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CONE MODEL PREDICTIONS OF DYNAMIC IMPEDANCE FUNCTIONS OF SHALLOW FOUNDATIONS

Dennis R. HILTUNEN [1], Patrick W. DUNN[2], and Ulas TOROS[2]

ABSTRACT

Cone model solutions for square and rectangular footings on the surface of and embedded in both homogeneous and layered profiles are compared with rigorous theoretical solutions. Also, cone model solutions for surface and embedded round, square, and rectangular footings are compared with actual measured behavior. It is demonstrated that even for foundations that significantly deviate from an axi-symmetric shape, sufficient engineering accuracy can typically be obtained via cone models.

Keywords: cone models, foundation dynamics, impedance functions, measured behaviour, predicted behavior

INTRODUCTION

To analyze the vibrations of a foundation on the surface of or embedded in a layered half-space, Wolf and Deeks (2004) have presented an approach using conical bars and beams, called cones. With these models, the complicated, exact formulation of three-dimensional elastodynamics is replaced by simple one-dimensional description of the theory of strength of materials, postulating the deformation behavior (plane sections remain plane). A half-space with linear elastic behavior and hysteretic material damping can consist of any number of horizontal layers either overlying a half-space or fixed at its base. Besides cylindrical foundations, axi-symmetric configurations of arbitrary embedment shape can be processed, with the wall and base of the embedded foundation assumed to be rigid. Dynamic-stiffness coefficients describing the interaction force-displacement relationship are calculated for all frequencies. Only approximations of the one-dimensional strength-of-materials approach based on wave propagation in cones apply, and no other assumptions are made. For each degree of freedom, only one type of body wave exists: for the horizontal and torsional motions, S-waves propagating with the shear-wave velocity; and for the vertical and rocking motions, P-waves propagating with the dilatational-wave velocity. The corresponding displacements can be formulated directly in closed form as a function of the depth of the site, without any spatial Fourier transformation into the wave number domain.

The paper will extend the evaluation of cone model predictions as presented by Wolf and Deeks (2004) beyond comparison of cylindrical foundation behavior with more rigorous analytical solutions. First, cone model solutions for square and rectangular footings on the surface of and embedded in both homogeneous and layered profiles are compared with theoretical solutions of Wong and Luco (1978, 1985) and Mita and Luco (1989) available in the literature. Second, cone model solutions for surface and embedded round, square, and rectangular footings are compared with actual measured behavior. While the database is limited, the well-documented work of Nii (1987) for small-scale foundations provides useful measured foundation behavior to compare with cone model predictions. These

[1]Associate Professor, Department of Civil and Coastal Engineering, University of Florida, USA, E-mail: dhilt@ce.ufl.edu
[2]Grad. Research Assistant, Department of Civil and Coastal Engineering, University of Florida, USA

evaluations will further demonstrate that sufficient engineering accuracy can typically be obtained via the strength-of-materials based analyses of cone models.

WOLF AND DEEKS (2004)

Wolf and Deeks (2004) have presented extensive evaluation of their methodology. In these evaluations, cone model predictions are compared with results from rigorous, three-dimensional, elastodynamic solutions, usually the thin-layer finite element method. The evaluations are presented for axi-symmetric foundations, for all degrees of freedom, and for dimensionless frequencies typically up to six. The half-space materials are assumed to be linear elastic with hysteretic material damping. Embedded foundations are assumed to be in full contact with surrounding soil. The specific cases examined are as follows:

- Both surface and embedded cylindrical foundations on/in a homogeneous half-space.
- Both surface and embedded cylindrical foundations on/in a homogeneous layer fixed at its base.
- Cylindrical foundations on the surface of a layered half-space.
- Cylindrical foundations embedded in a layered half-space.
- A cylindrical foundation embedded in an incompressible, layered half-space.
- A hemi-ellipsoid embedded in a homogeneous half-space.
- A sphere embedded in a homogeneous full-space.

For all of these cases, the deviations in impedance functions between cone model predictions and the rigorous solutions are within the range ±20%. Wolf and Deeks (2004) conclude that this is sufficient engineering accuracy.

WONG AND LUCO (1978)

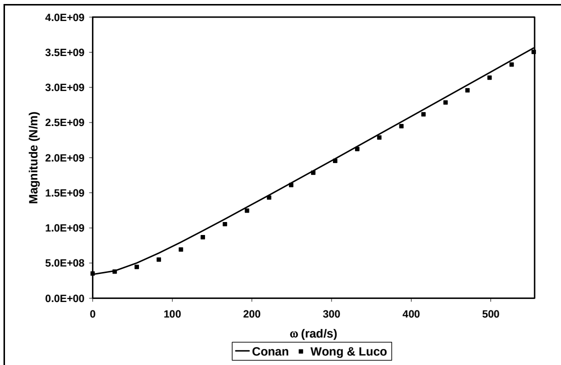
While the extensive evaluation results presented by Wolf and Deeks (2004) are convincing, it is also fair to observe that most real shallow foundations are not axi-symmetric. In fact, the majority of real shallow foundations are probably rectangular in shape. Hence, the first evaluation presented herein is to assess the ability of cone models to determine impedance functions for non-axi-symmetric foundations, in this case rectangular.

