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PARAMETRIC INVESTIGATION OF THE ROCKING OF FOOTINGS UNDER HARMONIC EXCITATION

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ABSTRACT

In this paper, the dynamic response of a rigid footing on an elastic tensionless Winkler foundation, which allows partial uplift to occur, is studied. A parametric investigation is performed for harmonic excitation. The parameters studied are expressed in dimensionless quantities and concern the foundation stiffness, the characteristics of the excitation and the loads from the superstructure applied to the footing. The maximum rocking response, the minimum length of contact after uplift and the maximum stress developed at the soil are used to measure the effect of each parameter. The outcomes of the dynamic analysis are compared to the ones of a simplified static approach, which is based on the maximum values of the applied loads, similarly to what is usually applied in practice. The results show that the static approach can, in general, predict the response satisfactorily, except of cases in which resonance happens or the dynamic part of the moment and/or the axial force of the column are large. In such cases, the static approach may underestimate or overestimate the response significantly.

Keywords: Winkler foundation, uplift, rocking, footings, harmonic response

INTRODUCTION

During strong earthquakes, partial uplift of shallow foundations is a common phenomenon and it is due to the inability of the soil to develop tensile stresses. Whether uplift will happen or not depends on the intensity of the ground motion, the stiffness of the foundation mat, the loads imposed to the footings from the superstructure and the dimensions of the footings. It is known that foundation uplift significantly affects the dynamic response of the superstructure, usually in a beneficial way, since, in most cases, it reduces its response. This phenomenon has been investigated by many researchers in the past and will not be considered in this paper. Concerning the foundation, uplift causes an increase to the stresses that are developed at the soil. In order to examine this phenomenon, the dynamic response of footings on an elastic Winkler foundation is studied here.

Modern seismic codes, as Eurocode 7 (European Committee for Standardization, 2004), FEMA 356 (American Society of Civil Engineers, 2000) and ATC 40 (Applied Technology Council, 1996), face this problem by static considerations, which are based on the forces and moments applied to the footing during the seismic design situation. In the approach of the American codes (a thorough investigation of this methodology has been presented by Allotey and El Naggar, 2003), a relation is established between the overturning moment and the footing rotation for each possible state: elastic behaviour of the soil and full contact of the base; inelastic behaviour of the soil and full contact of the base; elastic behaviour of the soil and partial uplift; inelastic behaviour of the soil and partial uplift. In Eurocode 7 (Appendix D), the effect of the base uplift is considered in the calculation of the bearing capacity of the soil by reducing the effective area of contact between the footing and the foundation mat. This reduction depends on the eccentricity of the loads applied, which is measured by the ratio M/N, where M and N are the moment and the axial force at the base of the column, respectively.

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The determination of the appropriate values of M and N, which should be used in such analyses, is not always a straight forward procedure. Usually in practice, the response of the superstructure is calculated by modal analysis and, thus, only the maximum, non-synchronous values of the dynamic components of M and N (i.e. the forces and moments caused by the seismic loading only) are known. Especially for the axial force, it is not clear whether it should be considered as tensile or compressive. According to the codes, the assumption of smaller axial force leads to a more conservative design and, for this reason, the dynamic part of N is usually considered tensile. In many cases, however, this assumption is unrealistically strict, leading to foundations with unnecessarily large dimensions.

In order to examine the real behaviour of uplifting footings, the dynamic response of a rigid footing on an elastic tensionless Winkler foundation is studied in this analysis. A parametric investigation, which covers not only the above-mentioned problem with the axial force, but all the parameters involved in the phenomenon, is performed for a harmonic excitation (base motion and loads applied to the footing from the superstructure). Harmonic excitation is used instead of earthquake motion in order to minimize the parameters concerning the excitation and also to investigate better any resonance phenomena. The results of the dynamic analysis are also compared to the outcomes of a static approach, which is based on the maximum values of the dynamic components of the loads, similarly to the usual procedure applied in practice.

METHOD OF ANALYSIS

The analysis is performed for a rigid footing sitting on an elastic tensionless Winkler foundation with subgrade modulus, k_0 and damping, c_0. The soil flexibility in the horizontal direction is not considered, because it does not affect the vertical stresses developed at the soil, which are of the main interest in this investigation. This model of foundation is widely used in Soil-Structure Interaction (SSI) problems, in spite of the fact that it cannot model precisely the dynamic response of footings if rocking and vertical vibrations occur simultaneously, because these modes are not independent of each other. In order to uncouple the vibrations, a higher order modelling should be applied, as the one proposed by Hall *et al*, 1979. The present analysis, however, aims to the qualitative investigation of the dynamic response and not to the exact calculation of the rotations and displacements and for this reason the simple Winkler foundation is used.

