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EFFECT of NONLINEARITY on the SITE RESPONSE ANALYSIS of GEO-STRUCTURE

Haydar ARSLAN [1], Bilge SIYAHI[2]

ABSTRACT

The objective of this study is to give a critical overview of the field of site response analysis. Some of the well known site response analysis methods will be summarized. Linear and nonlinear Site Response analysis will be summarized and evaluated numerically. Two layer soil column with the assumption of linear and rigid base bedrock (or viscoelastic half-space) will be analyzed by using linear and non-linear approaches. Nonlinear analysis will be compared with the linear method of analysis. Steps involved in ground response analyses to develop site-specific response spectra at a soil site will be briefly summarized. An appropriate nonlinear computational model for prediction of seismic site response will be proposed and compared with equivalent linear assumption.

Keywords: Earthquake**,** Site Response, Nonlinear, Seismic Excitation.

INTRODUCTION

Earthquakes are one of the most devastating natural hazards. Approximately 20 earthquakes of magnitude 7.0 or larger occur worldwide every year. Earthquake hazard mitigation requires researches in many areas including geotechnical earthquake engineering. In the past few decades significant advancement has been made to understand and develop physical and analytical modeling for aforementioned geotechnical earthquake engineering problems. Signs of soil nonlinearity include decreased spectral ratios of surface to input motion near the dominant frequency of the soil, decreased statistical uncertainty in prediction of peak acceleration, and increased effective period of surface motion (Yu et. al., 1992).

The strains generated by earthquakes can be large in the soil. These strains reduce the shear modulus (and consequently the shear-wave velocity) and increase the damping ratio (e.g. Seed & Idriss, 1969; Hardin & Drnevich, 1972). These effects lead to nonlinearity in the soil media under earthquake loadings. Such non-linear behavior may explain the difference between expected high amplifications due to site effects and the lower amplifications recorded in large earthquakes (Seed & Idriss, 1969).

The nonlinearity of soil behavior is known very well thus most reasonable approaches to provide reasonable estimates of site response is very challenging area in geotechnical earthquake engineering. Understanding of Site Response of geological materials under seismic loading is an important element in developing a well established constitutive model. A number of different techniques have been developed for site response analysis.

[1] Post Doctorate, Research Fellow, University of Colorado- Boulder Department of Civil Engineering and LASP 428 UCB Boulder, CO 80309-0428 Phone: (303)492-7112 E-mail: arslan@colorado.edu
[2] Associate Professor Bogazici Univ., Kandilli Observatory and Earthquake Research Institute, Department of Earthquake Engineering, 81220, Cengelkoy, Istanbul, Turkey, Email: bilge.siyahi@boun.edu.tr

To approximate the actual nonlinear, inelastic response of soil, an equivalent linear approach was proposed by Schnabel et al (1972). In the equivalent linear approach, linear analyses are performed with soil properties that are iteratively adjusted to be consistent with an effective level of shear strain induced in the soil. Yoshida (1994), Huang et al. (2001), Yoshida and Iai (1998) showed that equivalent linear analysis shows larger peak acceleration because the method calculates acceleration in high frequency range large. There are many issues, which require deeper understanding and better tools for the analysis and simulation of the underlying processes governing ground motion, site response and instability. In this paper, a nonlinear method of site response analysis will be compared with equivalent linear approach. Similarities and differences will be summarized with a numerical example.

Previous Studies

Chin & Aki (1991) simulated strong ground motion records from the Loma Prieta earthquake. The predicted PGA on soil is overestimated and the predicted PGA on rock is underestimated for hypocentral distances less than 50 km. Similar differences observed for all stations within 50 km so it is not due to the radiation pattern, near-source structure or topography. It is due to the fact that the soil sites behave non-linearly for PGA larger than 0.1–0.3 g and so the amplification due to the sediment layers is not as great as it is for weak motion.

Field *et al.* (1997) compared recordings from 184 aftershocks ($3.0 < M < 5.6$) of the Northridge (17/1/1994) earthquake. The main shock records were from the 21 sites including 15 alluvial sites. A generalized inversion for source, path and site effects using the shear-wave Fourier amplitude spectrum illustrated that during the aftershocks the amplification at the alluvial sites is about 1.4 at a period of 0.1 s, about 2.5 at a period of 0.3 s and about 3.1 at a period of 1 s, whereas for the mainshock the amplification is only about 0.8 at 0.1 s, about 1.3 at 0.3 s and about 1.9 at 1 s. These differences are significant at the 99% confidence level between 0.2 and 1.3 s.