Wong and Luco (1978) have presented a detailed tabulation of numerical values for the impedance functions for a massless, rigid, rectangular foundation perfectly bonded to the surface of a viscoelastic, homogeneous half-space. Impedance functions are provided in normalized form for four aspect ratios, namely length/width=1, 2, 3, and 4, and for dimensionless frequencies up to 10. The computations have been performed for two values of Poisson's ratio, ν=0.33 and 0.45, and for three values of hysteretic damping ratio, ξ=0, 0.02, and 0.05.

Five specific rectangular foundation models were constructed to compare cone model predictions with the tabulated results of Wong and Luco (1978). Models 1-4 consisted of foundations with aspect ratios of 1, 2, 3, and 4, respectively, and a Poisson's ratio of ν=0.33. Model 5 was a rectangular foundation with an aspect ratio of 4, and a Poisson's ratio of ν=0.45. All models were constructed using a foundation width of 2 m, a shear wave velocity of 125 m/s, a mass density of 1600 kg/m^3, and a hysteretic damping ratio, ξ=0.05.

For each of the five models, cone model impedance functions for vertical, horizontal, rocking, and torsional degrees of freedom were calculated using the CONAN executable program provided by Wolf and Deeks (2004). Please note that the cone models assume that the coupling impedance of a surface foundation is zero. Indeed, the coupling impedances tabulated by Wong and Luco (1978) are small in comparison to the other degrees of freedom. Also, since the cone model impedances are calculated for circular disk (axi-symmetric) foundations, each of the five rectangular foundations were converted to an equivalent disk for CONAN computations. The equivalency was based upon contact area for translational modes (vertical, horizontal), and moment of inertia for rotational modes (rocking, torsion).

By way of example, figures 1-2 present impedance function comparisons for Models 1 and 4, respectively. In each figure, impedance functions for each degree of freedom and as computed by CONAN are compared with the corresponding Wong and Luco (1978) data. Each graph plots the magnitude of the impedance function in dimensional form versus circular frequency. Impedance function magnitude is simply the vector summation of the real and imaginary components. It is observed that for all cases presented, the ±20% engineering accuracy documented by Wolf and Deeks (2004) is also maintained for these foundations on the surface of a homogeneous half-space. While it may not be surprising that results for a square shape are very similar to an axi-symmetric solution (figure 1), it is also evident that the cone model solutions are in good agreement for a rectangular footing with an aspect ratio as large as four (figure 2). It should also be noted that the comparison results for the remaining Models 2, 3, and 5 not shown herein are of equal or better quality.



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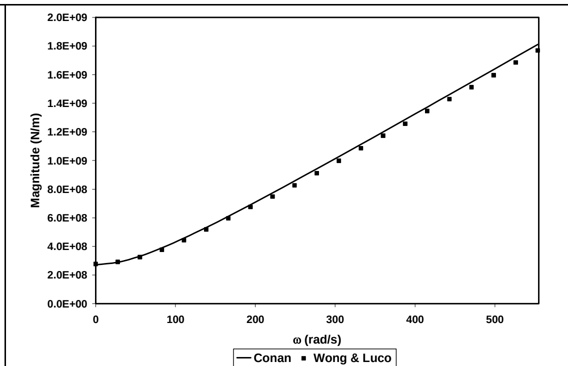
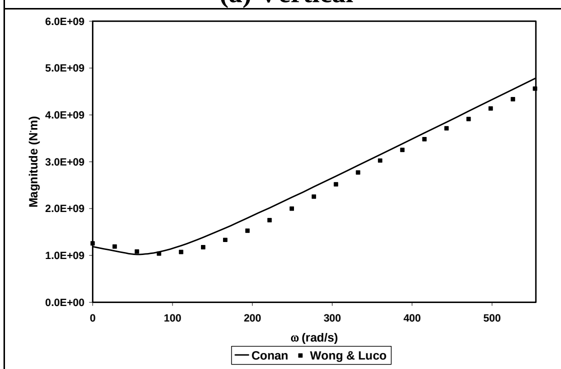
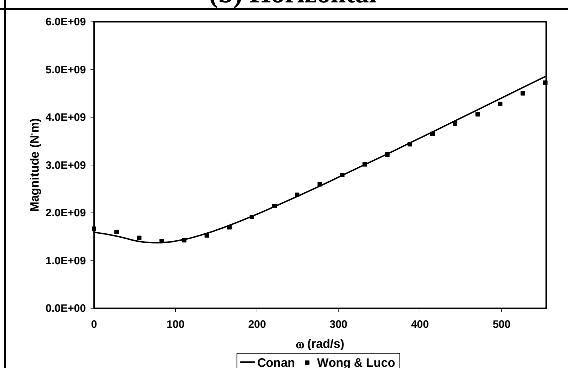