The system considered is shown in Fig. 1. The footing is shown rectangular, with dimensions $b \times a$. However, the analysis can be applied to any other shape that is symmetric about the vertical axis through the centre of the base, by substituting b with the width of the base, h with the height at which the centre of gravity of the footing is located and a with the total height of the footing, equal to the distance between the base of the column and the foundation mat.

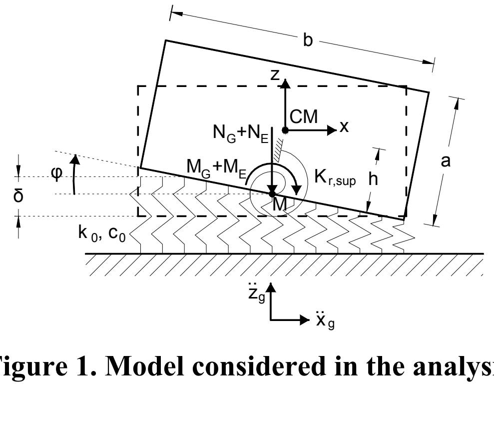

Figure 1. Model considered in the analysis

The analysis is performed in two dimensions, so a unit width is assumed in the direction normal to the figure. Also, in order to take under consideration the stiffness of the column that is connected to the footing, a rotational spring is placed at the centre of the base (point M). The stiffness, $K_{r,sup}$, of this spring is calculated from the rotational and the horizontal stiffness of the column (at its base), $K_{r,col}$ and $K_{x,col}$ respectively, using the relation

$$K_{r,sup} = K_{r,col} + a^2 \cdot K_{x,col} \tag{1}$$

Since the horizontal movement is restricted, there are only two independent variables: the rotation, φ and the vertical displacement of CM, z, (Fig. 1). The horizontal displacement of CM, x, can be expressed in terms of the angle of rotation as $x=h\cdot\varphi$ (for small rotations).

The superstructure loads, which are applied to the footing through the column, are considered acting at point M. If M_{col}, N_{col} and V_{col} are the moment, the axial force and the shear force at the base of the column, respectively, the loads at point M are: $M = M_{col} + a\cdot V_{col}$ and $N = N_{col}$. These loads consist of two terms: the static term, which is due to the gravity loads of the superstructure and is denoted by the subscript "G" and the dynamic term, which is caused by the earthquake response of the superstructure and is denoted by the subscript "E". Therefore, one can write:

$$M(t) = M_G + M_E(t) \tag{2a}$$

$$N(t) = N_G + N_E(t) \tag{2b}$$

Since the footing is examined by itself, $M(t)$ and $N(t)$ are considered as external loads and correspond to fixed base conditions for the superstructure. In this way, SSI effects are considered indirectly and in an approximate way: the actual moment at the base of the column is smaller than $M(t)$, relaxed by the term $K_{r,sup}\cdot\varphi$, but its calculation is not exact because it is based on the fixed-base dynamic characteristics of the superstructure.

With these assumptions, the equations of motion can be derived from the kinematics of the rocking block of Fig. 1. They are not presented here for space-saving reasons, but are similar to the ones for a free rocking block (Psycharis and Jennings, 1983) with the addition of the contribution of the mass of the superstructure, the rotational stiffness of the superstructure, $K_{r,sup}$ and the external loads $M(t)$ and N(t). The effective mass of the superstructure is considered equal to N_G/g, with g being the acceleration of gravity.

It should be noted that there is an initial angle of rotation, φ_0, which is caused by the gravity loads and is equal to $\varphi_0=M_G/K_{r,tot}$, in which $K_{r,tot}$ is the total rotational stiffness of the system, given by:

$$K_{r,tot} = K_{r,sup}+K_{r,f} \tag{3}$$

In this relation, $K_{r,f}$ is the rocking stiffness of the Winkler foundation during full contact, given by:

$$K_{r,f} = \frac{k_0 b^3}{12} \tag{4}$$

The system also possesses an initial vertical displacement, δ, equal to the static deflection caused by the gravity loads: $\delta = (N_G + mg)/k_0 b$. Uplift starts when the vertical displacement of any of the corner points of the base becomes greater than δ and full contact is reestablished when this displacement becomes smaller than δ. If z denotes the vertical displacement of CM (Fig.1), measured from the equilibrium position, the criteria for the initiation and the ending of uplift can be written as:

Initiation of uplift: $$z + \tfrac{1}{2} b|\varphi| \geq \delta \tag{5a}$$

End of uplift: $$z + \tfrac{1}{2} b|\varphi| \leq \delta \tag{5b}$$

The ground motion $\ddot{x}_g(t)$ and the dynamic components of the superstructure forces $M_E(t)$ and N_E(t) are assumed harmonic and of the same frequency ω. The assumption that the frequency of the superstructure response is the same with the frequency of the excitation is valid for the steady-state response only and is adopted here for simplicity. Also, 180° phase difference is considered between the base motion and the response of the superstructure, although this is not, in general, true. These approximations are not expected to alter the results significantly, since the base excitation affects the impulsive forces of the footing only, while the overall response is controlled by the much larger superstructure loads.

