

EFFECTS OF SOIL-STRUCTURE-EARTHQUAKE-INTERACTION ON SEISMIC SOIL LIQUEFACTION TRIGGERING

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ABSTRACT

Although there exist some consensus regarding seismic soil liquefaction assessment of free field soil sites, estimating the liquefaction triggering potential beneath building foundations still stays as a controversial and difficult issue. Assessing liquefaction triggering potential under building foundations requires the estimation of cyclic and static stress state of the soil medium. As part of these ongoing studies, structural-induced shear stresses under building foundations subjected to cyclic loading are attempted to be estimated. For this purpose, series of 3-D numerical simulations have been performed on generic soil-structure sites. A new controlling parameter, β , was defined to assess the effects of soil-structure-earthquake-interaction (SSEI) on liquefaction triggering. It is concluded that i) zones extending 0.2 width inside and 0.8 width outside of the edges are found to be relatively more critical from liquefaction triggering point of view as compared to the liquefaction potential of the zone in the vicinity of the foundation centerline, ii) either positive or negative, effects of the presence of the structure on liquefaction triggering potential of foundation soils diminishes rapidly at depths below 0.5 width and becomes almost none beyond the depths of 1.5 B, iii) the presence of the overlying structure decreases liquefaction triggering potential for systems with a β value greater than 1.4. More specifically, iv) for β values in the range of 0.5 to 1.0, due to the presence of the structure, CSR values corresponding to the soil, structure and earthquake interacting system (i.e.: CSR_{SSEI}) are estimated to be twice as much the free field soil values at the ground surface, v) for the β values in the range of 1.5 to 2.0, CSR_{SSEI} value is 25 to 40 % less than the free field values.

Keywords: soil-structure-earthquake interaction, cyclic stress ratio, liquefaction, sliding, rocking

INTRODUCTION

Evaluation of the in situ cyclic shear stress time history induced within any soil element (or stratum) is a key component of any well-based method for assessment of the likelihood of “triggering” seismically-induced soil liquefaction. The seismically-induced cyclic shear stress time history is, in most analysis methods, normalized by some measure of the initial normal effective stress in the soil, to give rise to the earthquake-induced cyclic shear stress ratio (CSR). At most soil sites, the cyclic shear stresses acting on horizontal planes due to seismic loading are largely dominated by cyclic shear stresses induced by vertically propagating, or nearly vertically propagating, shear waves. This gives rise to the “simplified” procedure for evaluation of induced cyclic shear stresses at depth (Seed and Idriss, 1971). When this “equivalent uniform cyclic shear stress” is normalized by the initial effective overburden stress, the result is an estimate of the “equivalent uniform cyclic stress ratio” (CSR) as:

$$CSR = 0.65 \cdot \frac{a_{max}}{g} \cdot \frac{\gamma \cdot h}{\sigma'_v} \cdot r_d \quad (1)$$

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The stress reduction coefficient, r_d , is a function of site stratigraphy, soil properties, and characteristics of the "input" motions (excitations). It has a value of 1.0 by definition at the ground surface. Cyclic stress ratio; CSR was chosen as demand term for liquefaction triggering assessment of Seed et al (1984). Since then, numerous researchers, still adopting CSR as the demand term (Liao et al., 1988, Liao and Lum, 1998, Youd and Noble, 1997, Toprak et al., 1999, Juang et al, 2002, Cetin et al., 2004) have attempted to improve the accuracy of liquefaction triggering relationships. However, all these methods founded on "simplified procedure" suffer from a major limitation that they are all applicable to the liquefaction triggering assessment of free field soil sites. Direct applicability of these methods to foundation soils underlying structural systems is not possible, unless, in the estimation of CSR, structure-soil-earthquake interaction is properly addressed. More specifically, compared to a free field soil site, under static conditions, due to the presence of an overlying structural system i) overburden stresses acting on the underlying foundation soils are higher, ii) nonzero shear stresses act on the horizontal soil planes. Similarly during earthquake shaking, presence of iii) foundation elements (e.g.: footings or mat) forms usually a sharp impedance contrast which causes deviations from the free field vertical propagation pattern of shear waves (i.e. : kinematic interaction), iv) an overlying structure due its inertia will exert additional cyclic shear stresses on to foundation soils. Issues referred to in (i) and (ii) can be handled by introducing series of correction factors formerly known as K_α , and K_σ , respectively applied on CSR_{eq} as given in Equation (2).

$$CSR_{eq, \alpha=0, \sigma_v'=100kPa} = \frac{CSR_{eq, \alpha, \sigma_v'}}{K_\alpha \cdot K_\sigma} \quad (2)$$

K_α factors as a function of relative density, D_R , and α as defined in Equation (3) were introduced by Seed (1983) to assess the effects of initial static shear stresses on liquefaction triggering resistance. Since then number of researchers have studied this issue and have shown that K_α is mainly dependent on soil stiffness and confining stress conditions. A review of the literature was presented by Harder and Boulanger (1997) as part of NCEER 1997. More recently, Boulanger (2003, 1) proposed a new set of K_α corrections as a function of relative dilatancy index (ξ_R). Based upon currently available K_α corrections, it can be concluded that from only K_α point of view, due to the presence of an overlying structure, decrease in liquefaction triggering resistance of loose foundation soils is expected especially near the foundation edges. Figure 1 presents a typical α and Figure 2 shows K_α fields developed along the center line of a 4 story residential structure founded on a mat underlain by a sand layer with relative densities of 30 % and 70 %, respectively. As can be interpreted from Figure 2, K_α effects increase seismic demand (i.e. CSR) for "loose" soils.

