

NON-LINEAR SSI EFFECTS ON THE SEISMIC PERFORMANCE EVALUATION

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ABSTRACT

When an earthquake occurs, the surrounding soil and the structural elements can exhibit non-linear behaviour. Usually, only non-linear behaviour of structural elements is evaluated. But, actually, the soil reaches the limit of its linear elastic behaviour before the structural elements.

In general, the soil-structure interaction effects are assumed beneficial and thus ignored. Nevertheless, a more precise knowledge of the expected structural seismic response can allow reduce the cost of the structure and improve the earthquake engineering practice.

This work concerns the assessment of the effects of non-linear soil behaviour on the seismic demand evaluation. For this purpose, numerical simulations of non-linear dynamic analysis are performed in order to study the role of several parameters on the seismic performance evaluation. This paper presents a summary of the main findings.

Keywords: Seismic Performance Evaluation, Capacity Spectrum Method, non-linear Soil-Structure Interaction

INTRODUCTION

In the present earthquake engineering practice, the capacity spectrum method is a widely used strategy for seismic performance evaluation of existing and new structures. Nevertheless, usually the effects of dynamic soil-structure interaction (SSI) and the non-linear behaviour of the surrounding ground are neglected.

Some simplified procedures taking into account the dynamics SSI effects on the determination of the design earthquake forces and the corresponding displacements exist. For instance, FEMA 356 [FEMA 356] and ATC-40 [ATC 40] documents give some provisions to include ground flexibility in the structural analysis model. Recently, FEMA 440 [FEMA 440] draft document proposes some techniques to improve the traditional non-linear static seismic analysis. Concerning soil-structure interaction effects, this document presents procedures to take into account kinematical effects as well as foundation damping effects. Kinematical effects are related to filtering the ground shaking transmitted to the structure i.e. a modification factor to input motion is applied. Foundation damping is combined with the structural damping to obtain a revised damping for the system. All these procedures are based on traditional soil-structure interaction expressions with linear-elastic soil behaviour assumption. However, it is well-known that the limit of linear-elastic soil behaviour is very low ($\gamma \leq 10^{-5}$). This strain limit is normally surpassed during a real motion.

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In this work the influence of SSI effects on the seismic performance evaluation is investigated. For this purpose, numerical simulations of pushover tests and non-linear dynamic analyses (i.e. non-linearity of the soil and the structure behaviour) are performed in order to study the role of several parameters on the seismic performance evaluation. This parametric study concerns the mechanical properties of the soil foundation (e.g. $V_{s,30}$ and fundamental soil deposit frequency) and the structure (i.e. fundamental period, effective height and mass) as well as the characteristics of the input motion (i.e. amplitude and frequency content). Thus, several 2D finite element computations are carried out using non-linear elastoplastic models to represent both the soil and the structure behaviour. Results obtained by simplified computations performed following a two-step approach (it will be described below), are compared with ones obtained from fully non-linear time-history finite element modeling analyses.

These results allow to identify and to quantify the differences between the two approaches. Thus, it is possible to establish the situations for which the approximate techniques might tend to overestimate or underestimate the displacement demand. We present a summary of the main findings including some recommendations to consider in a performance evaluation following the Capacity Spectrum Method (CSM).

PROPOSED APPROACHES

In order to investigate the effect of non-linear soil behaviour on seismic demand evaluation, a comparative dynamical analysis is carried out. First, a complete finite element model including soil and structural non-linear behaviour is used to assess the effect of non-linear dynamic soil-structure interaction on the structural response. Secondly, a two-step approach is carried out consisting in: a non-linear 1D wave propagation problem is solved for a simple soil column of the foundation soil. Next, the obtained free field motion is imposed as ground motion to a fixed base structural model. The two approaches are sketched in Figure 1.

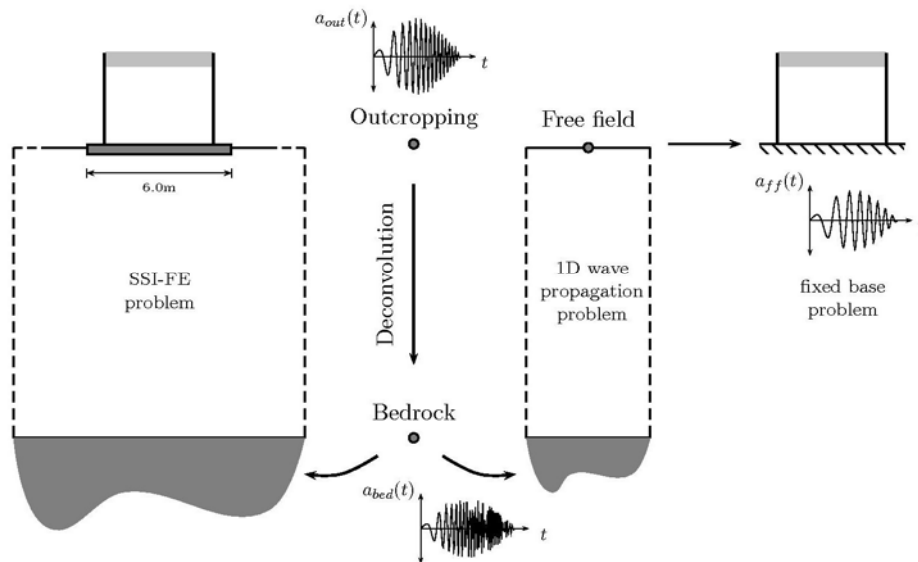


Figure 1. Summary of proposed approaches

The analysis is carried out for several non-linear SDOF models, with fundamental period varying from 0.1 to 0.4 seconds. The mass and height of each SDOF is obtained with typical weight and height values, relating its fundamental period to the number of levels of structure. The same infinitely rigid shallow foundation is considered for all SDOF's.

In order to simulate the soil foundation, two non-linear homogenous dense sandy soil deposits were used. The first one in dry condition and the second one fully saturated. The depth of the bedrock is 30 meters. Four Europeans earthquakes are considered, scaled to different maximum outcropping acceleration values.