There are two main groups of soil models to account for the soil nonlinearity: equivalent linear models, and nonlinear models. A number of studies have been conducted to compare the response of soil deposits using both equivalent linear and direct nonlinear methods and the common observation is that while both methods give similar response spectra, the equivalent linear method underestimates displacements and overestimates accelerations (Constantopoulos et. al., 1973; Finn et. al., 1977, 1978; Yu et. al., 1992). Nonlinear one-dimensional ground response analysis characterizes the stress-strain behavior of the soil by cyclic stress-strain models. In all these models, the nonlinear shear behavior is described by a shear stress-strain backbone curve.

The models that represent the nonlinear behavior of soils more accurately are based on advanced constitutive models that use basic principles of mechanics. These models generally require a yield surface that describes the limiting stress conditions for which elastic behavior is observed, a hardening law that describes changes in the size and shape of the yield surface as plastic deformation occurs, and a flow rule that relates plastic strain increments to stress increments (Kramer, 1996).

Quantitative studies have been conducted using strong-motion array data after 1970's. Several methods have been proposed for evaluating site effects by using ground-motion data , such as soil-to rock spectral ratios (e.g., Borchert, 1970), a generalized inversion (e.g., Iwata and Irikura, 1988; Boatwright *et al.*, 1991), and horizontal-to-vertical spectral ratios (e.g., Nakamura, 1988; Lermo and Chavez-Garcia, 1993; Field and Jacob, 1995; Yamazaki and Ansary, 1997, Bardet et al., 2000, Bardet and Tobita, 2001, Lam et al. 1978, Joyner and Chen 1975, Arslan and Siyahi 2006). One dimensional analysis has been used to predict the behavior of soil under earthquake loading for a practical and quick study. Equivalent linear approach (Schnabel et al. (1972) is widely used for one dimensional site response analysis. In the following section, equivalent linear approach and nonlinear approach proposed by Kramer (1996) will be compared to illustrate similarities and differences of Linear and Nonlinear approaches.

Background for Equivalent Linear and Nonlinear Site Response Analysis

Schnabel et al. (1972), Idriss and Sun (1992), Kramer (1996) explained that the actual nonlinear hysteretic behavior of cyclically loaded soil can be approximated by equivalent linear approximation. Linear approximation requires an equivalent shear modulus (G) and equivalent linear damping ratio (ξ). SHAKE (Schnabel et. al 1972) is the most known computer program that uses equivalent linear approximation, used widely. This code based on the multiple reflection theory, and nonlinearity of soil is considered by the equivalent linear method. Unlike the name of "equivalent", this is an approximate method.

SHAKE uses a frequency domain approach to solve the ground response problem. In simple terms, the input motion is represented as the sum of a series of sine waves of different amplitudes, frequencies, and phase angles (Schnabel et al.1972). A relatively simple solution for the response of the soil profile to sine waves of different frequencies (in the form of a transfer function) is used to obtain the response of the soil deposit to each of the input sine waves. The overall response is obtained by summing the individual responses to each of the input sine waves.

To illustrate the basic approach used in SHAKE, consider a uniform soil layer lying on an elastic layer of rock that extends to infinite depth, as illustrated in Figure 1. If the subscripts *s* and *r* refer to soil and rock, respectively, the horizontal displacements due to vertically propagating harmonic s-waves in each material can be written as:

$$u_s(z_s,t) = A_s e^{i(\omega t + k_s^* z_s)} + B_s e^{i(\omega t - k_s^* z_s)} \tag{1}$$

$$u_r(z_r,t) = A_r e^{i(\omega t + k_r^{*} z_s)} + B_s e^{i(\omega t - k_r^{*} z_s)} \tag{2}$$

where u is the displacement, ω is the circular frequency of the harmonic wave and k* is the complex wave number. No shear stress can exist at the ground surface (z_s=0), so

$$\tau(0,t) = G_s^* \gamma(0,t) = G_s^* \frac{\partial u_s(0,t)}{\partial z_s} = 0 \tag{3}$$

where $G_s^* = G(1+2i\xi)$ is the complex shear modulus of the soil.