$$\alpha = \frac{\text{Initial Static Shear Stress}}{\text{Vertical Effective Stress}} = \frac{\tau_{\text{static, horizontal}}}{\sigma_v'} \quad (3)$$

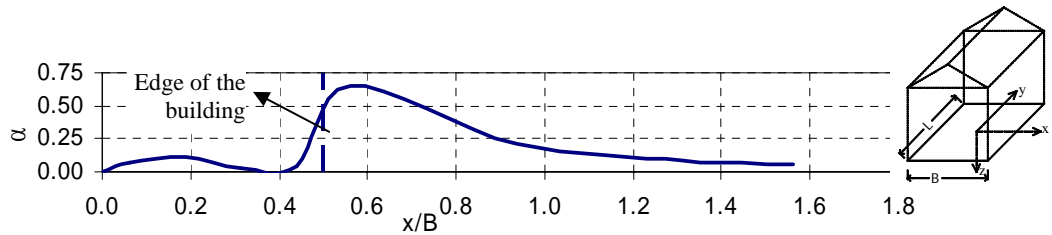


Figure 1. Variation of "α" along the width of the structure at $z/B = 0$ and $y/L = 0$

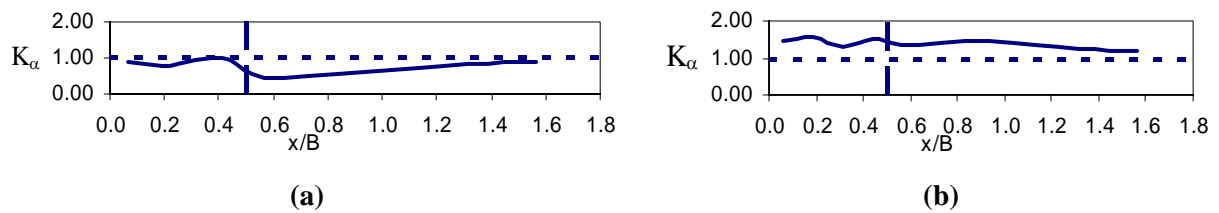


Figure 2. K_α along the width of the structure, for (a) $D_R = 30\%$, (b) $D_R = 70\%$ $z/B = 0$ and $y/L = 0$

The additional effect of reduction of normalized liquefaction resistance with increased initial effective overburden stress (σ'_v) has been demonstrated by means of laboratory testing, and is a manifestation of “critical state” type of behavior (suppression of dilatancy at increased effective confining stress). K_σ concept has been addressed by a number of researchers (e.g.: NCEER Working Group, Youd et al., 2001; Boulanger, 2003 (2). Cetin et al. (2004); Idriss and Boulanger, 2006). Equation (4) presents the recommendations of Cetin et al. (2004) regarding the correction factor K_σ .

$$K_\sigma = \exp\left(\frac{-3.2 \cdot \ln(\sigma'_v/P_a)}{13.32}\right) \quad (4)$$

As evident by the available literature, even though there exist some disagreements on the actual values of K_α , and K_σ factors, yet it is still possible to compare the free field soil CSR (i.e.: CSR_{FF}) values to the CSR value estimated for a soil, structure and earthquake interacting system (i.e.: CSR_{SSEI}) through currently available K_α , and K_σ correction factors, if both kinematic and inertial interaction components are put aside.

The kinematic and inertial interactions defined in iii) and iv) are rather complex and deserve further attention on the soil structure and earthquake interaction (SSEI) problem from liquefaction triggering point of view. Veletsos and Meek (1974) discovered that a) σ (ratio of structure-to-soil stiffness) as defined in Equation (5), b) the ratio of structure height to foundation radius (width) and c) the interaction of the fixed-base natural frequency of the structure to the frequency regions of the design spectrum were the critical factors controlling SSEI. By Veletsos and Meek, it was concluded that for σ values in the range of 3-20, soil-structure interaction becomes critical.

$$\sigma = \frac{V_{s,final} \times T_{str}}{h_{effective}} \quad (5)$$

Yoshimi and Tokimatsu (1977) have shown that as the width ratio, (B/D) of a structural system increases, the settlement ratio, (S/D) decreases. Finn and M. Yodengrakumar (1987) have performed centrifuge model tests as well as finite element simulations and revealed that seismically-induced pore pressure generation in foundation soils especially near the external wedges was faster than the one in the free field. Rollins and Seed (1990) based on a number of shaking table and centrifuge model tests, concluded that usually pore pressure generation was slower beneath the buildings than the one in the free field. They have also concluded that if spectral acceleration ratio, S_a/a_{max} was larger than about 2.40, the induced cyclic stress ratio value would be higher beneath the building than the one in free field. Popescu and Prevost (1993) have performed centrifuge model tests and numerical simulations; however, lack of pore pressure transducers installed at the edges of the model structure during centrifuge tests disabled them quantifying the interaction effects on pore pressure generation. Jun-Tsai Hwang et al. (1994) have conducted a two dimensional effective stress based analysis of soil-structure-earthquake interaction and concluded that the soils outside the structural influence zone liquefy first, followed by the zone under the structure. Yoshiaki et al. (1997) have performed shaking table tests, using sand box with a footing model, along with numerical simulation of the test setup. They have concluded that, soils directly beneath the footing would be harder to liquefy than free field soils, whereas the region near the external wedge of footing was more susceptible to liquefaction than the free field soil region. Similarly, Liu and Dobry (1997) have performed shaking table tests with sand box containing model footings, and suggested that a weak zone of liquefaction resistance locating around the line starting from the external end point of footing base and inclining 45° to the horizontal direction exists. Stewart et al. (1999) presented a summary of the existing literature on soil structure interaction problems and described analysis procedures and system identification techniques for evaluating inertial soil structure interaction effects on seismic structural response. Two sets of analyses, aiming to estimate period lengthening ratio and foundation damping factors, as well as soil structure system identification procedures were presented for the evaluation of vibration parameters.

