h$ . Compared with the case of submerged backfill, the effect of vertical acceleration for the case of dry backfill appears to be more significant. The downward inertial force is critical for earth pressure whereas the upward inertial force is to reduce the pressure on the wall.



**Figure 4. Effect of vertical inertial force on factor of safety against sliding: (a) partially submerged backfill; (b) dry backfill**



**Figure 5. Effect of vertical inertial force on factor of safety against overturning: (a) partially submerged backfill; (b) dry backfill**

The factor of safety of the wall against sliding is calculated and shown in Figure 4. Again, the effect of vertical inertial force appears to be significant at large values of  $k_h$ . However, contrary to the effect on earth pressures, the upward inertial force is critical for the factor of safety against sliding whereas the vertical inertial force acting downward is to increase the factor of safety. Comparison of the two graphs in Figure 4 suggests that the effect of vertical inertial force is more significant for the case of submerged backfill than the case of dry backfill. As far as the factor of safety against overturning is concerned, the inclusion of vertical acceleration has almost no effect for the retaining wall model considered (Figure 5). It should be mentioned, however, that the effect for the factor of safety against overturning may become apparent for other types of retaining walls (e.g. caisson type walls).

The yield acceleration is conventionally defined as the horizontal acceleration that results in the factor of safety against sliding of unity. Table 2 gives the values of yield acceleration calculated for different ratios of  $k_v/k_h$ . Only positive values of  $k_v$  are considered as they are critical as compared with the negative  $k_v$ . In general, the inclusion of vertical inertial force does not cause a substantial change of the yield acceleration if the ratio of  $k_v/k_h$  is less than the rule-of-thumb value, 0.5. For example, for the wall retaining dry backfills under the horizontal inertial force only, the yield acceleration is estimated to be 0.283g. If the vertical inertial force is taken into account by assuming that  $k_v/k_h$  is 0.5, the yield acceleration is reduced by about 13%. However, for an extremely high value of  $k_v/k_h$  (=1.5), the yield acceleration is reduced by as large as 30%.

The effect of vertical acceleration on the sliding displacement of the wall is summarized in Table 3 for both the cases of dry and submerged backfills. The acceleration time history used in the calculation was recorded at a retaining wall site during the 2003 Tokachi-oki earthquake in Japan (Figure 6), with the peak acceleration of 136 cm/s<sup>2</sup>. The effect appears to be significant on the wall displacement although it is relatively small on the yield acceleration. As an example, Figure 6 illustrates the calculation of the permanent wall displacement for submerged backfill under the condition of  $k_v/k_h=1.0$ .

**Table 2. Effect of vertical inertial force on yield acceleration**

Yield acceleration (g)	$k_v/k_h=0$	$k_v/k_h=0.5$	$k_v/k_h=0.67$	$k_v/k_h=1.0$	$k_v/k_h=1.5$
$a_y$ (submerged)	0.094	0.085	0.082	0.077	0.070
$a_y$ (dry)	0.283	0.247	0.238	0.221	0.198

**Table 3. Effect of vertical inertial force on wall displacement**

	$k_v/k_h=0$	$k_v/k_h=0.5$	$k_v/k_h=0.67$	$k_v/k_h=1.0$	$k_v/k_h=1.5$
Wall displacement (cm)	0.51 (1.0)	1.24 (2.43)	1.48 (2.90)	2.08 (4.08)	3.14 (6.16)