SOIL CONSTITUTIVE MODEL

The ECP's elastoplastic multi-mechanism model [Aubry et al., 1982; Hujeux, 1985], commonly called Hujeux model is used to represent the soil behaviour. This model can take into account the soil behaviour in a large range of deformations. The model is written in terms of effective stress. The representation of all irreversible phenomena is made by four coupled elementary plastic mechanisms: three plane-strain deviatoric plastic deformation mechanisms in three orthogonal planes and an isotropic one. The model uses a Coulomb type failure criterion and the critical state concept. The evolution of hardening is based on the plastic strain (deviatoric and volumetric strain for the deviatoric mechanisms and volumetric strain for the isotropic one). To take into account the cyclic behaviour a kinematical hardening based on the state variables at the last load reversal is used. The soil behaviour is decomposed into pseudo-elastic, hysteretic and mobilized domains.

The model's parameters of the soil are obtained using the methodology suggested by Lopez-Caballero et al. [Lopez-Caballero et al., 2003]. In order to verify the model's parameters, the behaviour of the sand must be studied by simulating drained (DCS) and undrained cyclic shear tests (UCS). The Figure 2a shows the responses of these DCS tests obtained by the model of the sand at an effective stress of 100kPa. The tests results are compared with the reference curves given by Iwasaki [Iwasaki et al. 1986].

In saturated conditions, the evolution of pore pressure can be observed during the UCS. The Figure 2b shows the pore pressure evolution for a stress controlled shear test with the same model's parameters. The increment of pore pressure reduces the effective stress inducing cyclic mobility without liquefaction.

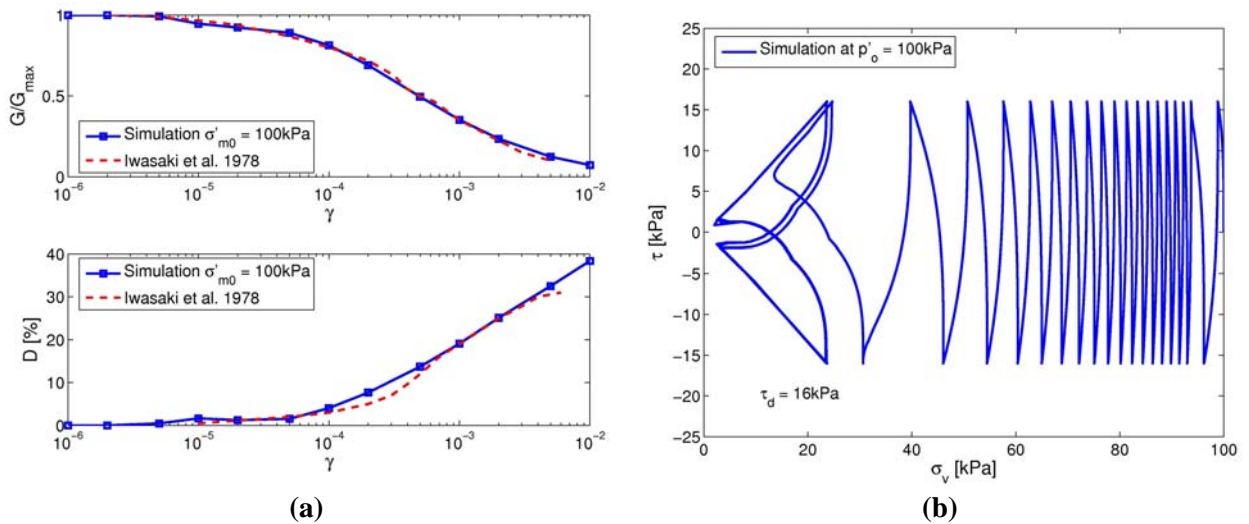


Figure 2. Soils test simulations

Two levels of water table were considered: at bedrock level (dry) and at surface level (saturated or wet). The shear wave velocity of the soil increases with depth (Figure 5a). The shear wave velocity profile gives an average shear wave velocity in the upper 30m ($V_{s,30}$) of 232.8 (m/s) for dry conditions and equal to 204.3 (m/s) for saturated condition, corresponding to a site category C of Eurocode 8 (deep deposit of dense or medium dense soil) in both cases.

STRUCTURAL MODEL

The typical one-story frame chosen to represent each SDOF structure is shown in Figure 3a. The mass of the building is assumed to be uniformly distributed along beam elements and the columns are supposed massless. Non-linear material behaviour is taken account through a elastic-perfect plastic strain-stress relation. Figure 3b shows a normalized moment-curvature ($M-\Psi$) diagram obtained from the computation of a simply fixed beam with this model behaviour. As can be noticed, the stiffness decreases when the elastic limit is reached (at M_y or Ψ_y), and under load reversal the curve forms hysteresis loops. The maximum resisting moment remains constant under increasing deformation and the member rotates as a hinge with this constant resisting moment. The value of stress yield is supposed to be the same for all computations.