In the equivalent linear approach, the shear modulus is taken as the secant shear modulus which, as shown to the right, approximates an “average” shear modulus over an entire cycle of loading. Because the transfer function is defined as the ratio of the soil surface amplitude to the rock outcrop amplitude, the soil surface amplitude can be obtained as the product of the rock outcrop amplitude and the transfer function.

Schnabel et al. (1972) explained that within a given layer (layer j), the horizontal displacements for the two motions (motion A and motion B) may be given as:

$$u_r(z_j,t) = (A_j e^{ik_j^* z_j)} + B_j e^{-ik_j^{*'} z_j)})e^{i\omega t} \tag{4}$$

Thus, at the boundary between layer *j* and layer *j*+1, compatibility of displacements requires that

$$A_{j+1} + B_{j+1} = A_j e^{ik_j^* h_j} + B_j e^{-ik_j^* h_j} \tag{5}$$

Continuity of shear stresses requires that

$$A_{j+1} + B_{j+1} = \frac{G_j^* k_j^*}{G_{j+1}^* k_{j+1}^*}(A_j e^{ik_j^* h_j} - B_j e^{-ik_j^* h_j}) \tag{6}$$

The effective shear strain of equivalent linear analysis is calculated as:

$$\gamma_{eff} = R_\gamma \gamma_{\max} \tag{7}$$

where $\gamma_{\max}$ is the maximum shear strain in the layer and R_γ is a strain reduction factor often taken as:

$$R_\gamma = \frac{M-1}{10} \tag{8}$$

in which *M* is the magnitude of earthquake.

While the equivalent linear approach allows the most important effects of nonlinear, inelastic soil behavior to be approximated, it must be emphasized that it remains a linear method of analysis. It is based on the continuous solution of the wave equation, adapted to use with transitory movements by means of the Fast Fourier Transform algorithm. The strain-compatible shear modulus and damping ratio remain constant throughout the duration of an earthquake - when the strains induced in the soil are small and when they are large. Permanent strains cannot be computed and pore water pressures cannot be computed. However, the equivalent linear approach has been shown to provide reasonable estimates of soil response under many conditions of practical importance.

Maximum shear modulus of a layer is calculated by:

$$v_s = \sqrt{G/\rho} \quad = \sqrt{\frac{Gg}{\gamma}} \tag{9}$$

in which G_{max} is maximum shear modulus, ρ is density of the soil, γ is unit weight, and g is the acceleration of gravity.

As Finn et al. (1978) and Kramer (1996) explained the method is incapable of representing the changes in soil stiffness that actually occurs under cyclic loadings. In addition, the behavior of geological materials under seismic loading is nonlinear.

Nonlinear Site Response Analysis:

Main reason using linear approach is the method is computationally convenient and provides reasonable results for some practical cases (Kramer 1996). However, the nonlinear and inelastic behavior of soil is well established in geotechnical engineering. The nonlinearity of soil stress-strain behavior for dynamic analysis means that the shear modulus of the soil is constantly changing. The inelasticity means that the soil unloads along a different path than its loading path, thereby dissipating energy at the points of contact between particles. Both time domain and the frequency-domain analyses are used to account for the non-linear effects in site-response problems. Non-linear and equivalent-linear methods are utilized respectively in the time and frequency domain for the one-dimensional analyses of shear wave propagation in layered soil media. When compared with earthquake observation, nonlinear analyses are shown to agree with the observed record better than the equivalent linear analysis.

Kramer (1996) developed a nonlinear approach as by this method a nonlinear inelastic stress-strain relationship is followed in a set of small incrementally linear steps.The soil medium is divided into sub-layers with absolute displacements u_j, defined at the *j* th sub layer, interface and with shear stress, τ_j , defined at the mid-points of each interface. As Kramer (1996) explained, the response of soil deposit under dynamic loading is governed by the equation of motion:

$$\frac{\partial \tau}{\partial z} = \rho \frac{\partial^2 u}{\Delta t^2} \tag{10}$$