(b) Horizontal



(c) Rocking

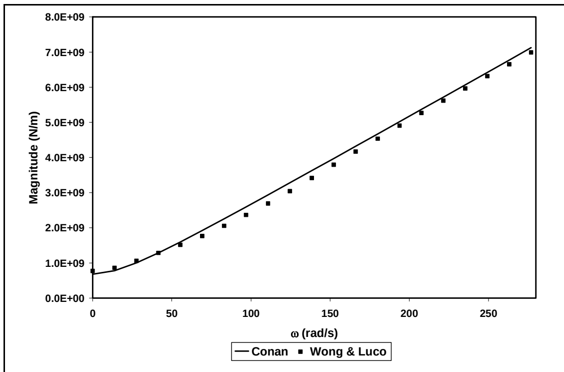


(d) Torsion

Figure 1. Impedance Functions for Wong and Luco (1978) Model 1: Square

MITA AND LUCO (1989)

To explore the influence of foundation embedment, Mita and Luco (1989) have presented a detailed tabulation of numerical values for the impedance functions for a massless, rigid, square foundation embedded in and perfectly welded to a viscoelastic, homogeneous half-space. Impedance functions for horizontal, coupling, rocking, vertical, and torsional degrees of freedom are provided in normalized form for embedment to half-width ratios up to 1.5, and for dimensionless frequencies up to three. The computations have been performed for three values of Poisson's ratio, ν=0.25, 0.33 and 0.4. All computations assumed hysteretic damping ratios of 0.001 for shear waves, and 0.0005 for compression waves. The numerical results were calculated using a hybrid approach described by Mita and Luco (1987).



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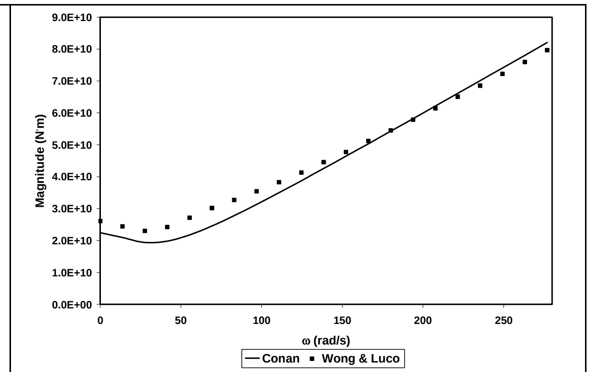
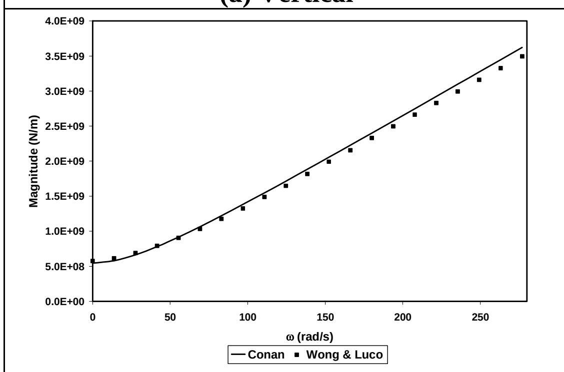
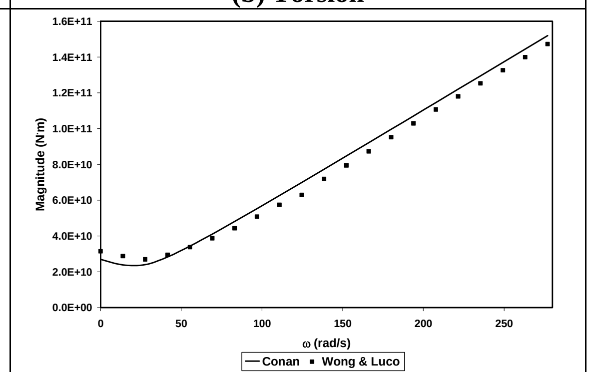