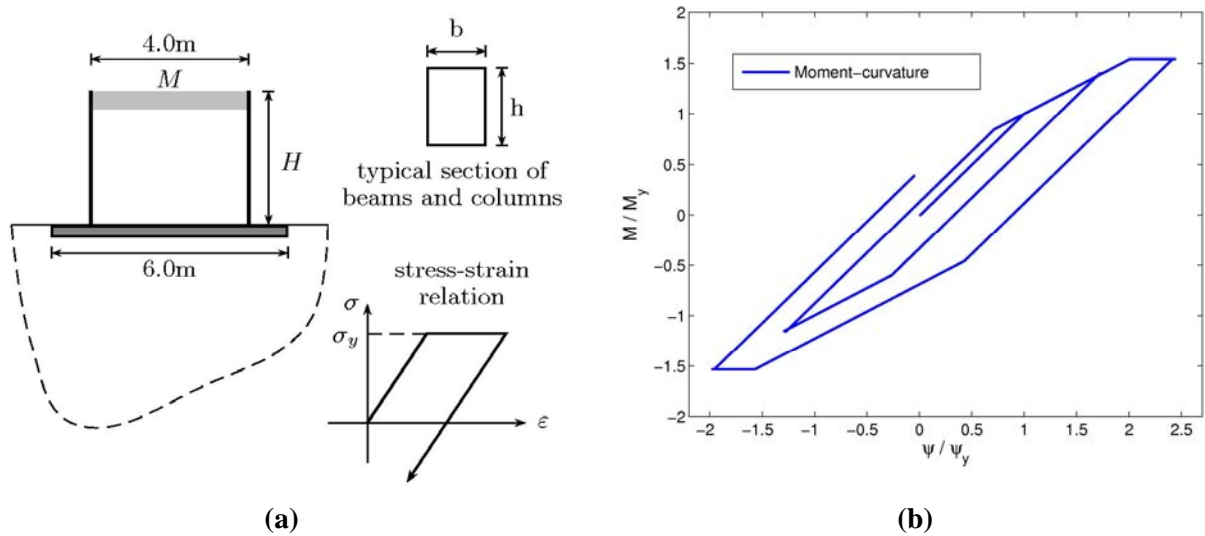


Figure 3. Structural model description

The elastic modulus (E) of structural elements is equal to 25.5GPa. The mass and height of each SDOF is obtained with typical weight and height values. A typical value of 20000kg is assumed for each level. A constant value of 2.5m is considered for each interstory height. Thus, the equivalent SDOF corresponding to a building of “n” levels is computed assuming a mass of 20000*n (kg), a fixed base period of $T_0 = n/10$ (s) and equivalent height $H = 2/3 * 2.5n$ (m), and finally solving for b and h (lateral stiffness). The Table 1 shows the basic properties for the used SDOF.

Table 1. Properties of equivalent SDOF

T_0 (s)	Mass (kg)	Equivalent height (m)
0.10	20000	1.66
0.15	30000	2.50
0.20	40000	3.33
0.25	50000	4.17
0.30	60000	5.00
0.40	80000	6.67

A viscous damping of $\beta_i=0.02$ was considered for all computations. The same infinitely rigid shallow foundation with a characteristic length of 6 (m) was used for alls SDOF.

INPUT EARTHQUAKE MOTION

The used seismic input motions are the acceleration records of Friuli earthquake - San-Rocco site (Italy-1976), Superstition Hills earthquake - Supers. Mountain site (USA-1987), Kozani earthquake (Greece - 1995) and Aegion earthquake (Greece - 1995). The frequency content was characterised

with the mean period (T_m) [Rathje et al., 1998] (Table 2). All signals are consistent with the response spectra of Type A soil of Eurocode8.

Table 2. Used input earthquake motions

Earthquake Records	$T_{m,out}$ (s)
Kozani (Greece)	0.28
Superstition Hill (USA)	0.38
Friuli (Italy)	0.46
Aegion (Greece)	0.56

FINITE ELEMENT APPROACH (SSI-FE)

The Finite Element model is composed of: the structure, the soil foundation and a part of the bedrock. The considered structure is a one-story, one bay frame. The 30 (m) thick homogenous soil deposit is modelled by 4 node linear elements. In the bottom, a layer of 5 (m) of elastic bedrock is added to the model. The finite element mesh used for modelling this problem is showed in Figure 4. Plane strain condition is assumed for the soil deposit and the bedrock.

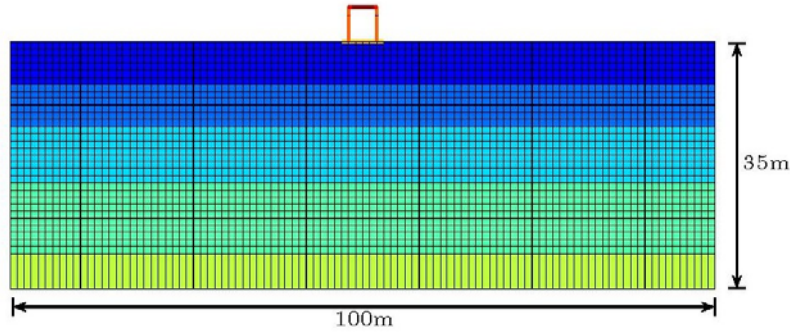


Figure 4. Finite element mesh

For the bedrock's boundary condition, paraxial elements simulating a "deformable unbounded bedrock" have been used. The incident waves, defined at the outcropping bedrock are introduced into the base of the model after deconvolution. In the analysis, as the lateral limits of the problem are considered to be far enough so that periodic conditions are verified. Then, only vertically propagating shear waves are studied resulting in the free field response. Thus, equivalent boundaries have been imposed on the nodes of these boundaries (i.e. the normal stress on these boundaries remains constant and the displacements of nodes at the same depth in two opposite lateral boundaries are the same in all directions). Thus, the obtained movement at the bedrock is composed of the incident waves and the reflected signal. The computations are carried-out in the time domain.

The simulations were performed with the Finite Element code GEFDYN [Aubry et al. 1985; Aubry and Modaressi 1996]. A numerical validation of the soil-structure interaction phenomenon assuming linear elasticity behaviour for both the soil and structure was performed comparing FE computations with a numerical BE-FE code [Saez et al. 2006].

TWO-STEP APPROACH

The first step is to solve a non-linear one-dimensional wave propagation problem for a simple soil column. The mesh consists of one column of solid elements obeying the same constitutive model as in the SSI-FE approach. The same boundary conditions have been imposed. The incident waves, defined at the outcropping bedrock are introduced into the base of the model after deconvolution. In the second step, the obtained free field motion is imposed as ground motion to a fixed base structural model. This two-step approach neglects all SSI effects, but takes into account the effect of non-linearity of both soil and structure.

SOIL ANALYSIS AND RESULTS

In order to define the input motion for the two-step approach (corresponding to first step), a free field dynamic analysis of the soil profile, was performed. The response of the free field soil profile was analysed for the four earthquake records (Table 2) as outcropping input. The Figure 5b shows the simulation values and a tendency curve representing the PGA (Peak Ground Acceleration) at surface with respect to maximum acceleration on rock ($a_{\max \text{ bedrock}}$). This curve is compared with the one corresponding to an AB deep soil profile according to the classification proposed by Dickenson and Seed [Dickenson and Seed, 1996]. It is possible to see that for weak base acceleration the behaviour of both soil deposits is similar. It is noted that an amplification of the ground response for moderate range of $a_{\max \text{ bedrock}}$ is obtained. For strong base acceleration the soil softening attenuates the seismic motion. In saturated conditions, the water acts as a frequency filter and the soil de-amplifies the input motion for $a_{\max \text{ bedrock}}$ large values.