The differentiation for a soil divided to *N* sublayers of thickness *Δz* and proceeding for the small time increment (*Δt*) *is* calculated by using finite difference method as:

$$\frac{\partial \tau}{\partial z} = \frac{\tau_{i+1} - \tau_{i-1}}{\Delta z} \tag{11}$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\dot{u}_{i,t+\Delta t} - \dot{u}_{i,t}}{\Delta t} \tag{12}$$

where $\dot{u} = \frac{\partial u}{\partial t}$ is the velocity of the motion and $\frac{\partial^2 u}{\partial t^2} = \frac{\partial \dot{u}}{\partial t}$ is the acceleration. If we combine equation 10 equation 11 and equation 12, the equation yields ;

$$\frac{\tau_{i+1} - \tau_{i-1}}{\Delta z} = \rho \frac{\dot{u}_{i,t+\Delta t} - \dot{u}_{i,t}}{\Delta t} \tag{13}$$

The equation can be simplified as:

$$\dot{u}_{i,t+\Delta t} = \dot{u}_{i,t} + \frac{\Delta t}{\rho \Delta z}(\tau_{i+1,t} - \tau_{i,t}) \tag{14}$$

It should be noted that for the soil surface the shear stress is equal to zero and boundary condition for each sublayer must be satisfied. Joyner and Chen (1975) proposed an equation for soil rock boundaries as:

$$\tau_{r,t} = \rho_r v_{sr} (2\dot{u}_r (t + \Delta t) - \dot{u}_{N+1,t+\Delta t}) \tag{15}$$

By using equation 14 and equation 15, boundary conditions are satisfied. Kramer (1996) gave the shear for each layer as:

$$\gamma_{i,t} = \frac{\partial u_{i,t}}{\partial z} \approx \frac{u_{i+1,t} - u_{i,t}}{\Delta z} \qquad (16)$$

As can be seen from the above equations, the shear stress is calculated by using current shear strain and stress-strain history ($\tau_{i,t} = G_i \gamma_{i,t}$). Thus the proposed method satisfies the nonlinear and inelastic behavior of soil under cyclic loading. The nonlinear method is implemented into commercial software MATLAB and the results are compared with SHAKE for two layers soil deposit.

Numerical Example

Nonlinear and linear solution techniques are compared in a two layers soil deposit as shown in figure 1. The thickness of the soil deposits is 150m.The top soil is silty sand (SM) with 50 m thickness and 19.0 kN/m^3 mass density (ρ_1) and 420m/sec shear wave velocity $V_{S1} = 450 m/\sec$. Lower layer is silty gravel (GM) with mass density, $\rho_2 = 19.0\ \text{kN}/m^3$ and the shear wave velocity and $V_{s2} = 600 m/\sec$ 105 m thickness. The shear wave velocity of the half-space interface is 1000m/s.

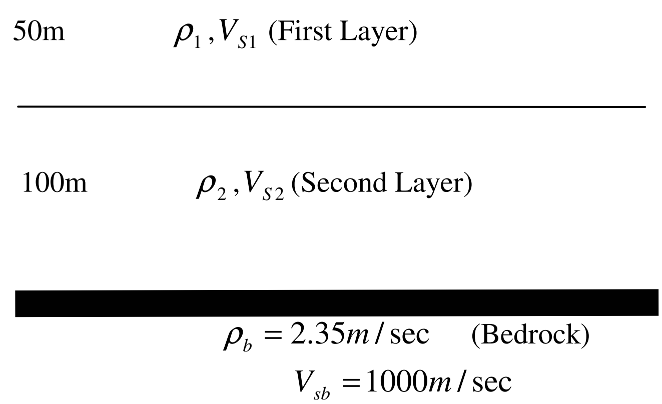
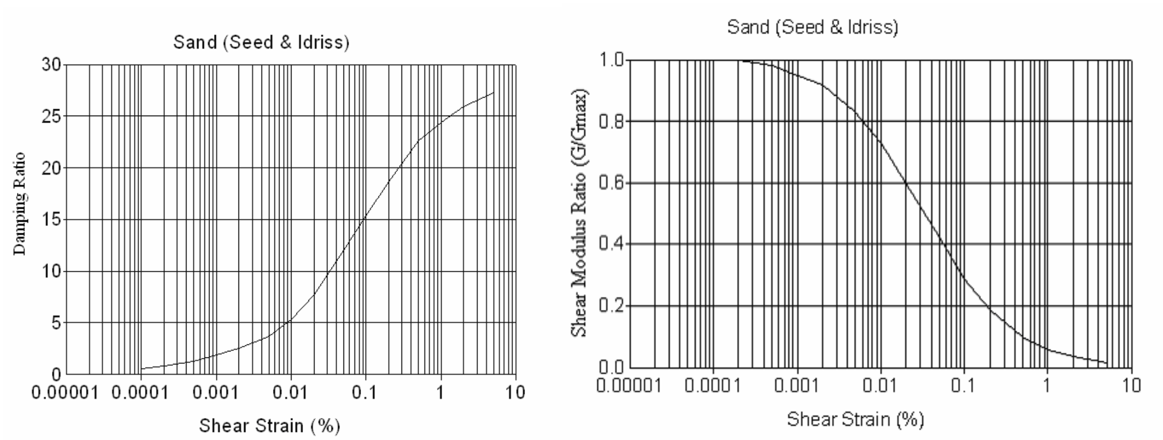