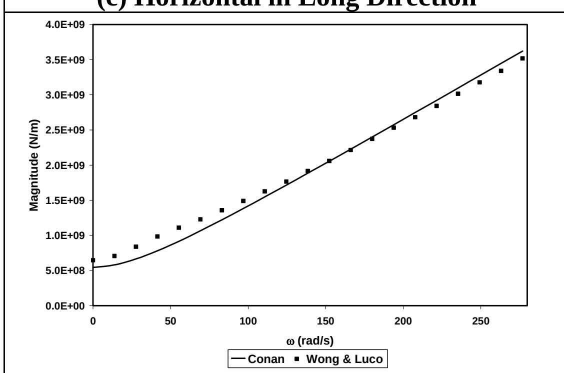
(b) Torsion



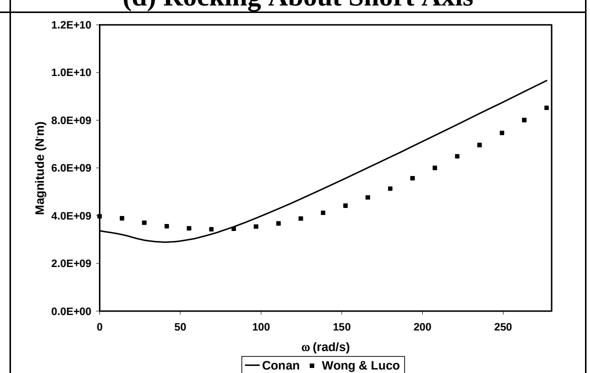
(c) Horizontal in Long Direction



(d) Rocking About Short Axis



(e) Horizontal in Short Direction

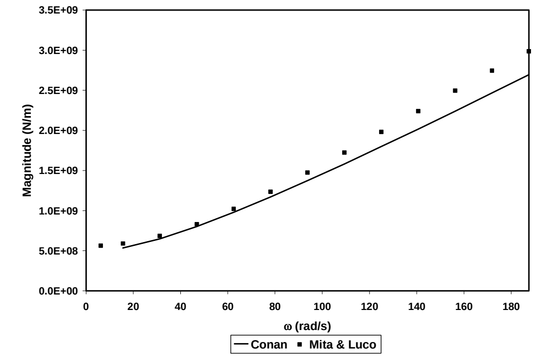


(f) Rocking About Long Axis

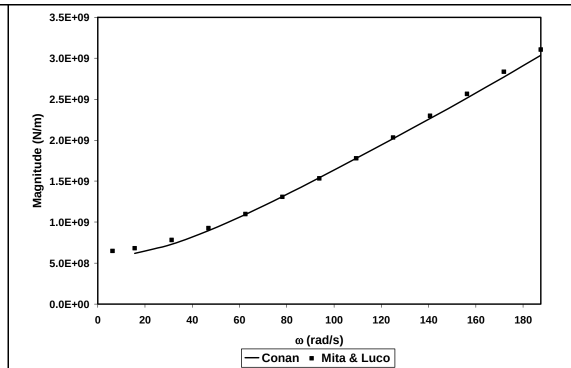
Figure 2. Impedance Functions for Wong and Luco (1978) Model 4: Length/Width = 4

Five specific square foundation models were constructed to compare cone model predictions with the tabulated results of Mita and Luco (1989). Models 1-3 consisted of foundations with embedment to half-width ratios of 0, 0.5, and 1.5, respectively, and a Poisson's ratio of ν=0.33. Model 4 was a square foundation with an embedment to half-width ratio of 1.5, and a Poisson's ratio of ν=0.25, while Model 4 was a square foundation with an embedment to half-width ratio of 1.5, and a Poisson's ratio of ν=0.4. All models were constructed using a foundation width of 2 m, a shear wave velocity of 125 m/s, a mass density of 1600 kg/m^3, and hysteretic damping ratios as noted above for the Mita and Luco (1989) results. As with the comparisons presented above, cone models for these square footings were based upon equivalent circular disks.

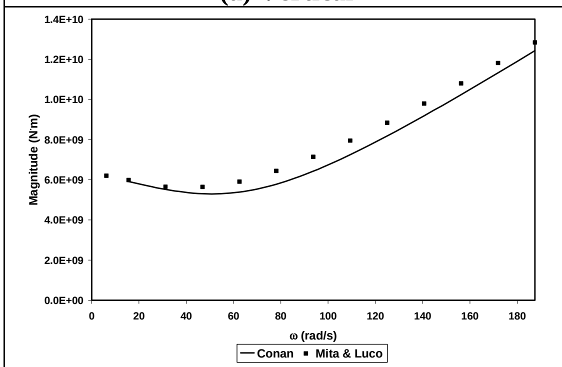
By way of example, figure 3 presents impedance function comparisons for Model 3, a case with maximum embedment. In each figure, impedance functions for each degree of freedom and as computed by CONAN are compared with the corresponding Mita and Luco (1989) data. As in the previous sections, plotted in each graph is the magnitude of the impedance function in dimensional form versus circular frequency. It is observed that for all cases presented, the ±20% engineering accuracy documented by Wolf and Deeks (2004) is also maintained for these square foundations embedded in a homogeneous half-space. It should also be noted that the comparison results for the remaining Models 1, 2, 4, and 5 not shown herein are of equal or better quality.