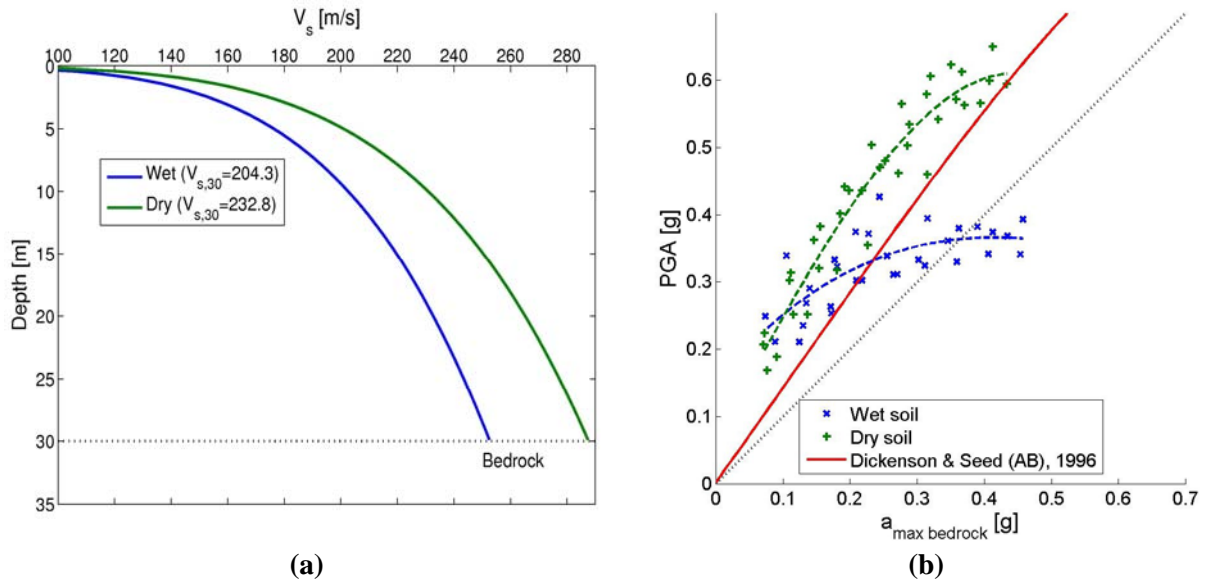


Figure 5. Effect of the water on soil response

The influence of the inelastic behaviour of the soil deposit on the structural response can be directly related with to pseudo-acceleration response spectra (PSA) at the structure base level. The comparison between outcropping and obtained structure's base normalized PSA for different acceleration levels using Friuli earthquake and $T_0=0.3$ (s) SDOF are showed in Figure 6., for dry and saturated conditions.

The Figure 6 indicates that the stiffness degradation of the soil induces a reduction of short period (i.e. periods less than about 0.4s) spectral accelerations. However, for periods larger than 0.4 (s), surface spectral accelerations are greater than outcropping accelerations. Taken into account that the fixed base fundamental structural period of SDOF is $T_0=0.3$ (s) in this case, the spectral accelerations are slightly smaller at this range of periods. Consequently for this record smaller displacements must be obtained.

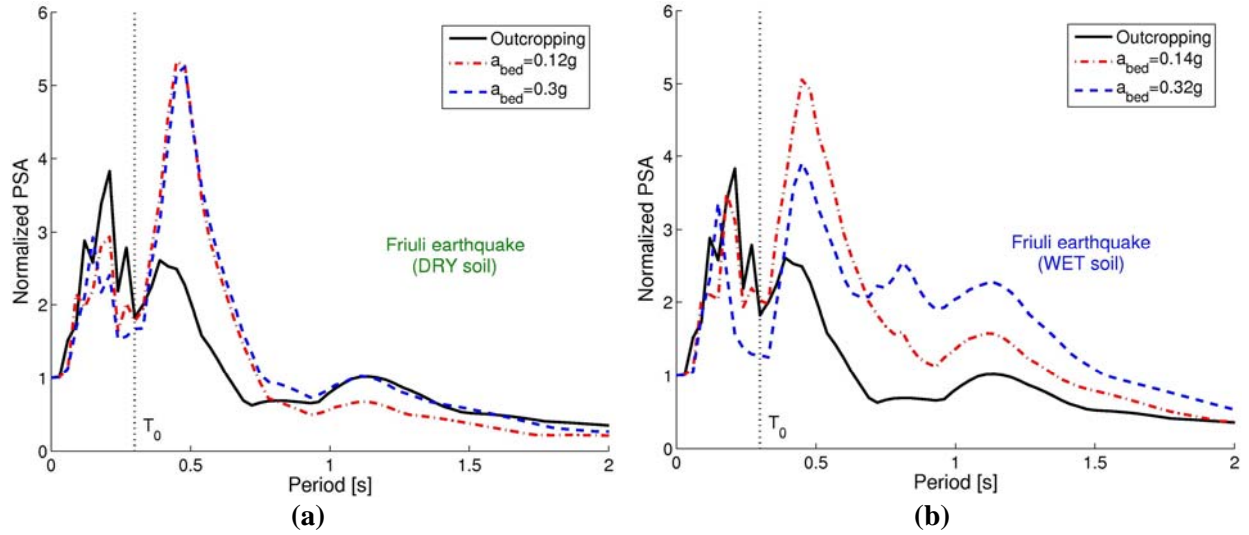


Figure 6. Effect of the water on PSA

In dry condition (Figure 6a), the frequency content of the surface PSA does not change significantly with the bedrock amplitude. However, in saturated condition the frequency content varies significantly (Figure 6b). According to previous results (Figure 5b), the wet soil undergoes a significant rigidity degradation with the acceleration increase. Then, this degradation modifies the soil fundamental period and thus the frequency content of the acceleration.