Figure 1. Bedrock half-space interface

The results of site response analyses were presented in terms of acceleration time history and response spectra. As explained in previous sections, SHAKE uses linear equivalent approaches with an iterative procedure to obtain soil properties compatible with the deformations developed in each stratum. The method of analysis used in SHAKE cannot allow for nonlinear stress-strain behavior because its representation of the input motion by a Fourier series and use of transfer functions for solution of the wave equation rely on the principle of superposition - which is only valid for linear systems.

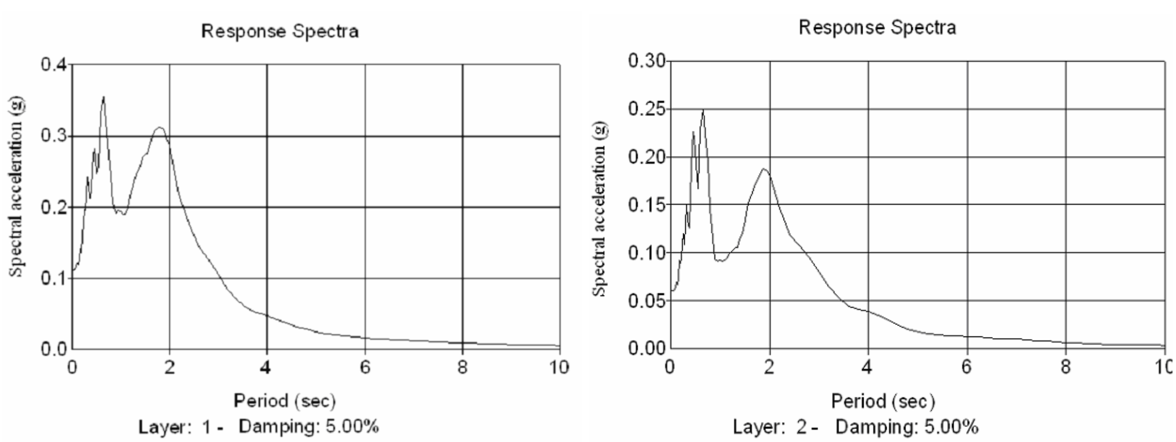
The input and output motion of the soil medium is given through Figures 2 and 3. The comparison of linear elastic numerical analysis by using SHAKE and nonlinear analysis are given in figures 4 and figures 5, and the results are summarized in table1.



(a) (b)

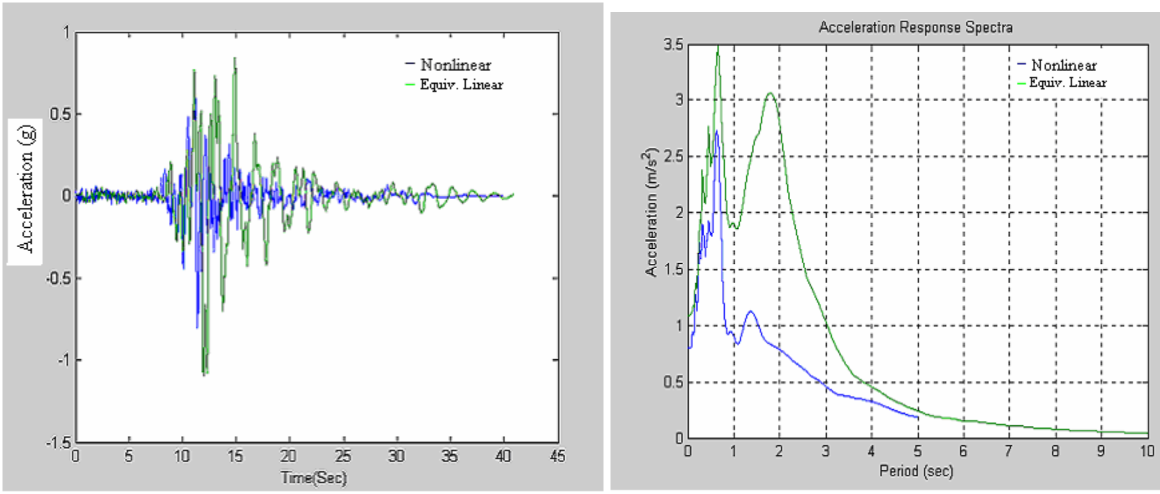
Figure 2) a: Strain-dependent damping ratio, b: The normalized strain-dependent shear modulus ratio