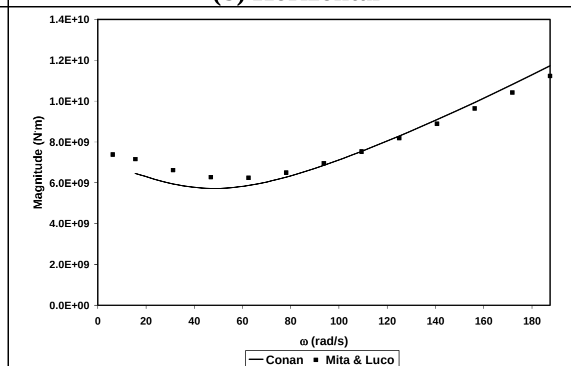
(a) Vertical



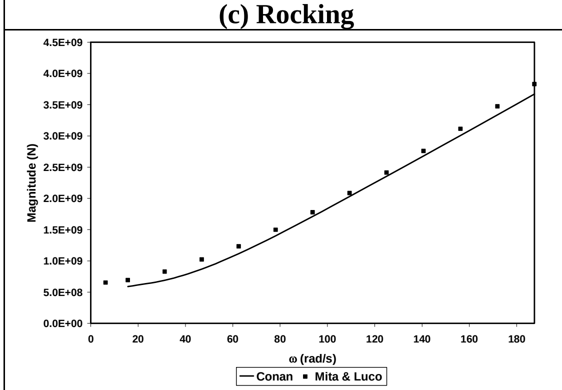
(b) Horizontal



(c) Rocking



(d) Torsion



(e) Coupling

Figure 3. Impedance Functions for Mita and Luco (1989) Model 2: Embedded

WONG AND LUCO (1985)

To explore the influence of soil layering, Wong and Luco (1985) have presented a detailed tabulation of normalized horizontal, coupling, rocking, vertical, and torsional impedance functions for a rigid, massless, square foundation resting on two types of layered viscoelastic soil models. The first soil model consists of a uniform layer over a uniform half-space, while the second model consists of a layer with linearly varying properties over a uniform half-space.

The impedance functions have been calculated based upon an extension to the case of layered media of the approach proposed by Wong and Luco (1976). The approach computes a numerical solution of an integral equation involving the Green's functions for a layered medium. The rigid, square foundation is assumed to be perfectly bonded to the surface of the layered half-space.

The impedance function results are presented for two Poisson's ratio combinations: 1) ν_1=0.33 for the layer, and ν_2=0.33 for the half-space, and 2) ν_1=0.45 for the layer, and ν_2=0.33 for the half-space. Three values for the contrast in shear wave velocity between layer and half-space are provided, namely V_{s1}/V_{s2}=0.3, 0.6, and 0.8. The density ratio between the half-space and the layer is fixed at ρ_2/ρ_1=1.13, and the material damping constants are fixed at 0.03 and 0.05 for the half-space and layer, respectively.

In each of the tables, the real and imaginary parts of the normalized impedance functions are presented for 21 dimensionless frequencies ranging from 0.1 to 5.0, and for five values for the ratio of layer thickness (H) to foundation half-width (a): H/a=0.5, 1, 2, 3, and 4.

Eight specific square foundation models were constructed to compare cone model predictions with the tabulated results of Wong and Luco (1985). For simplicity, all eight models consisted of a uniform layer over a half-space. In addition, all cone models were assigned a fixed density ratio of 1.13, and fixed material damping values of 0.03 and 0.05, as noted above for the Wong and Luco (1985) results. As with the comparisons presented above, cone models for these square footings were based upon equivalent circular disks. Additional parameters for the eight models are shown in table 1.

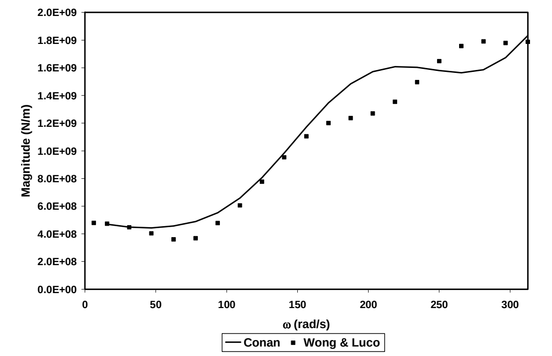
Table 1. Parameters for Eight Wong and Luco (1985) Comparisons

Model No.	Parameters*	Comment
1	V_{s1}/V_{s2} = 0.6, ν_1 = 0.33, H/a = 2 a = 2 m, V_{s1} = 125 m/s, ρ_1 = 1600 kg/m^3	Basis model
2	a = 1 m	Influence of scale: 1/2 footing size
3	V_{s1} = 250 m/s	Influence of scale: 2x shear wave velocity
4	H/a = 0.5	Influence of layer thickness: 1/4 layer thickness
5	H/a = 4	Influence of layer thickness: 2x layer thickness
6	V_{s1}/V_{s2} = 0.3	Influence of stiffness contrast: higher
7	V_{s1}/V_{s2} = 0.8	Influence of stiffness contrast: lower
8	ν_1 = 0.45	Influence of Poisson's ratio: higher