The results of the dynamic analysis are compared to the ones of a simplified pseudo-static approach, based on the maximum values of the applied loads. This is a standard procedure in the codes (e.g. Eurocode 7) when it is checked whether the bearing capacity of the soil has been exceeded. For this approach, the force and moment that act at the middle point of the base, M, are:

$$M_{tot,max} = M_G + M_{E,max} + mA_g h \tag{6a}$$

$$N_{tot,max/min} = N_G \pm N_{E,max} + mg \tag{6b}$$

where m is the mass of the footing, A_g is the amplitude of the base acceleration, M_G and N_G are the static components of the moment and the axial force of the superstructure loads, respectively and $M_{E,max}$ and $N_{E,max}$ are the amplitudes of the dynamic components of these loads.

Due to the rotation of the footing, part of the moment $M_{tot,max}$ is undertaken by the column connected to it, relaxing the fixed-base moment considered at its base. The remaining moment is undertaken by the foundation. Thus, the maximum moment and axial force applied to the foundation are:

$$M_{f,max} = M_{tot,max} - K_{r,sup} \cdot \varphi_{max} \tag{7a}$$

$$N_{f,max/min} = N_{tot,max/min} \tag{7b}$$

In order to calculate $M_{f,max}$ from equation (7a), φ_{max} must be determined first using the relation:

$$\varphi_{max} = \frac{M_{tot,max}}{K_{r,tot}} \tag{8}$$

in which $K_{r,tot}$ is the total rotational stiffness, defined in equation (3) and equal to the sum of the foundation and the superstructure stiffnesses. If uplift does not happen, the foundation stiffness, $K_{r,f}$, can be calculated from equation (4). If uplift occurs, a reduced value of $K_{r,f}$ should be used, in order to take under consideration the reduced length of contact. Therefore, the calculation of $K_{r,tot}$ would require an iterative procedure, since the minimum length of contact depends on φ_{max}.

In practice, even when SSI is considered, uplift is usually neglected in the analysis, but it is considered in the calculation of the bearing capacity of the soil, by assuming a reduced area of contact. Following this hybrid approach, equation (8) is used here without any reduction in the value of $K_{r,f}$, even if uplift happens. This approximation leads to an underestimation of φ_{max}, which, however, does not seem to affect significantly the rest of the results.

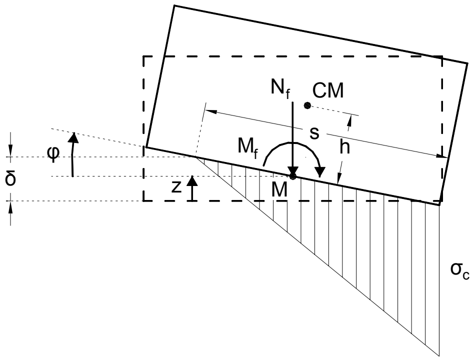

Figure 2. Length of contact and soil stresses after uplift

For each case examined, the maximum angle of rotation, φ_{max}, the minimum length of contact, s_{min}, and the maximum corner stress, $\sigma_{c,max}$, developed at the soil (Fig. 2) are determined. For the dynamic analysis, the corresponding time histories are calculated, using the following relations:

Full contact:

$$s(t) = b \tag{9a}$$

$$\sigma_c(t) = k_0\left[\delta - z(t) + \tfrac{1}{2}b\left|\varphi(t)\right|\right] \tag{9b}$$

With uplift:

$$s(t) = \frac{b}{2} + \frac{\delta}{\left|\varphi(t)\right|} - \frac{z(t)}{\left|\varphi(t)\right|} \tag{10a}$$

$$\sigma_c(t) = k_0\, s(t)\, \varphi(t) \tag{10b}$$

Note that complete separation is possible to occur for stiff foundations. If this happens, s becomes zero and the analysis stops. In such cases, the values of φ_{max} and $\sigma_{c,max}$ are calculated from the corresponding time histories up to the time of complete separation.