More recently, Travasarou et al. (2006) have performed 2-D soil-structure interaction analyses for two representative buildings in Adapazari, Turkey and showed that for stiff structures on shallow foundations, seismic demand was considerably higher than free field adjacent to the perimeter of the structure and conversely for typical static building loads, the demand was reduced by 50% directly underneath the foundation of the structure.

As clearly presented by the summarized literature, there exist contradicting arguments regarding the effects and the assessment of liquefaction triggering potential of foundation soils overlain by structural systems. Within the confines of this paper, it is attempted to present the preliminary results of our ongoing research studies aiming to comparatively quantify the effects of SSEI on liquefaction triggering potential, more specifically on CSR. The scope of our studies includes 3-D numerical simulation of SSEI.

NUMERICAL SIMULATIONS

3-D finite difference-based simulations were performed for the purpose of assessing SSEI. For verification purposes additional 1-D soil site response analyses were performed by SHAKE91 software. The numerical assessment scheme included the followings : i) 3-D static assessment of soil-structure system for the purpose of estimating horizontal shear stresses and vertical effective stresses, which will be further used in the calculation of K_α and K_σ correction factors ii) 3-D seismic assessment of soil-structure system for the purpose of estimating both structural and soil mass-induced shear stresses acting on the horizontal plane, which will be further used in the calculation of CSR, iii) 3-D seismic assessment of the free field soil site (without the structural system) for the purpose of enabling direct comparisons with 1-D SHAKE 91 simulations, useful for the calibration of the 3-D model. In the development of the 3-D finite difference mesh, mesh window and element sizes were selected in such a way that the results of FLAC 3-D free field simulations match with SHAKE 91 results, as illustrated in Figure 3, Figure 4 and Figure 5.

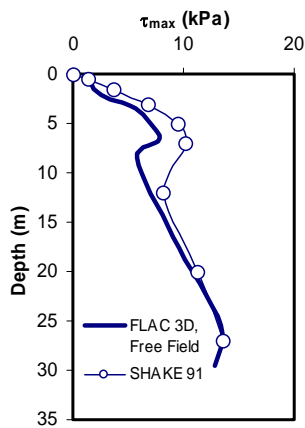


Figure 3. Variation of shear stress (τ) with depth (d)

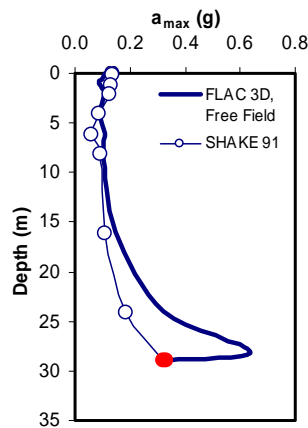


Figure 4. Variation of a_{max} with depth (d)

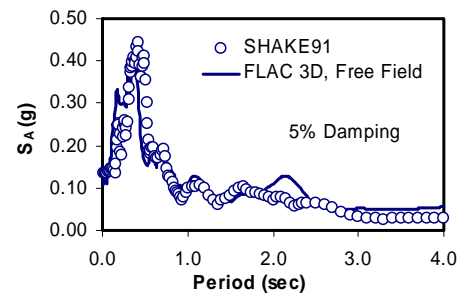


Figure 5. Comparison of elastic response spectra

Figure 6 presents a typical mesh adopted for the numerical simulations. In FLAC 3-D simulations, to be consistent with the available liquefaction triggering methods (e.g.: NCEER, 1997, Cetin et al. 2004) equivalent linear model with hysteretic degradation and damping curves as recommended by Vucetic and Dobry (1991) for clean sands (i.e.: $PI = 0$) was adopted for the purpose of estimating induced shear stresses. The spatial element size is selected to be smaller than approximately one-tenth of the wavelength associated with the highest frequency component of the input wave, which is 15 Hz, to prevent numerical distortion of the propagating wave in dynamic analyses. The boundary condition at the sides of the model is selected as free field which accounts for the free field motion that would exist in the absence of the structure.