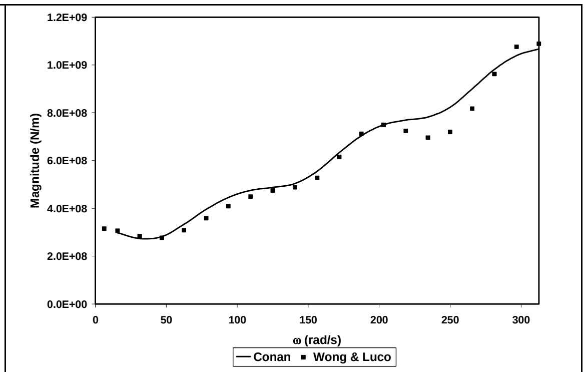
*Note: For Models 2-8, the parameters shown are those that are different from the parameters for the Basis Model 1.

Model 1 was chosen as a basis model with the parameters indicated. For Models 2-8, the parameters shown in table 1 are those that are different from Model 1, while all other parameters remain the same. Models 2 and 3 briefly examine the issue of scale, since the Wong and Luco (1985) results are presented in dimensionless format. Model 2 has a foundation half the size of Model 1, while Model 3 has increased the shear wave velocity of the layer by a factor of two. Models 4 and 5 address the influence of layer thickness, Models 6 and 7 the influence of stiffness contrast, and Model 8 the influence of Poisson's ratio. While these eight models certainly do not exhaust the full range of possibilities presented by the Wong and Luco (1985) data, these models explore the ranges in principal parameters.

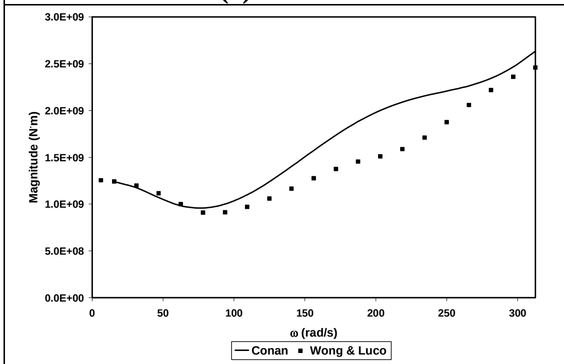
By way of example, figures 4-7 present impedance function comparisons for Models 1, 4, 6, and 8, respectively. Results for the remaining Models 2, 3, 5, and 7 are also discussed below. In each figure, impedance functions for each degree of freedom and as computed by CONAN are compared with the corresponding Wong and Luco (1985) data. As in the previous sections, plotted in each graph is the magnitude of the impedance function in dimensional form versus circular frequency. It is observed that for the vast majority of cases presented, the ±20% engineering accuracy documented by Wolf and Deeks (2004) is also maintained for these square foundations on the surface of a layered half-space. The only notable exceptions are possibly the rocking mode in Model 4, and the vertical