For large periods, the spectral amplitude of saturated soil is greater than that of dry soil. This amplification of the saturated soil with respect to dry soil can be explained by the pore water pressure built up phenomenon properly simulated by the soil constitutive model. It can be noted that it is not possible to identify this feature of soil behaviour using a simplified approach such as equivalent linear method.

NON-LINEAR SSI ANALYSIS AND RESULTS

Concerning the seismic demand evaluation, the maximum top displacement and its corresponding base shear (in terms of spectral acceleration) are plotted for each studied SDOF structure following the two approaches for dry soil. For each SDOF, the corresponding capacity curve is also plotted (dashed lines in Figure 7a).

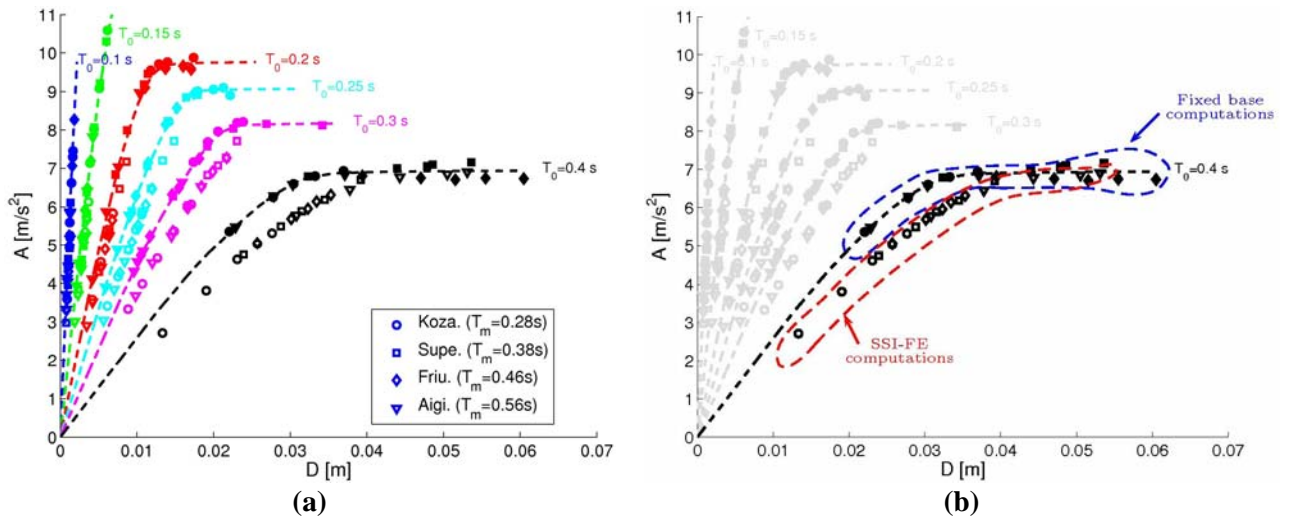


Figure 7. Summary of computations

To visualize the SSI effect on seismic demand evaluation it is possible to take for example the $T_0=0.4$ (s) fundamental period SDOF (Figure 7b). Solid points correspond to the two-step approach. Each point represents a response obtained by one input motion scaled to a specific value.

It is well-known that the stiffness degradation of the soil of the foundation introduces additional damping in the system, modifying the structural response. Additionally, radiation damping appears. According to our computations, the predicted top displacement value given by the two-step approach is conservative, i.e. larger than the obtained one in the SSI-FE approach.

Figure 7 shows that, even for relatively weak motion, the dynamic response of the structure is not placed on the pushover curve (hollow symbols in Figure 7). Then, the linear elastic limit of soil is surpassed inducing a significant variation of effective period of the total system and increasing the damping of the system. In order to explain this behaviour, it is possible to see the distribution of plastic strains in the neighboring soil of the structure during the Friuli earthquake scaled to 0.25g at outcropping acceleration.