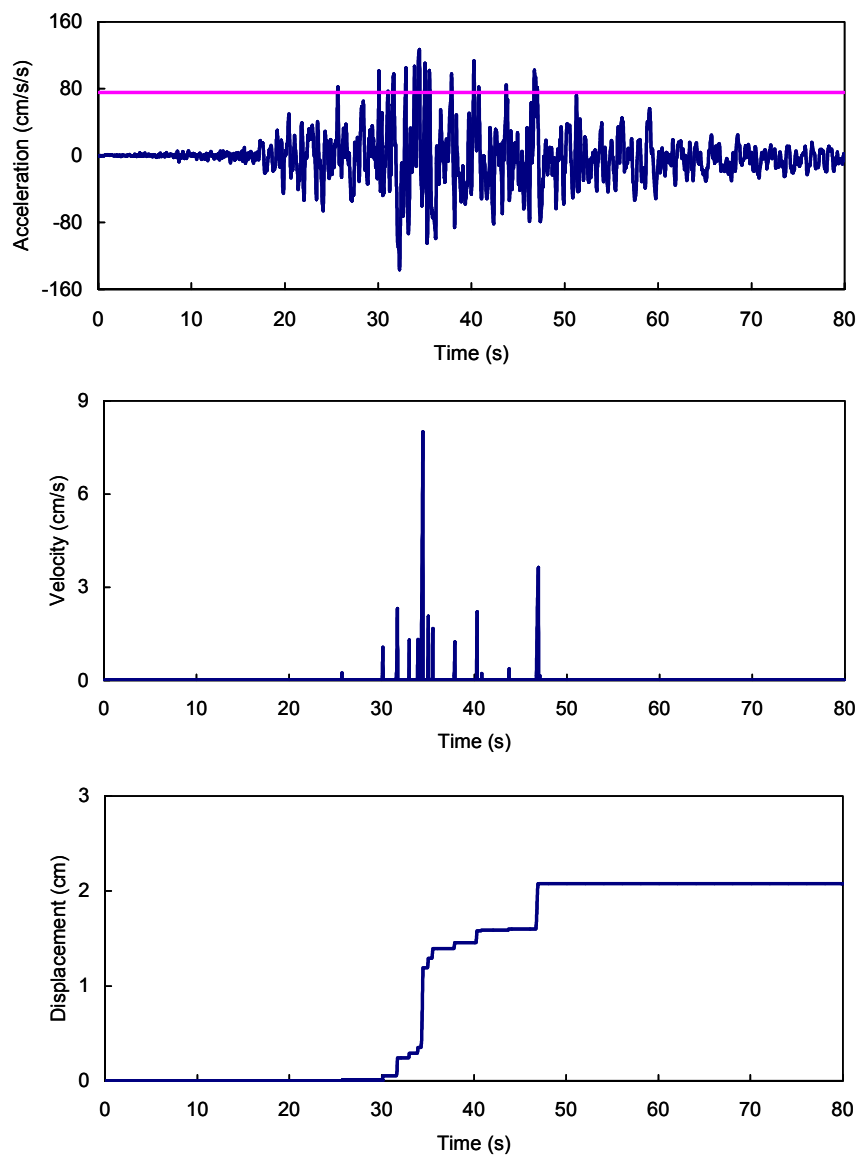
The number in the bracket is the displacement ratio with reference to the case of zero  $k_v$ .

## CONCLUSIONS

This paper discusses the possible effect of vertical ground acceleration that has been simply ignored in the current practice of seismic design of retaining structures. Using the well-accepted pseudostatic approach combined with the Newmark sliding block theory, the effect on the active earth pressure, factors of safety against sliding and overturning, and permanent wall displacement has been identified. The investigation shows that the effect is associated with the groundwater conditions in backfills and

the magnitudes of the vertical and horizontal accelerations. The effect of vertical acceleration is generally small when the horizontal seismic coefficient is less than 0.1, but tends to become significant at large values of horizontal seismic coefficient. Compared to the case of dry backfills, the effect appears to be more significant for the wall retaining submerged backfills.

Current practice is mainly based on the pseudostatic approach and Newmark sliding block theory. While they provide a simple way of screening for various stability problems, the methods do not adequately account for some complex conditions in real earthquakes (e.g. time-varying amplitudes and frequencies of ground motion, deformations associated with significant buildup of excess pore pressures). When there is a need to go beyond pseudostatic analyses, comprehensive procedures such as dynamic nonlinear finite element analyses can be used to provide more insight and information as to the performance of retaining walls (although these procedures are not considered viable in practical applications due to the difficulty and uncertainty associated with defining various input parameters).



**Figure 6. Evaluation of permanent displacement of retaining wall**

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