The solution algorithm used in SHAKE assumes viscous soil damping which represents using a complex shear modulus. Viscous damping implies behavior that would be characterized by elliptical stress-strain loops. Because actual stress-strain loops are seldom elliptical, an equivalent damping ratio is used – the equivalent damping ratio is equal to the damping ratio that would be computed based on the area within the hysteresis loop, the secant shear modulus, and the maximum shear strain. The relationship between this equivalent damping ratio and shear strain is characterized by means of a damping curve. 5% damping ratio is used in this study.



(a) (b)

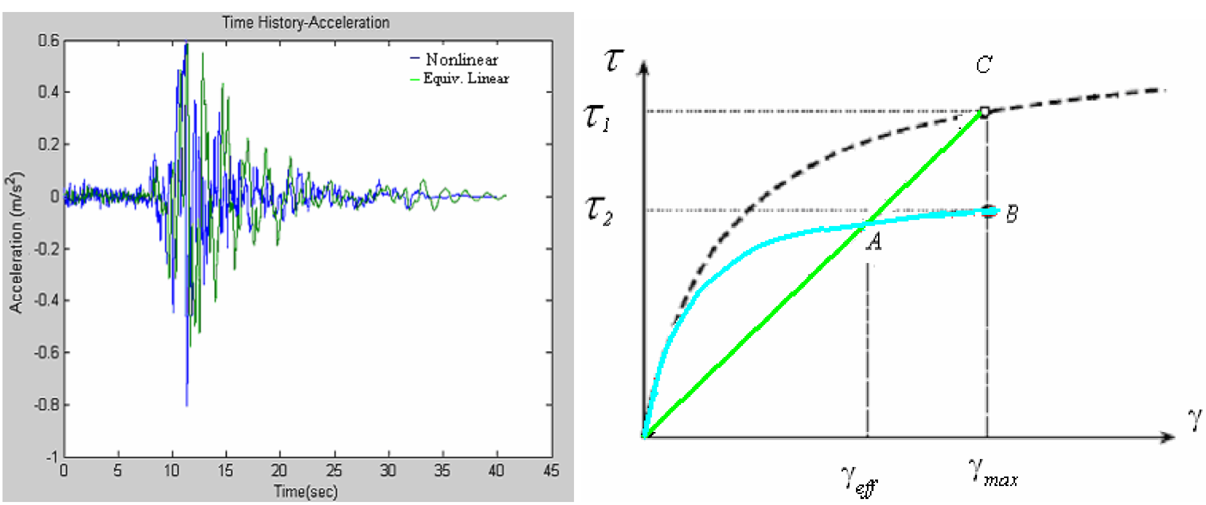
Figure 3) a: Output Motions Top Layer Response Spectra (SHAKE); b: Output motions: second layer response spectra (SHAKE)



(a) (b)

Figure 4) a: Comparison of acceleration & time history relation for the top layer; b: Comparison of acceleration response spectra for the top layer

As Table 1 and Figure 4 and Figure 5 show, the equivalent linear approach gives a higher acceleration. The reason of the high acceleration can be explained graphically in Figure 5 b (Kramer 1996). If the solid line in this figure is a stress-strain curve for the analysis and γ_{max} is a maximum strain, then linear relation used in the equivalent linear analysis is a line OAC. Therefore the shear stress (τ_2) at point B is not the peak shear stress that lies on the specified stress-strain curve, but τ_1. Similarly, when specified stress-strain curve is a solid line, then the peak stress-peak strain relationship may be expressed to be a dashed line; as it is seen in the figure, the shear stress is always overestimated. It should be noted that larger acceleration begins to appear as nonlinear behavior becomes predominant. However, it doesn't mean that nonlinear solution gives larger acceleration. In fact nonlinear analysis techniques give lower acceleration as shown in Table 1.