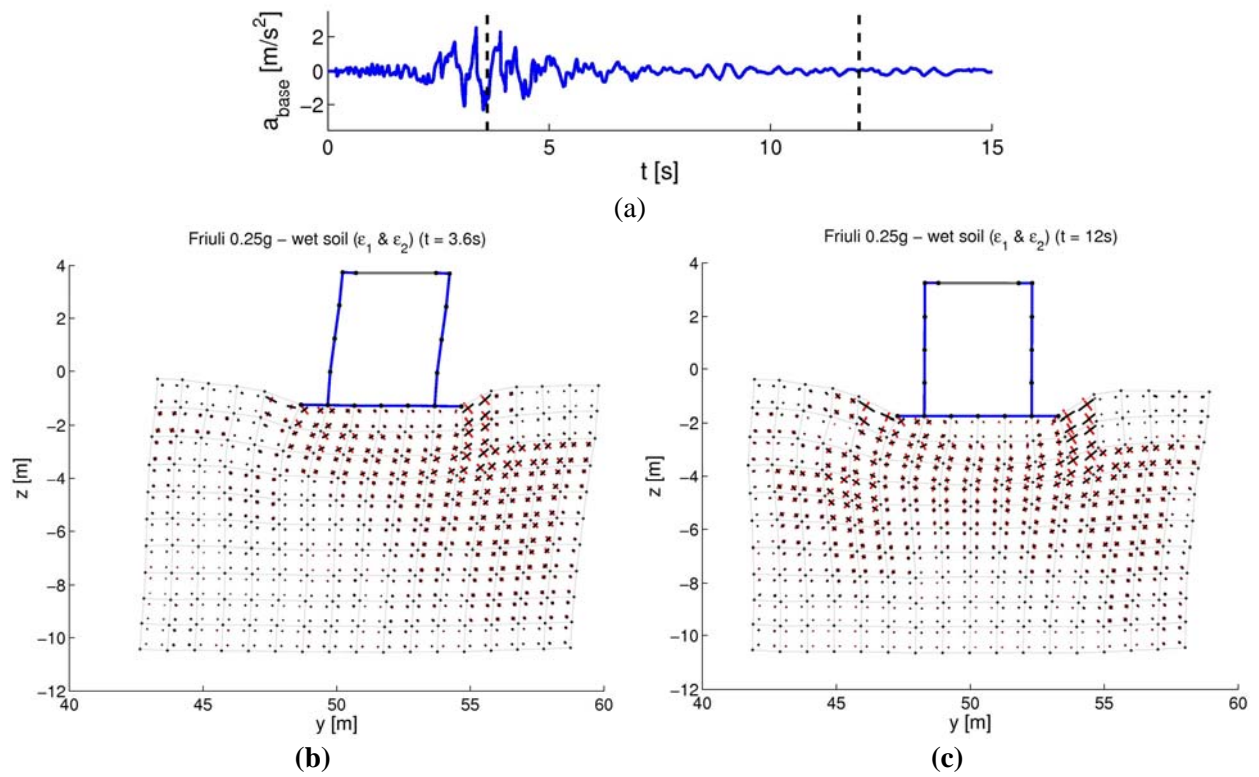


Figure 8. Principal strain directions and the deformed mesh (x50)

Figure 8 shows the principal strain directions with the same scale in two different steps of the analysis. After the first part of strong motion ($t=3.6$ s) the soil is extensively yielding, then for the subsequent part of the motion the stiffness and damping of the system differs considerably from the initial values. After the strong motion ($t=12$ s), an asymmetrical distribution of irreversible deformations is found. Permanent settlements, are also generated. This soil deformation induces a high material soil damping. This damping has a direct influence on the seismic response of the structure and it cannot be properly evaluated following a fixed based approach or even if elastic SSI is taken into account. Therefore, the total seismic demand is highly controlled by the surrounding non-linear soil behaviour. For motions able to induce damage into a structure, the soil behaviour will be certainly non-linear.

To complete the previous analysis, the saturated soil results are also included in Figure 9a for the $T_0=0.4$ (s) SDOF. The tendency of the results is the same. The computed results are clearly aligned following a greater effective period. This value of T_{eff} can be calculated from a linear fitting. After this approximately linear portion, the computed values of seismic demand approach asymptotically the

fixed base capacity curve. The plateau of the curve does not change because it depends only upon the strength of column elements. For a given motion, it can be noticed that the Performance Point (P.P.) from two-step dynamic computation is approximately placed on the capacity curve, what indicates that capacity spectrum method is adequate for fixed base analysis. However, when SSI effects are taken into account, the P.P. from SSI-FE dynamic computation is placed approximately on the modified capacity spectrum (Figure 9b).

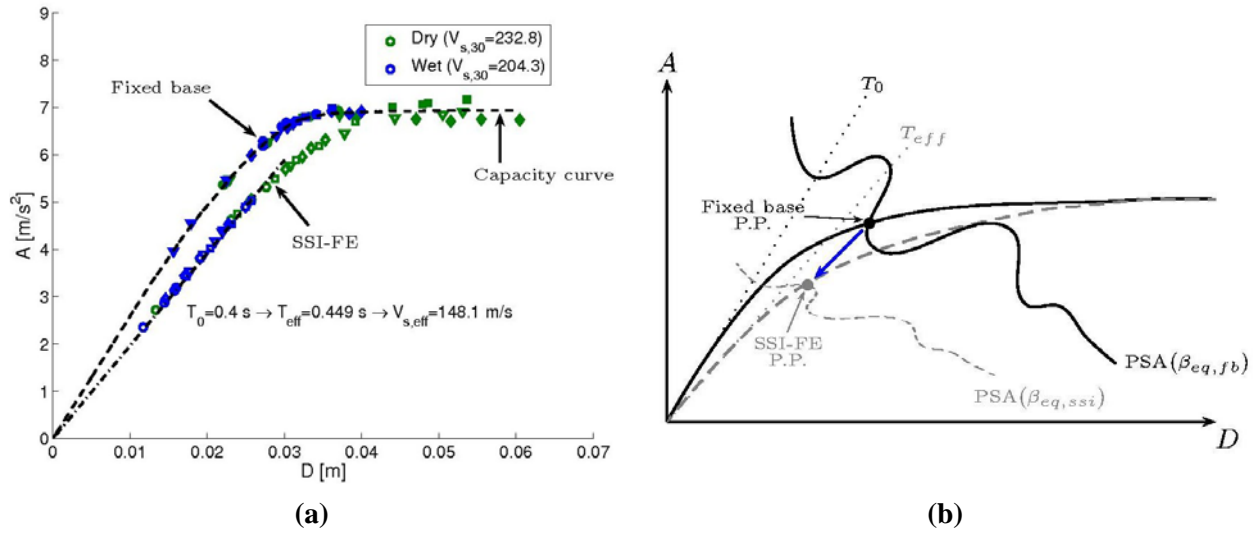


Figure 9. Summary of results.

Period lengthening due to SSI

The computed effective period (T_{eff}), may be related to the height (h), mass (m) and foundation characteristic length (a) of the SDOF structure by traditional linear elastic soil-structure interaction expressions for rigid shallow foundations. With these expressions, an effective shear wave velocity can be computed ($V_{s,eff}$):

$$\left(\frac{T_0 \cdot V_{s,eff}}{2\pi h}\right)^2 = \frac{\frac{a^2(2-\nu)}{h^2} + 3(1-\nu)}{\frac{8\rho a^3}{m} \left(\left(\frac{T_{eff}}{T_0}\right)^2 - 1\right)} \quad (1)$$

where ν is the Poisson ratio and ρ is the mass per unit volume of the soil.

For this case, the effective shear wave velocity corresponds approximately to two thirds of $V_{s,30}$ Eurocode 8 parameter. Repeating the computations for the other structures, it is possible to obtain for each SDOF the value of effective period and effective shear wave velocity (Figure 10).

According to Figure 10a, it is possible to conclude that non-linear soil-structure interaction effects seem important only for structures with periods placed between the two first elastic periods of the soil deposit (T_{soil}^1 and T_{soil}^2). For periods larger than the second period of soil the effective periods approach quickly that of the fixed base value. The ratio between the fixed base value and effective value is near to 90% for this type of soil.