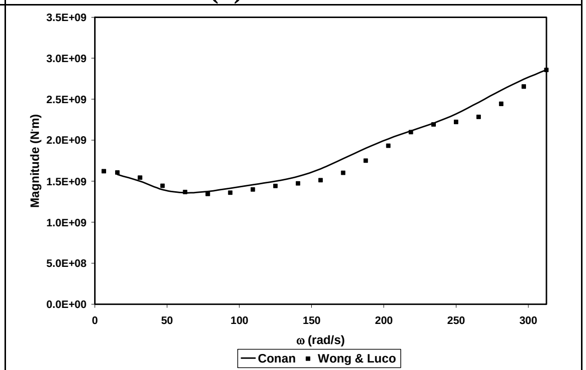
(a) Vertical



(b) Horizontal

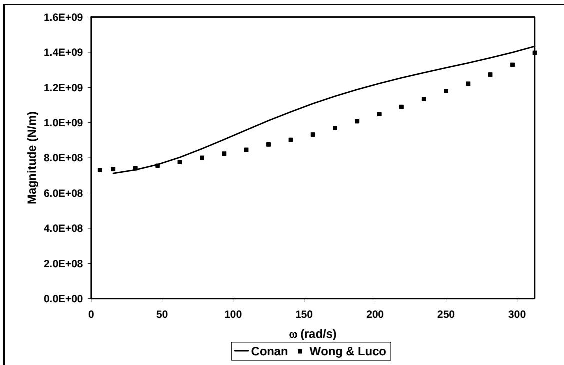


(c) Rocking

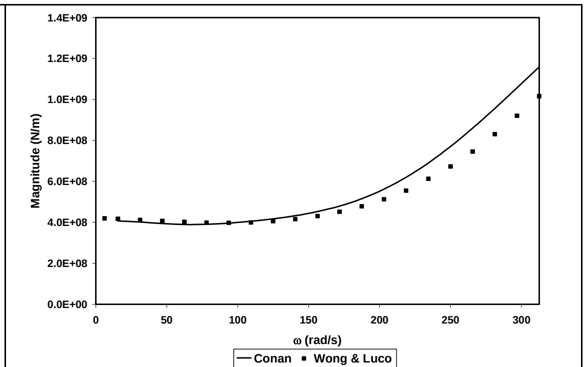


(d) Torsion

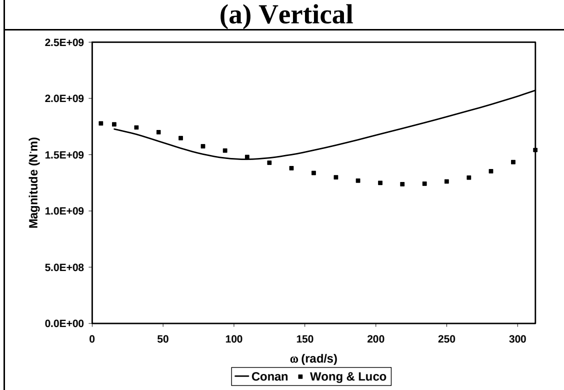
Figure 4. Impedance Functions for Wong and Luco (1985) Model 1: Basis Model



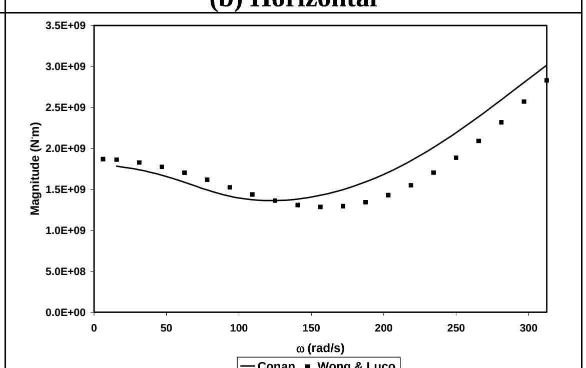
(a) Vertical



(b) Horizontal



(c) Rocking



(d) Torsion

Figure 5. Impedance Functions for Wong and Luco (1985) Model 4: ¼ Layer Thickness

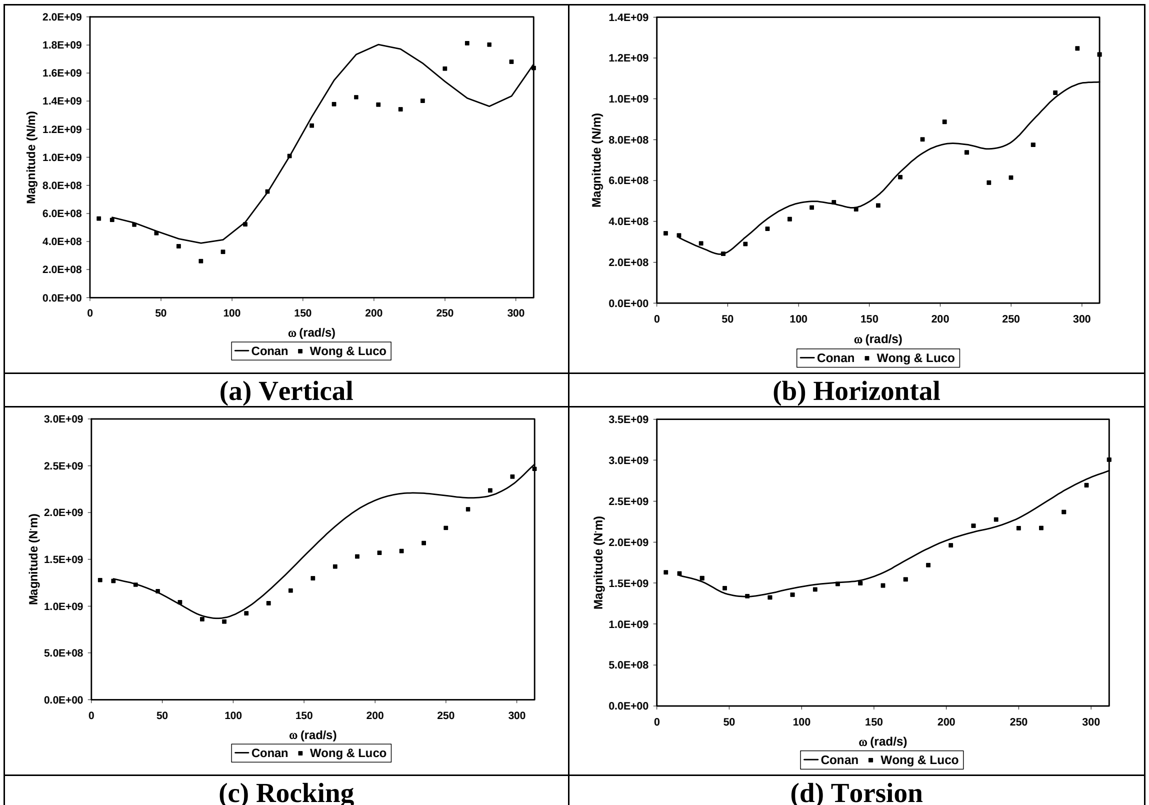


Figure 6. Impedance Functions for Wong and Luco (1985) Model 6: Higher V_s Contrast