(a) (b)

Figure 5) a: Comparison of acceleration &time history relation for the second layer; b: The reason why linear approximations exhibit larger shear strain than specified

Table 1 Summary of the Site Response Analysis

	Max. Acceleration (m/s^2)	
	Nonlinear	Shake
First Layer	**0.62**	**0.82**
Second Layer	**0.81**	**1.07**
Spectral acceleration(m/s^2)	**2.8**	**3.6**

Summary and Conclusions:

Linear and nonlinear solution techniques for a site respon analysis have been evaluated in this short paper. Equivalent linear solution technique by SHAKE and a nonlinear technique have been used to predict the site response of a two layer soil medium under earthquake loading. The numerical results showed that the equivalent linear analysis calculates larger peak acceleration. The reason of this larger acceleration prediction is due to the fact that linear site response analysis calculates acceleration in high frequency range Thus, the linear solution techniques give higher acceleration. In addition to the effect of depth and properties of the soil, the method that used for a site response analysis has a great importance on the prediction of soil response under an earthquake loading.

REFERENCES

Arslan, H**.**, Siyahi, B., “ A Comparative Study on Linear and Nonlinear Site Response Analysis”, Environmental Geology Journal, Springer Publishers, ISSN: 0943-0105, Volume 50, No. 8, pp 1193-2000, 2006.

Bardet, J.P., Ichii, K., Lin, C.H., 2000. EERA: A computer program for Equivalent–linear Earthquake site Response Analyses of layered soil deposits. Department of Civil Engineering, University of Southern California, Los Angeles, California, 37 pp, 2000.

Bardet, J.P., Tobita, T., 2001. NERA: A computer program for Nonlinear Earthquake site Response Analyses of layered soil deposits. Department of Civil Engineering, University of Southern California, Los Angeles, California, 43 pp, 2001.

Constantopoulos, I. V., Roesset, J.M., Christian, J. T.. “A comparison of linear and exact nonlinear analyses of soil amplification,” 5th World Conf. Earthquake Eng., Rome, sess. 5B, pp. 225, 1973.

Field, E. H., Johnson, P. A., Beresnev, I.A., Zeng, Y. (1997). “Nonlinear ground-motion amplification by sediments during the 1994 Northridge earthquake,” Nature, Vol. 390, pp. 599-602.

Finn, W. D. L., Lee, M. K. W., Martin, G. R.. An effective stress model for liquefaction, J. Earthquake Eng. ASCE, 103, pp.517-533, 1977.

Hardin, B. O., & Drnevich, V. P.. Shear modulus and damping in soils: Design equations and curves. Journal of the Soil Mechanics and Foundations Division, ASCE, 98(SM7), 667–692, 1972.

Schnabel, P. B., Lysmer, J. & Seed, H. B.. SHAKE A Computer program for Earthquake response analysis of horizontally layered sites, Report No. EERC72-12, University of California, Berkeley, 1972.

Seed, H. B., & Idriss, I. M.. Influence of soil conditions on ground motions during earthquakes.Journal of the Soil Mechanics and Foundations Division, ASCE, 95(SM1), 99–137, 1969.

Idriss IM, Sun JI. “User’s Manual for SHAKE91: A computer program for conducting equivalent linear seismic response analyses of horizontally layered soil deposits.” University of California: Center for Geotechnical Modelling, Dept. of Civil and Environmental Engineering, 1992.

Joyner WB, Chen ATF,. Calculation of nonlinear ground response in earthquakes. Bull Seismol Soc Am, 65:1315–36, 1975.

Kramar, S.L., Geotechnical Earthquake Engineering, Prentice-Hall International Series, New Jersey, 1996.

Seed HB, Idriss IM, Soil moduli and damping factors for dynamic response analysis. Report No. EERC70-10, University of California, Berkeley, 1970.

Yu, G., Anderson, J.G., and Siddharthan R.. "On the characteristics of nonlinear soil response," Bulletin of the Seismological Society of America, Vol.83, No 1, pp.218-244, 1992.