For the pseudo-static approach, first the loads applied to the soil are calculated using equations (7a) and (7b) and then the following relations are applied:

Full contact:

$$s = b \tag{11a}$$

$$\sigma_c = \frac{N_f}{b} + \frac{6M_f}{b^2} \tag{11b}$$

With uplift:

$$s = \frac{3b}{2} - \frac{3M_f}{N_f} \tag{12a}$$

$$\sigma_c(t) = \frac{2N_f}{s} \tag{12b}$$

Note that the length of contact after uplift, as defined by equation (12a), is different than the effective length of contact, b', used for the calculation of the bearing capacity of the soil according to Eurocode 7. The latter is defined as $b' = b - 2M_f / N_f$. Using equation (12a), the effective length of contact of the Eurocode can be written as: $b' = 2s/3$.

PARAMETRIC INVESTIGATION

The parametric investigation is performed for an orthogonal footing with base width b=4.00m and unit length in the transverse direction. The specific weight is taken equal to γ=25 kN/m^3 (concrete).

Table 1. Typical values of the parameters considered in the analysis

Dimensionless parameter	Value
ω_φ/p	30.00
$K_{r,sup}/K_{r,f}$	2.00
ζ_φ (ζ_z)	0.15 (0.05)
a/b	0.20
e_G/b	0.05
e_E/b	1.00
N_G/mg	7.50
N_E/N_G	0.20
ω/ω_φ (ω/ω_z)	0.50 (2.50)
A_g/g	0.50

Nine dimensionless parameters, related to the stiffness and the damping of the foundation, the stiffness of the superstructure, the dimensions of the footing, the frequency of the excitation and the superstructure loads, are examined. The effect of each parameter is investigated by varying its value, while all the other dimensionless terms are kept constant, equal to their typical value given in Table 1. The investigation is focused on the variation of: the maximum angle of rotation, φ_{max}; the minimum length of contact, s_{min}; and the maximum corner stress of the soil, $\sigma_{c,max}$, which are calculated as explained above. The angle φ_{max} is measured in degrees, s_{min} is normalized with respect to the width of the base, b and $\sigma_{c,max}$ is normalized with respect to the corresponding stress from gravity loads only, $\sigma_{c,G}$, which is equal to:

$$\sigma_{c,G} = \frac{N_G + mg}{b} + \frac{6M_G}{b^2} \tag{13}$$

For the dynamic analysis, the equations of motion are integrated numerically, using the Runge-Kutta method with a small time step, equal to 0.0002 sec. The criteria for uplift or contact are checked at each time step.

Stiffness and damping of the foundation

First, the effect of the total rotational stiffness is examined. This stiffness is measured by the rocking eigenfrequency for the full-contact state, which can be expressed as

$$\omega_\varphi = \sqrt{\frac{K_{r,tot}}{I_M}} \tag{14}$$

in which I_M is the mass moment of inertia of the footing with respect to point M. For the parametric investigation, ω_φ is normalized with respect to the characteristic frequency p, which depends on the geometry of the footing and is defined by

$$p = \sqrt{\frac{mgh}{I_M}} \tag{15}$$

The characteristic frequency, p, is similar to the one defined by Housner (Housner, 1963) for a block rocking on a rigid foundation, with the difference that the pole of rotation is located at the middle of the base and not at the corner.

The effect of the ratio ω_φ/p is illustrated on the plots of the top row of Fig. 3. Note that larger values of ω_φ/p correspond to a stiffer soil and a stiffer superstructure, since the ratio $K_{r,sup}/K_{r,f}$ is kept constant, as shown in Table 1. In Fig. 3, the variation of the subgrade modulus, k_0, which corresponds to the range of values of ω_φ/p considered, is shown on the horizontal axis on the top of the diagrams.

As it was expected, the rotation of the footing decreases as ω_φ/p increases, since the total rotational stiffness increases. However, s_{min} and $\sigma_{c,max}$ are practically not affected by the variation of ω_φ/p, except of the case of very soft soils. This happens because these quantities depend mainly on the forces applied to the footing (which remain constant) and thus, they are practically unaffected by the stiffness of the foundation. Note that the frequency of the harmonic excitation is changing whith ω_φ, since the ratio ω/ω_φ is kept constant (see Table 1) and thus, resonant phenomena do not happen.

The accuracy of the static approach is relatively good in this case. The results show that the approximate method predicts very well the length of contact and underestimates the soil stress by only 10% in most cases. The error in the angle of rotation is larger and exceeds 40% for soft soils; this underestimation, however, was expected, since uplift is not considered in the calculation of φ_{max}, as explained earlier.