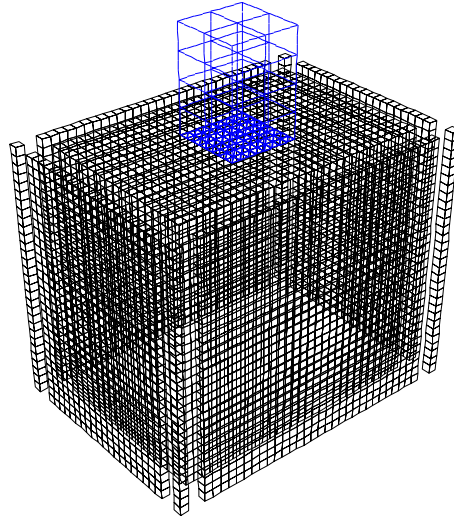


Figure 6. Finite Difference 3D Mesh

As shown in Figure 7, four generic 30 m thick homogeneous soil profiles with shear wave velocity values selected as $V_s=100$ m/sec, $V_s=150$ m/sec, $V_s=200$ m/sec and $V_s=300$ m/sec, overlain by two different structural systems with fixed base natural periods of 0.38 and 0.22 seconds are shaken by 1999 Kocaeli Earthquake, $M_w = 7.4$, Sakarya (SKR) record, 1989 Loma Prieta Earthquake, $M_w = 7.0$ Santa Cruz USCS Lick Observatory Station (LP) record and 1995 Kobe Earthquake, $M_w = 6.9$, Chihaya Station (CHY) record. Foundation systems are defined as 30 cm thick surfacial mats, overlain by a regular concrete column-beam system with column and beam dimensions of 40 x 40 and 30 x 30 cm, respectively. Column spacing and storey heights were selected as 4 and 3 m respectively. The storey masses were concentrated at the floor levels.

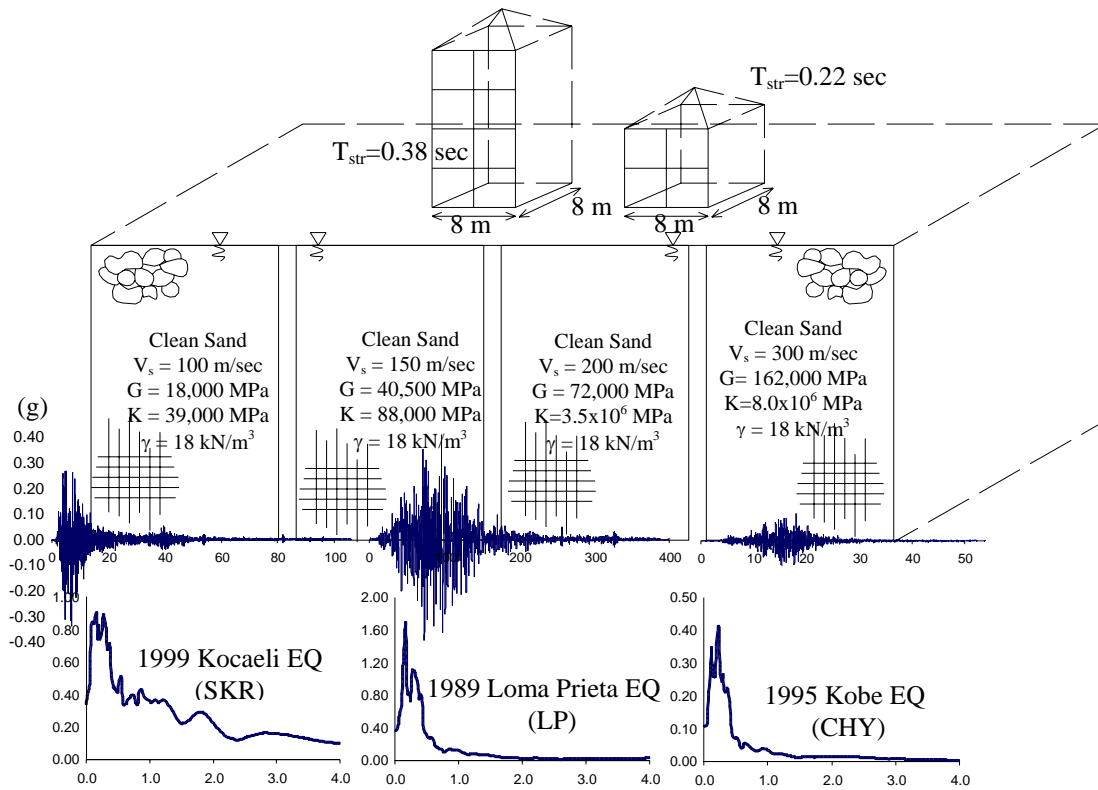


Figure 7. Schematic view of soil-structure and earthquake combinations

As presented in Table 1, 19 sets of numerical simulations were performed corresponding to different combinations of generic soil sites, structures and earthquake records. As shown in Figure 8, the results of these simulations are presented in the form of corrected CSR values. As shown in Equation (6), in estimating the CSR values of the soil-structure-earthquake interacting model, initial static horizontal shear stresses, $\tau_{static,h}$, were subtracted from the estimated maximum horizontal cyclic shear stresses, $\tau_{SSEI,h}$, further normalized by static vertical effective stresses leading to CSR_{SSEI} values. CSR_{SSEI} was then corrected by K_α and K_σ factors to eliminate the variabilities due to initial static vertical effective and horizontal shear stresses.

$$CSR_{SSEI, \alpha=0, \sigma_v'=100kPa} = \frac{0.65 \cdot (\tau_{SSEI,h} - \tau_{static,h})}{K_\alpha \cdot K_\sigma \cdot \sigma_{v',static}} \quad (6)$$

The maximum cyclic shear stresses developed within the free field soil medium (simulated by using the exact same numerical model, but without the structure) were normalized by static vertical effective stress, and the free field CSR values (CSR_{FF}) were estimated. These values were also corrected by K_σ factors but not K_α factors due to lack of horizontal shear stresses in free field level soil sites. Then the ratio of CSR_{SSEI} to CSR_{FF} was defined as CSRR, a direct measure of the kinematic and inertial interaction of the soil, structure and earthquake.