From Figure 10b, it can be noticed that the effective shear wave velocity is approximately constant for structures with fundamental periods between the two first ones of the soil. This value can be considered like approximately constant and equal to two third of $V_{s,30}$ Eurocode 8's parameter. Then, according to our results, a typical value of $2/3V_{s,30}$ into traditional elastic SSI relations can be used to compute an effective period for structures placed on soils C supported by a rigid shallow foundation.

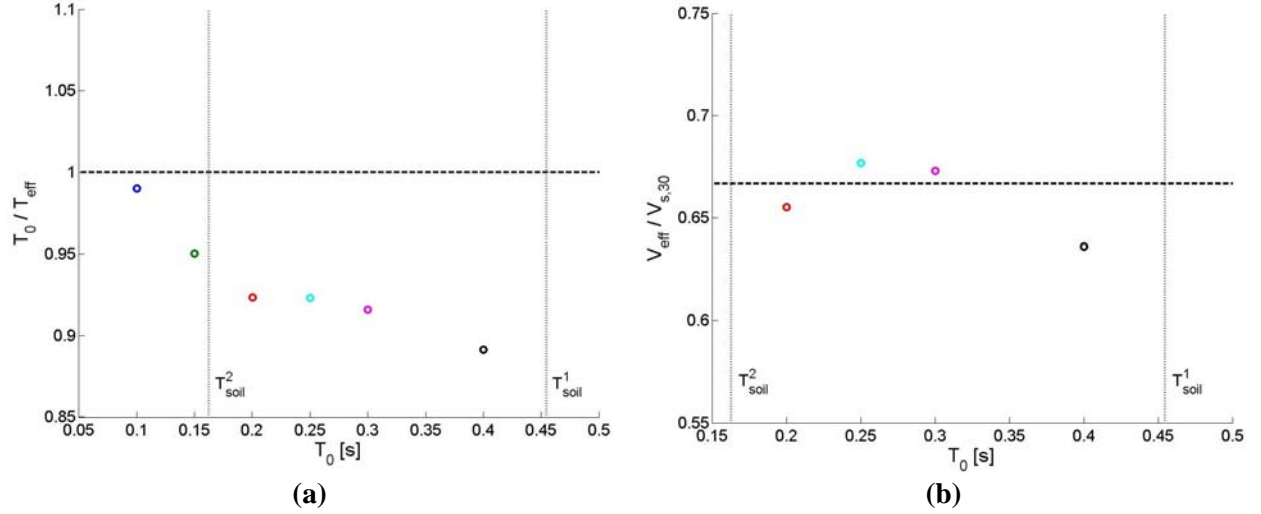


Figure 10. Effective period and shear wave velocity values

Structural damping quantification

The application of the CSM procedure, implies the computation of an equivalent viscous damping coefficient at the performance point (β_{eq}). This parameter includes the inherent structural damping (β_i) and the damping related to the damage of the structure (β_0).

A bilinear representation of the capacity spectrum is constructed following ATC-40 guidelines to estimate β_0 . For fixed base computations, the dynamic fixed base maximal response is taken as the performance point. The values of β_0 are computed using fixed base capacity spectrum (solid magenta symbols on Figure 11a). For SSI-FE computations, the capacity spectrum curve fitted using the obtained results of the dynamic SSI computations was used. With this capacity spectrum, the equivalent viscous damping β_0 values are also computed using the bilinear approximation suggested by ATC-40 (hollow magenta symbols on Figure 11a).

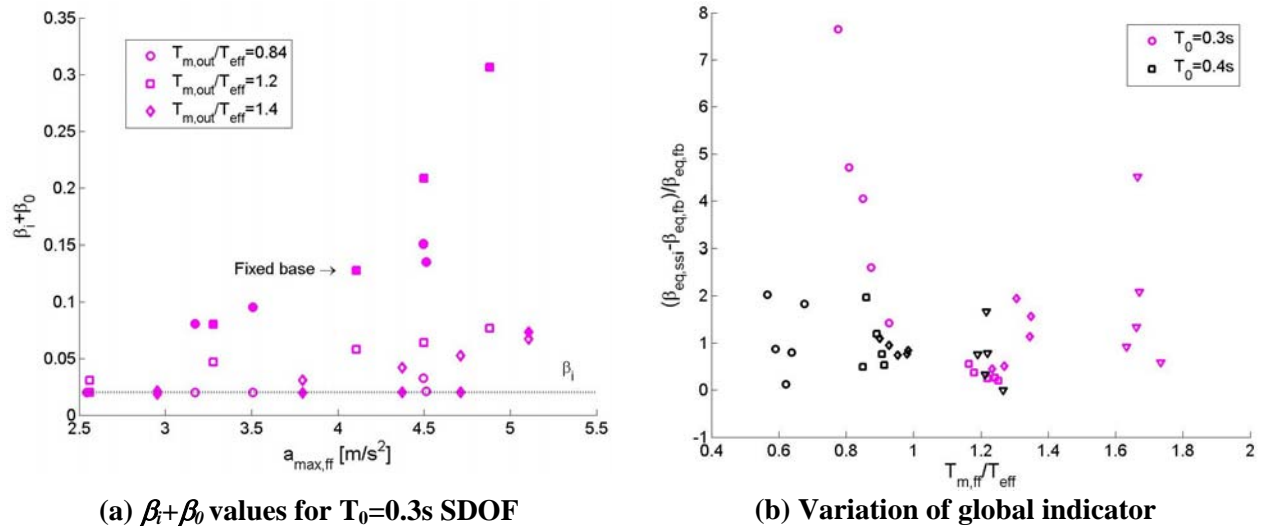


Figure 11. Equivalent damping

It can be noticed that the damping developed in the structure is significantly reduced when SSI effects are included in computations. According to our results, for motions with a frequency content near to the fundamental period of the fixed base structure (i.e. $T_{m,out}/T_0 \approx 1$) the damping attempts a maximum, i.e. a higher level of damage. The damping added to the system by nonlinear soil behaviour increases

the energy dissipation mechanisms, then the expected damage in the structure is reduced. When the ratio $T_{m,out}/T_{eff}$ is near to 1.4, the structural behaviour for fixed based condition is approximately elastic. But, when SSI effects are taken into account, the structure develops nonlinear behaviour and undergoes damage. In this case, the lengthening of fundamental period approaches the effective period value to resonance condition and induces plasticity in the structure for moderate values of acceleration thus increasing the damping.