The normalized rocking frequency, ω_φ/p, varies with the total rotational stiffness of the system and the results illustrated on the top row of Fig. 3 are derived assuming that the ratio of the stiffness of the superstructure to the stiffness of the foundation, $K_{r,sup}/K_{r,f}$, remains constant. The results concerning the variation of this ratio are shown on the bottom-row diagrams of Fig. 3. It is evident that $K_{r,sup}/K_{r,f}$ affects significantly the length of contact and the soil stress. For small values of $K_{r,sup}/K_{r,f}$, which correspond to a soft superstructure or a stiff foundation, the length of contact obtains small values and, as a consequence, the corner stress of the soil becomes large. This happens, because a reduction in $K_{r,sup}$ leads to a larger portion of the total moment transferred to the footing, as it is evident from equation (7a).

The angle of rotation is not affected significantly by the ratio $K_{r,sup}/K_{r,f}$, because the dimensionless parameter ω_φ/p does not change and, therefore, the total rotational stiffness remains constant. For the static approach, the rotation is calculated from equation (8) and, thus, φ_{max} is independent of $K_{r,sup}/K_{r,f}$. In the dynamic analysis, the maximum angle of rotation occurs during uplift, when the total stiffness is reduced and therefore, φ_{max} is influenced by $K_{r,sup}/K_{r,f}$, especially for small values of this ratio, when uplift is more pronounced.

Concerning the accuracy of the static approximation with respect to the variation of $K_{r,sup}/K_{r,f}$, it predicts the length of contact and the soil stress quite satisfactorily, although it underestimates the angle of rotation.

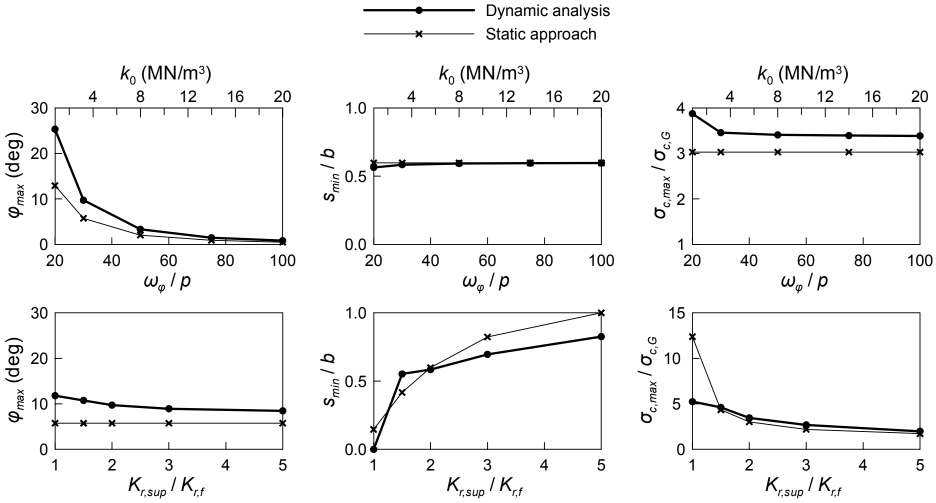

Figure 3. Effect of the stiffness of the foundation on the response

The results concerning the effect of the damping of the foundation to the response (not presented here) show that this parameter is not important.

Dimensions of the footing

The dimensions of the footing are measured by the aspect ratio a/b. However, only the height of the footing is varying, since the analysis is performed for a constant width of the base, b=4.00m. The results show that the ratio a/b has a very small influence on the length of contact and on the stress of the soil. This happens because the dimensions of the footing affect the inertia forces only, which are usually small, compared to the loads from the superstructure.

It is known that, for a rocking block on an elastic foundation, the size plays an important role to the response (e.g. Psycharis and Jennings, 1985), with smaller blocks experiencing larger rotations than bulkier ones with the same proportions. For the problem under consideration, the absolute dimensions affect only the characteristic frequency p and, thus, the size of the footing must not be important. In order to check this assumption, the plots of the top row of Fig. 3 were reproduced for the same values of the dimensionless parameters and for b=2.00m instead of 4.00m. The results obtained were exactly the same as the ones shown in Fig. 3.