Table 1. Input and Output Parameters of Numerical Simulations

$V_{s,initial}$ (m/sec)	$V_{s,final}$ (m/sec)	T_{str} (sec)	$CSRR_{peak,z/B=0}$	EQ	PGA (g)	S_A at $T=T_{str}$	S_A/PGA	σ	T_{soil}	T_{str}/T_{soil}	T_{soil}/T_p	β
100	60	0.38	2.41	SKR	0.15	0.22	1.45	2.87	1.20	0.32	4.62	0.46
100	60	0.22	2.44	SKR	0.15	0.41	2.78	1.66	1.20	0.18	4.62	0.51
150	103	0.38	1.64	SKR	0.18	0.38	2.15	4.93	0.80	0.48	3.08	1.02
150	103	0.22	2.36	SKR	0.18	0.38	2.14	2.85	0.80	0.28	3.08	0.59
200	149	0.38	1.18	SKR	0.26	0.54	2.10	7.11	0.60	0.64	2.31	1.33
200	149	0.22	1.50	SKR	0.26	0.42	1.61	4.10	0.60	0.37	2.31	0.59
300	235	0.38	0.42	SKR	0.36	0.68	1.87	11.21	0.40	0.95	1.54	1.78
300	235	0.22	0.61	SKR	0.36	0.81	2.25	6.47	0.40	0.55	1.54	1.24
100	62	0.22	2.33	LP	0.11	0.30	2.77	1.71	1.20	0.18	7.50	0.51
100	62	0.38	1.78	LP	0.11	0.39	3.63	2.97	1.20	0.32	7.50	1.15
150	97	0.22	2.05	LP	0.15	0.43	2.82	2.68	0.80	0.28	5.00	0.78
150	97	0.38	0.70	LP	0.15	0.53	3.45	4.64	0.80	0.48	5.00	1.64
200	136	0.22	2.31	LP	0.23	0.56	2.39	3.74	0.60	0.37	3.75	0.88
200	136	0.38	0.39	LP	0.23	0.66	2.84	6.47	0.60	0.64	3.75	1.80
100	79	0.38	2.34	CHY	0.05	0.14	2.61	3.76	1.20	0.32	5.45	0.83
150	122	0.22	0.77	CHY	0.08	0.40	5.00	3.37	0.80	0.28	3.64	1.38
150	122	0.38	0.73	CHY	0.08	0.35	4.38	5.83	0.80	0.48	3.64	2.09
200	163	0.22	0.53	CHY	0.12	0.54	4.44	4.49	0.60	0.37	2.73	1.63
200	163	0.38	0.47	CHY	0.12	0.22	1.81	7.77	0.60	0.64	2.73	1.15

Figure 8 present the variation of CSRR with depth and parallel to the width of the structure across the centerline of the structure. Depending on the soil, structure and earthquake characteristics, CSR_{SSEI} was estimated to be higher or lower than the CSR_{FF} values.

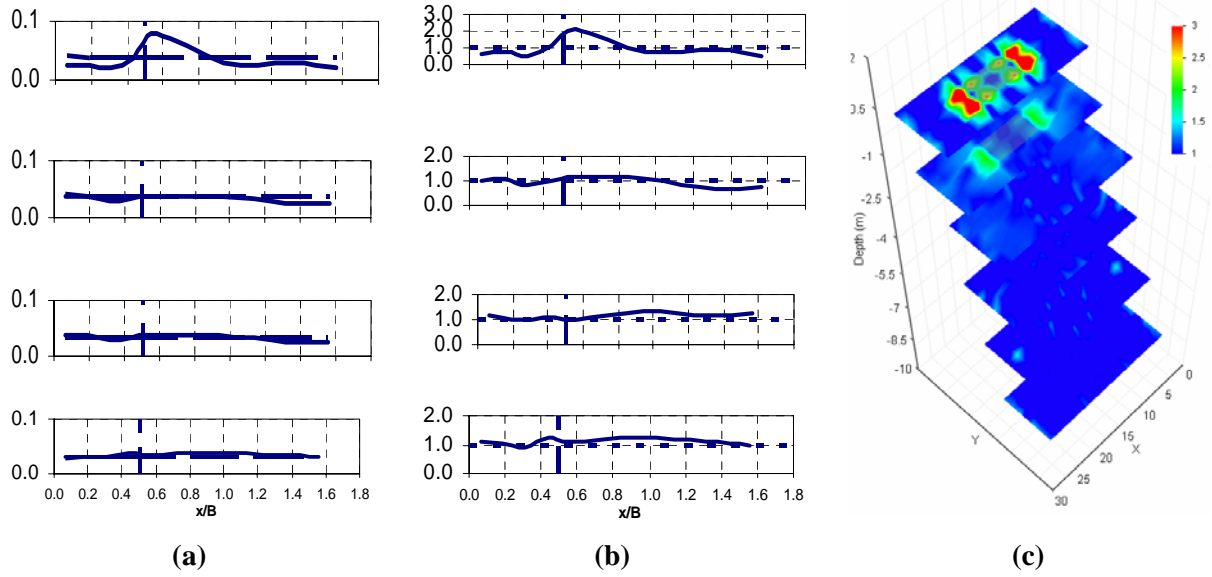


Figure 8. Variation of (a) CSR_{SSEI} and CSR_{FF} (b) and (c) $CSRR$ through Depth

The variation of $CSRR$ along the distance parallel to the structure width follows two patterns as also illustrated in Figure 9 (a) and (b). For case (b), it is concluded that SSEI is beneficial and reduces the seismic liquefaction triggering demand from the nearby foundation soils and opposite is true for case (a).