Global damping indicator

When an elasto-plastic constitutive model is used for the soil, it is not easy to evaluate a global value of hysteretic damping developed in the soil during a motion. In fact, that is shown in Figure 8, the plasticity distribution in the surrounding soil is not homogenous and consequently the hysteretic damping changes from one point to other. In order to obtain a global indicator of the SSI effects on damping, an equivalent viscous damping β_{eq} is computed from dynamic maximum response. For each dynamic computation, the required damping β_{eq} for the elastic PSA of the free field acceleration motion is computed to match the dynamic response with a CSM procedure: $\beta_{eq,fb}$ for fixed base and $\beta_{eq,ssi}$ for SSI-FE approach. The graphic meaning of these values is presented on Figure 9b.

Figure 11b shows the variation of the global damping indicator $((\beta_{eq,ssi} - \beta_{eq,fb}) / \beta_{eq,fb})$ for $T_0=0.3s$ and $0.4s$ SDOF structures in terms of the ratio between the free field mean period ($T_{m,ff}$) and the computed effective period of the structure ($T_{m,ff}/T_{eff}$). It can be noticed that the variation of the indicator can be up to twice the fixed base damping. However, for many cases there is no important variation of the damping $\beta_{eq,ssi}$ compared to the fixed base value ($\beta_{eq,fb}$). These results highlight the role of the frequency content on the seismic response when the SSI effects are taken into account.

In general it is difficult to identify global tendencies from the results, specially concerning the damping added to the system by the soil (material and radiation damping). The major challenge to quantify the non-linear SSI effects in seismic demand evaluation is to predict an accurate global damping taking into account several parameters related to SSI phenomena. Further investigations in this way will be needed in order to obtain more general conclusions concerning the influence of different SSI effects on the seismic response.

CONCLUSIONS

The influence of the inelastic behaviour of soil deposit on the amplification of ground seismic accelerations and on the soil-structure interaction effects has been highlighted in this work. The main conclusion of this study is that the soil-structure interaction with a non-linear soil model varies significantly the response of the structure with respect to one with fixed base condition. Then, the simple procedures specified in design codes are not sufficient to asses properly the local soil influence on the structural response.

It is well-known that the soil exhibits an elastic behaviour only in a very small range of distortion. This range is certainly exceeded for a motion able to induce inelastic deformations in a structure. Thus, a coupled approach using non-linear structural behaviour with linear soil hypothesis is not consistent. In fact, the results show that when non-linear SSI is properly taken into account, the seismic demand is not on the capacity curve.

A first approximation for T_{eff} may be obtained with $2/3V_s,30$ and with traditionally elastic SSI expressions. Nevertheless, the major challenge to quantify the non-linear SSI effects in seismic demand evaluation is to predict an accurate global damping, able to be related to a simpler approach.

The results of the study illustrate clearly the importance of accounting properly the non-linear soil behaviour. In this case, the non-linear SSI has a favourable effect related to decreasing of the maximum top displacements and base forces. However, the non-linear SSI could increase or decrease

the seismic demand depending on the type of the structure (e.g. m , h and T_0), the input motion (e.g. T_m), and dynamic soil properties (e.g. T_{soil} , V_s , 30). Furthermore, there is an economic justification to take into account the modification effects due to non-linear soil behaviour. Further investigations in this way will be needed in order to obtain more general conclusions for diverse structure and soil typologies.

ACKNOWLEDGEMENTS

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REFERENCES

- Applied Technology Council (ATC). “Seismic Evaluation and Retrofit of Concrete Buildings”, ATC 40, Report N° SSC 96-01, 1996.
- Aubry D, Hujeux J.C., Lassoudiere F, Meimon Y. “A double memory model with multiple mechanisms for cyclic soil behaviours”. In: International Symposium on Numerical Models in Geomechanics, Zurich, vol. 1, pp 3-13, 1982.
- Aubry D, Chouvet D, Modaressi H, Modaressi A. “GEFDYN 5: Logiciel d’analyse du comportement statique et dynamique des sols par éléments finis avec prise en compte du couplage sol-eau-air”. Rapport scientifique, Ecole Centrale Paris, 1985.
- Aubry D, Modaressi A. “GEFDYN, Manuel Scientifique”, Ecole Centrale Paris, 1996.
- Dickenson, S.E. & Seed, R. “Non-linear Dynamic Response of Soft and Deep Cohesive Soil Deposits”, Proc. of the International Workshop on Site Response Subjected to Strong Earthquake Motions, Yokosuka, Japan, vol. 2, 67-81, 1996.
- FEMA 356, “Prestandard and Commentary for the Seismic Rehabilitation of Buildings”, American Society of Civil Engineers for the Federal Emergency Management Agency, 2000.
- FEMA 440, “Improvement of Nonlinear Static Seismic Analysis Procedures (Draft)”, Applied Technology Council (ATC-55 Project) and by the American Society of Civil Engineers for the Federal Emergency Management Agency, 2004.
- Hujeux J.C. “Une loi de comportement pour les chargements cycliques des sols”, Génie Parasismique, Presse ENPC, 1985.
- Iwasaki T., Tatsuoka F. and Takagi Y. “Shear moduli of sands under cyclic torsional shear loading”, Soils and Foundations, vol. 18, n°1, 39-56, 1978.
- Lopez-Caballero F., Modaressi A. and Elmi F. “Identification of an elastoplastic model parameters using laboratory and in-situ tests”. Deformation Characteristics of Geomaterials ISLyon 03., 1183-1190, 2003.
- Rathje E.M. et al. “Simplified frequency content estimates of earthquake ground motions”, J. of Geotechnical and Geoenvironmental Engineering, 124 (2), 150-159, 1998.
- Saez E., Lopez-Caballero F. and Modaressi A. “Effects of SSI on the Capacity Spectrum Method”, Proc. First European Conference on Earthquake Engineering and Seismology, id 1073, 2006.