Frequency of the excitation

The frequency ω of the excitation (concerns the loads from the superstructure, $M_E(t)$ and $N_E(t)$, and the base acceleration, $\ddot{x}_g$) is normalized with respect to the rocking eigenfrequency during full contact, ω_φ, which is defined by equation (14). Note that the system possesses another natural frequency, ω_z, related to the vertical vibrations and defined by

$$\omega_z = \sqrt{\frac{k_0 b}{m + N_G/g}} \tag{16}$$

For an orthogonal footing and for constant values of N_G/mg, $K_{r,sup}/K_{r,f}$ and a/b, it can easily be proven that ω_z and ω_φ are related to each other, according to the following expression:

$$\frac{\omega_\varphi}{\omega_z} = \sqrt{\frac{(1+N_G/mg)(1+K_{r,sup}/K_{r,f})}{1+4(a/b)^3}} \tag{17}$$

The effect of the excitation period, T, on the response is shown in Fig. 4. The ratios T/T_φ and T/T_z are shown on the bottom and the top axes of the diagrams. Resonance occurs when $T \approx T_\varphi$ and $T \approx T_z$ resulting in a significant increase in the angle of rotation and the soil stress and a significant decrease in the length of contact. Actually, resonance happens for T/T_φ and T/T_z a little greater than unity, because the effective rocking period and the effective vertical period after uplift are longer than T_φ and T_z, respectively. Also, complete separation happens at resonance, as can be seen on the middle diagram of Fig. 4 (s_{min} becomes zero).

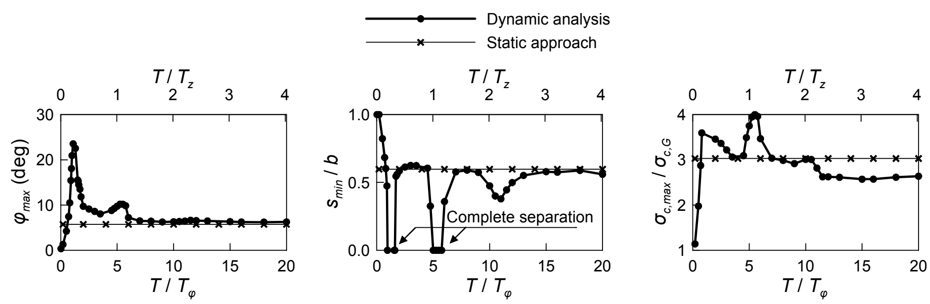

Figure 4. Effect of the excitation period on the response

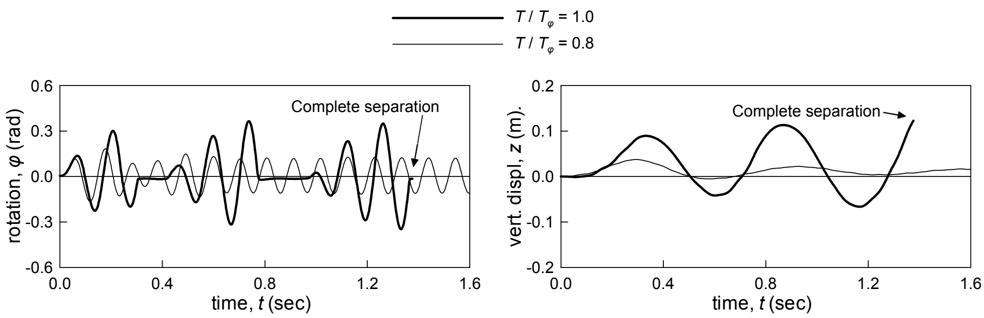

Figure 5. Time-histories of the rotation and the vertical displacement for T/T_φ=1.0 and T/T_φ=0.8

The response of the system for T close to T_φ is better shown in Fig. 5, in which the time-histories of the rotation and the vertical displacement are given for T/T_φ=0.8 and T/T_φ=1.0. In the latter case, complete separation occurs. It is interesting to note that, although resonance concerns the rocking response (T=T_φ), the vertical displacements also increase significantly, compared to the case of T/T_φ=0.8, showing transfer of energy from the rocking mode to the vertical mode. This transfer of energy between the two modes is a known phenomenon for rocking blocks on an elastic foundation (Psycharis and Jennings, 1985). It is interesting to note that in our case, complete separation happens when the vertical displacement becomes large and not when the angle of rotation is large. Actually, complete separation occurs for a quite small value of φ, close to zero. Also, note that at resonance (T/T_φ=1.0), the rocking response becomes irregular and is close to zero during long intervals of time.

The static approach is independent of the frequency of the excitation and, thus, the corresponding curves on the plots of Fig. 4 are straight lines. It should be mentioned, however, that, in this analysis, resonance concerns only the frequencies of the system related to the vibration of the foundation and not to the response of the superstructure.

Superstructure loads

The static and the dynamic terms of the superstructure loads (axial force and base moment of column) are considered separately. The moment due to the gravity loads, M_G, is measured by the eccentricity e_G, defined as: $e_G = M_G/(N_G + mg)$. Similarly, the moment due the dynamic response of the superstructure, M_E, is measured by the eccentricity e_E, defined by the ratio of the moment M_E to the static axial force: $e_E = M_E/(N_G + mg)$. Note that e_E is not the real eccentricity of the dynamic part of the moment, because the dynamic part of the axial force, N_E, is not included in the denominator. This is done in order to normalize M_E with a quantity that is independent of the other dynamic loads. The eccentricities e_G and e_E are normalized with respect to the width of the base, b, and thus the ratios e_G/b and e_E/b are considered in the parametric investigation.