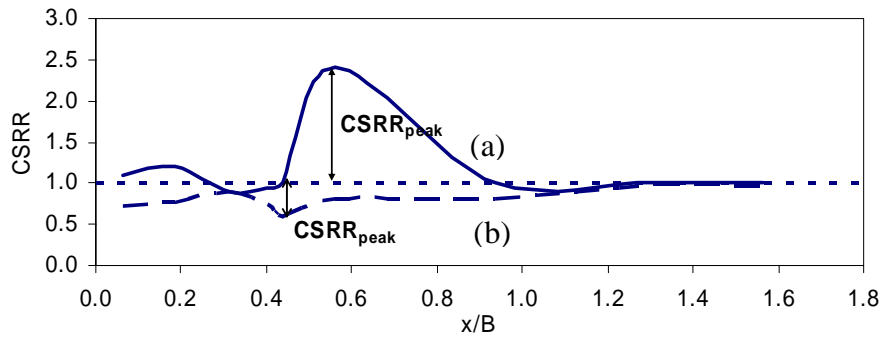


Figure 9. Schematic representation of $CSRR_{peak}$

DISCUSSION ON NUMERICAL SIMULATIONS RESULTS

The analyses results were summarized in Figure 10 in the form of variation of $CSRR$ with normalized depth (i.e.: depth/width of structure; z/B) and distance along the centerline of the building (x/B). As can be interpreted from the results provided, SSEI effects on seismic soil liquefaction triggering can not be concluded to be consistently beneficial or detrimental. Usually under the structure (i.e.: $x/B < 0.5$), $CSRR$ is less than 1.0, stating that generally under buildings, SSEI effects reduce the seismic soil liquefaction triggering demand from the foundation soils. However, starting with the edges of the structure to a distance of $1.5B$ from the center of the building, $CSRR$ values are higher than 1.0 (i.e.: $CSR_{SSEI} > CSR_{FF}$), which states that presence of the overlying structure negatively affects the soils in $1B$ neighborhood of the structure. As illustrated by Figure 11, SSEI effects, either positive or negative become less significant beyond a z/B ratio of 1. (i.e. CSR_{SSEI} approaches to the value of CSR_{FF}).