Concerning the axial force, the effect of the ratios N_G/mg and N_E/N_G is investigated. The first ratio compares the static axial force from the superstructure to the weight of the footing, while the latter gives the relation between the dynamic and the static part of the axial force. Since the dynamic axial force can be compressive or tensile, this ratio attains both positive and negative values.

The results concerning the static loads (not shown here) were obtained for values of the ratios e_G/b and N_G/mg that are expected to be encountered in practice ($0 \le e_G/b \le 0.15$ and $2 \le N_G/mg \le 15$) and show that the effect of these parameters on the response is not significant. In both cases, the accuracy obtained by the static approach was satisfactory, except of the expected underestimation in the angle of rotation.

In the contrary, the dynamic loads affect the response in a great extend, as shown in Fig. 6. The results concerning the dynamic eccentricity are given on the plots of the top row of Fig. 6 for values of e_E up

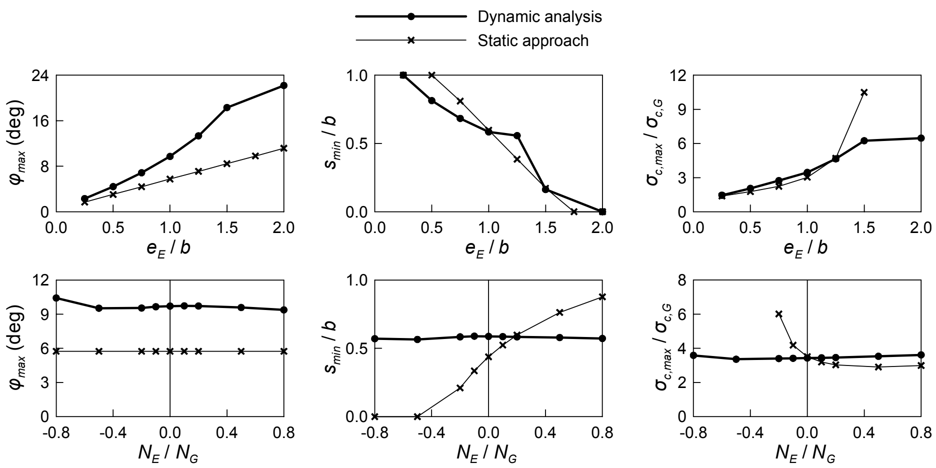

Figure 6. Effect of the dynamic eccentricity and the dynamic axial force on the response

to 2*b*. It is evident that the angle of rotation and the soil stress increase almost linearly with the dynamic eccentricity, while the length of contact decreases rapidly. The static approach predicts the results quite well for values of e_E/b up to 1.25. For larger eccentricities, it leads to an almost exponential increase in the soil stress, which is not observed in the dynamic analysis. In this sense, it can be said that the static approach may be very conservative for large eccentricities. It should be noted that complete separation occurs for $e_E/b \geq 1.75$ for both the dynamic and the static analyses. The points that are plotted on the diagrams of Fig. 6 for $e_E/b > 1.75$ correspond to the maximum or the minimum value of each parameter before complete separation happens.

An interesting result of this investigation concerns the effect of the dynamic part of the axial force, N_E and is illustrated on the plots of the bottom row of Fig. 6. The dynamic analysis shows that the minimum length of contact and the maximum soil stress are practically independent of the value of N_E and its sign. This happens because N_E affects significantly the vertical oscillations, but the rocking response is affected much less. Since the vertical displacements are generally small, the length of contact and the soil stress remain almost unaffected. Note that these results correspond to ω/ω_φ=0.5 and ω/ω_z=2.5, i.e. away from any resonance.

In the contrary, the static approach shows that the response is highly affected by N_E. For strong earthquake excitations, this force can be large, either positive (compressive) or negative (tensile), for some columns. In Fig. 6, results for N_E up to $\pm 0.8 N_G$ are shown; in practice, even larger values can be encountered. In such cases, if N_E is assumed negative (tensile) in the pseudo-static analysis, it leads to small lengths of contact and high soil stresses, showing that the dimensions of the footing should be increased. For the footing considered here, the length of contact becomes null and the soil stress reaches infinity for $N_E/N_G \leq -0.5$.