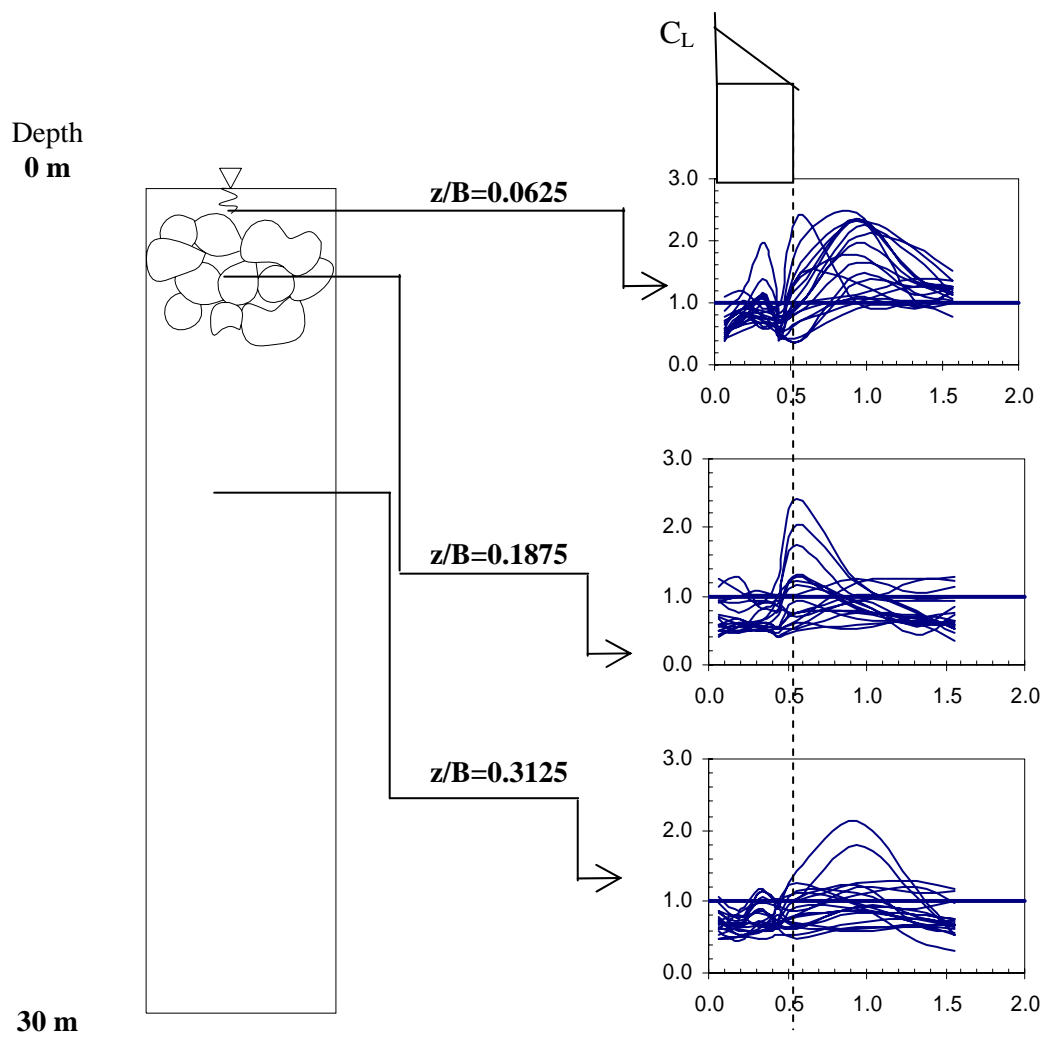


Figure 10. All CSRR values for $z/B=0.0625$, $z/B=0.1875$ and $z/B=0.3125$

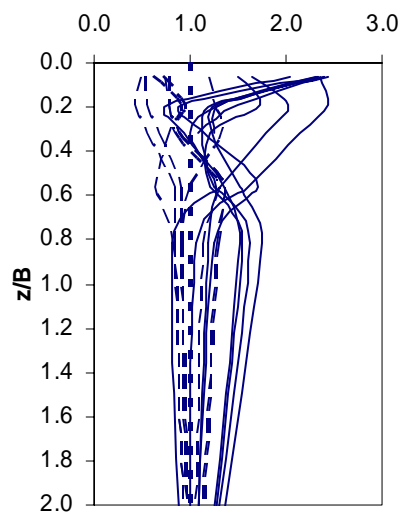


Figure 11. Variation of $CSRR_{peak}$ with depth along centerline

To quantify the effects of SSEI, $CSRR_{peak}$ values at the foundation levels were drawn against the SSI parameter of σ (Veletsos and Meek), S_A (Seed and Rollins). As presented in Figure 12, the correlation between $CSRR_{peak}$ vs S_A was found to be weak. The R^2 terms are calculated as 0.63 and 0.22, for σ and S_A parameters respectively.

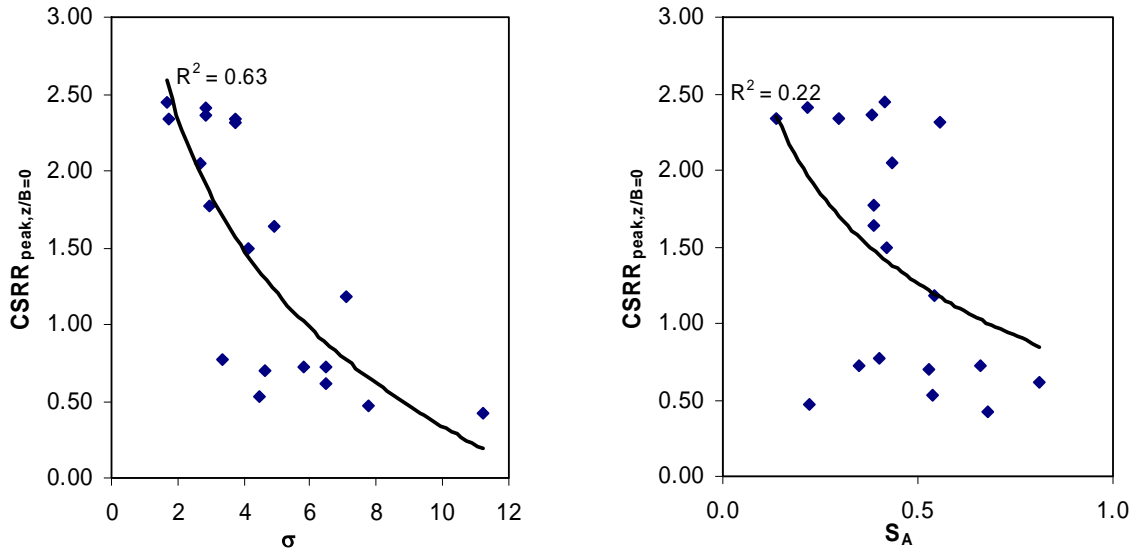


Figure 12. $CSRR_{peak,z/B=0}$ vs. σ and S_A

Inspired by these, a new term, “ β ”, that will model the interaction among soil, structure and earthquake is introduced, as given in Equation (7), where T_{str} is the fixed base period of the structure, T_{soil} is the pre-earthquake period of the soil which is found by (8), S_A is the spectral acceleration value corresponding to the fixed base period of the structure, and PGA is the peak ground acceleration of the free field soil site. For the ease of use, pre-earthquake periods of the structure and soil are taken into account in this new term “ β ”. As shown in Figure 13, the correlation between $CSRR_{peak,z/B=0}$ vs. this new term, β , is found to be stronger, as evidenced by an R^2 of 0.75.