In practice, the actual sign of N_E that corresponds to M_E is not known, since these quantities are usually derived separately from a modal analysis, after an SRSS or a CQC combination of the modal responses. For this reason, the static approach is usually applied for a negative value of N_E (tensile), in favour of safety. The results presented here show that this procedure may lead to a very conservative design. The use of positive values for N_E (compressive) gives results much closer to the dynamic response but to the unsafe side (see last diagram of the bottom row of Fig. 6 for positive values of N_E/N_G).

It should be noted that these conclusions are verified by the response of frames with uplifting footings to earthquake excitations (Paspatis, 2005). An example is presented in Fig. 7, in which the comparison of the dynamic analysis and the pseudo-static approach is made for a single-storey, single-

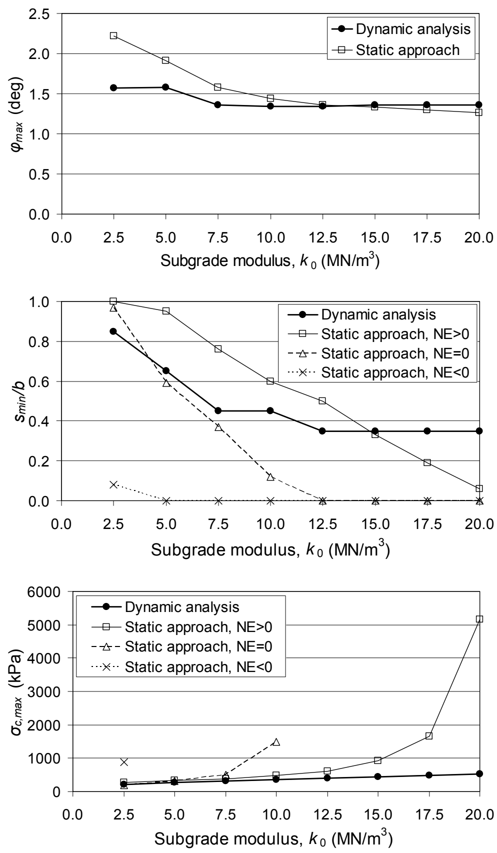

Figure 7. Comparison of the static approach to the dynamic analysis for a single-storey, single-bay frame with uplifting footings and for the El Centro, 1940 earthquake (after Paspatis, 2005)

bay frame subjected to the El Centro, 1940 earthquake. The results were obtained using SAP and assuming non-linear springs under the footings, which can only take compression, in order to capture partial uplift. In these plots, the maximum angle of rotation, the minimum length of contact and the maximum soil stress are given versus the subgarde modulus, k_0 of the soil. For s_{min} and $\sigma_{c,max}$, results for the pseudo-static approach are given for $N_E>0$, $N_E<0$ and $N_E=0$. The angle of rotation is independent of N_E. It is evident that the assumption of $N_E<0$ (tensile) is unrealistic, leading to the false conclusion that complete separation ($s_{min}=0$) occurs in almost all cases. The assumption of $N_E>0$ (compressive) underestimates the amount of uplift, but the soil stress is predicted well in most cases. The results for $N_E=0$ are between these two cases, being more conservative than $N_E>0$ and more realistic than $N_E<0$.

CONCLUSIONS

The dynamic response of footings sitting on an elastic tensionless Winkler foundation is examined. A parametric investigation on the effect of various dimensionless terms, including the foundation stiffness and damping, the superstructure stiffness, the frequency of the excitation and the magnitude of the superstructure loads, is performed for harmonic excitations. The investigation concerns the effect of each dimensionless parameter on the maximum rocking response, the minimum length of contact after uplift and the maximum stress of the soil, assuming that all the other parameters remain constant. The results of the dynamic analysis are compared to the ones of a simplified static approach, which is based on the maximum values of the applied loads to the footing. The most important conclusions can be summarized as follows:

The stiffness of the foundation, practically, does not affect the length of contact and the soil stress, as far as the ratio of the superstructure stiffness to the foundation stiffness is kept constant and resonance does not occur. Similarly, the damping of the foundation does not seem to be significant. In the contrary, the relation between the superstructure stiffness and the foundation stiffness is important.

The static approach can, in general, predict the response satisfactorily. Its accuracy, however, decreases significantly in two cases: if the foundation vibrations are close to resonance and if the dynamic parts of the moment and/or the axial force of the superstructure become large. In the first case, the static analysis underestimates, in general, the response. In the latter, the accuracy of the static analysis greatly depends on the sign of the dynamic part of the axial force: for tensile forces (N_E<0) it is unrealistically conservative; for compressive forces (N_E>0) the results are close to the ones of the dynamic analysis, but they might be against safety in some cases. The assumption of N_E=0 is more realistic than N_E<0 and it seems to be on the safety side, but it might also be quite conservative in some cases.

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