$$\beta = \frac{T_{str}}{T_{soil}} \cdot \frac{S_A}{PGA} \quad (7)$$

$$T_{soil} = \frac{4H_{soil}}{V_{s,initial}} \quad (8)$$

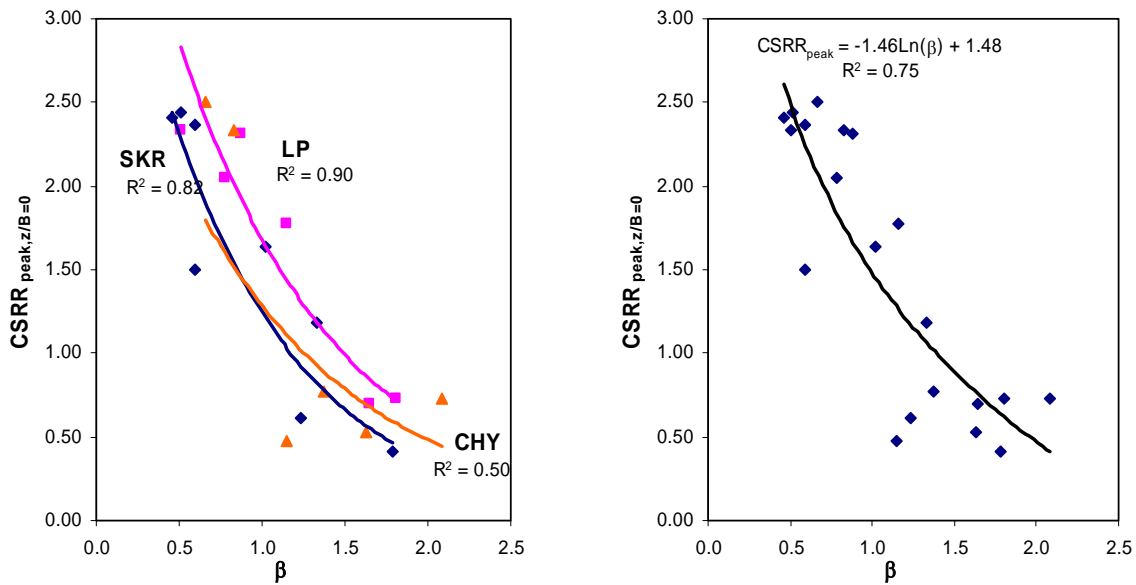


Figure 13. $CSRR_{peak,z/B=0}$ vs. β

For practical purposes, $CSRR_{peak,z/B=0}$ can be estimated as given in Equation (9). Based upon Equation (9), it is concluded that the presence of the overlying structure decreases liquefaction triggering potential for systems with a β value of greater than 1.4. For β values in the range of 0.5 to 1.0, due to the presence of the structure, CSR values corresponding to the soil, structure and earthquake interacting system (i.e.: CSR_{SSEI}) are estimated to be twice as much the free field soil values at the ground surface. Similarly, it can be concluded that for the β values in the range of 1.5 to 2.0, CSR_{SSEI} value is 25 to 40 % less than the free field values.

$$CSRR_{peak,z/B=0} = -1.46 \ln \beta + 1.48 \quad (9)$$

SUMMARY AND CONCLUSIONS

Within the confines of this paper, a review on seismic soil structure interaction problem, emphasis on its effects to seismic soil liquefaction triggering, is presented. Motivated by lack of available models for the purpose of assessing liquefaction triggering potential of soils overlain by a structure, it is intended to develop an empirically-based relationship to estimate CSR values of such soil structure, earthquake interacting system as compared to the values of a free field soil site. Thus, series of 3-D numerical simulations of generic soil, structure systems shaken by three earthquake records were performed. The results were summarized in the form of K_a , K_σ corrected CSR values. Then the ratio of the CSR values estimated for the SSEI system and free field soil site was defined as CSRR. A new soil, structure, earthquake interaction parameter β , was defined to assess the independency of CSRR. It was concluded that the proposed parameter, β can capture the effects of SSEI on liquefaction triggering well as evident by a high R^2 value of 0.75. Based upon analyses results following conclusions were made:

- i) Zone extending 0.2 width inside and 0.8 width outside of the edges are found to be relatively more critical from liquefaction triggering point of view as compared to the liquefaction potential of the zone in the vicinity of the foundation centerline.
- ii) Either positive or negative, effects of the presence of the structure on liquefaction triggering potential of foundation soils diminishes rapidly at depths below 0.5 width and becomes almost none beyond the depths of 1.5 B.
- iii) The presence of the overlying structure decreases liquefaction triggering potential for systems with a β value greater than 1.4.

More specifically,

- iv) For β values in the range of 0.5 to 1.0, due to the presence of the structure, CSR values corresponding to the soil, structure and earthquake interacting system (i.e.: CSR_{SSEI}) are estimated to be twice as much the free field soil values at the ground surface.
- v) For the β values in the range of 1.5 to 2.0, CSR_{SSEI} value is 25 to 40 % less than the free field values.

As the concluding remark, we would like to clarify our intention. All these preliminary findings were based on limited number of numerical simulations as part of our ongoing research studies. More simulations are going to be performed with i) natural soil profiles as opposed to generic soil sites, ii) structures of larger natural periods in the range of 0.4 to 1.0 seconds with and without basements, iii) a suite of earthquake records sampled from near-field, mid-field and far-field motions where our current database has a gap. After these, we believe that the resulting model will be more general and powerful. However, even these limited simulation results revealed that the proposed controlling parameter, β for the assessment of liquefaction triggering potential of foundation soils is promising.

ABBREVIATIONS

α	Ratio of static shear stresses to effective vertical stress
β	Soil-structure-earthquake interaction factor
CSR	Cyclic stress ratio
CSR_{eq}	Equivalent cyclic stress ratio
CSR_{FF}	CSR for free field soil sites
CSR_{SSEI}	CSR for soil-structure-earthquake interaction
CSRR	Ratio of structure CSR to free field CSR
$CSRR_{peak}$	Maximum/minimum CSRR beneath the foundation
$CSRR_0$	CSRR along midline of the structure
$h_{effective}$	Effective height of structure (generally taken as two thirds of the total height)
K_α	Correction factor for static shear stresses
K_σ	Correction factor for overburden
σ	Ratio of structure to soil stiffness
PGA	Peak ground acceleration
S_A	Spectral acceleration at the period corresponding to T_{str}
P_a	Atmospheric pressure
T_{str}	Fixed base natural period of the structure
T_p	Predominant period of earthquake
T_{soil}	Period of soil site
$V_{s,initial}$	Initial shear wave velocity of the site
$V_{s,final}$	Final shear wave velocity of the site

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