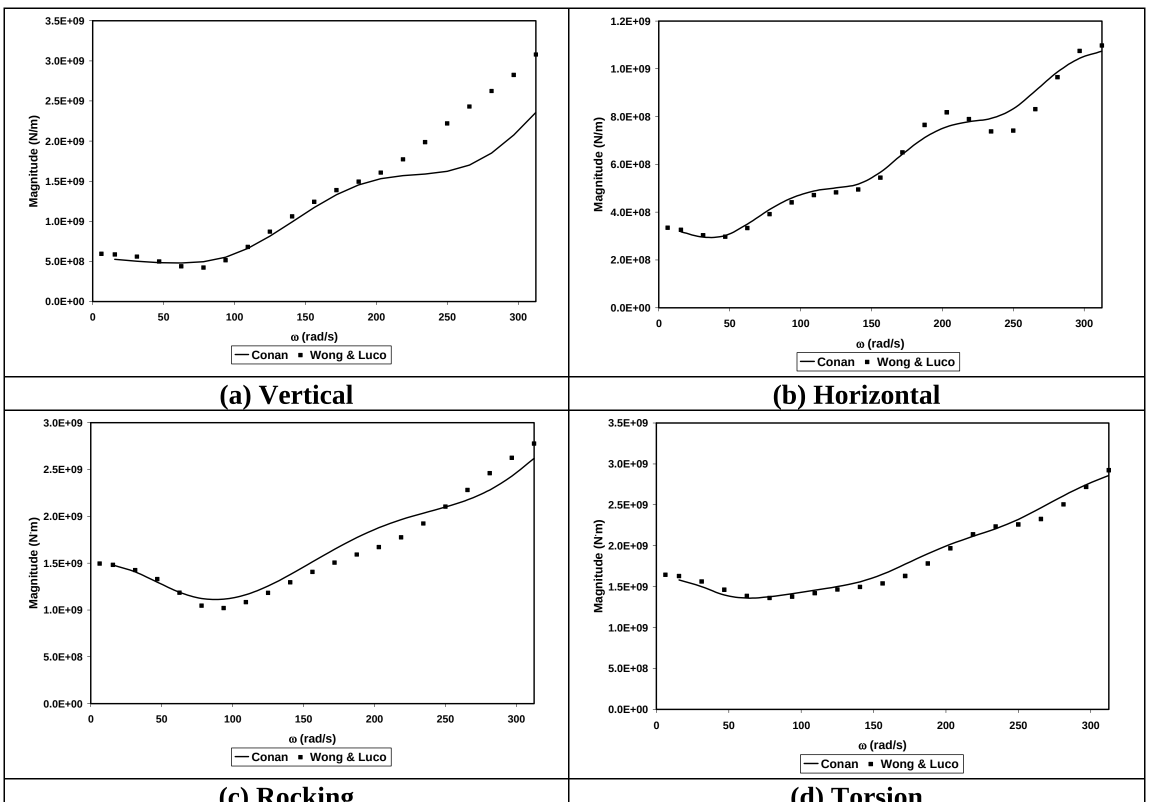


Figure 7. Impedance Functions for Wong and Luco (1985) Model 8: Poisson's Ratio = 0.45

mode in Model 8. It should also be noted that the remaining Models 2, 3, 5, and 7 that are not presented herein, display equal if not better comparisons than shown in figures 4-7. The results for Models 2 and 3 indicate that scale is not a factor in these comparisons. Indeed, while the impedance function magnitudes change with scale, the comparisons between cone model and Wong and Luco (1985) are identical to those shown in figure 1. Also, the results for Models 5 and 7 are better than for Models 4 and 6, respectively. This is to be expected since Model 5 has a much thicker layer than either Models 1 or 4, and Model 7 has a lower stiffness contrast than either Models 1 or 6.

NII (1987)

While all of the impedance function solutions presented above are based upon theoretical models, Nii (1987) reported experimental results for vertical dynamic impedance of small-scale footings attached to or embedded in a model half-space. The half-space was constructed of room temperature vulcanizing silicone rubber, and was cast in a steel tank 1.7 m long, 1.4 m wide, and 0.4 m high. The mechanical properties of the half-space are reported as follows:

- V_s = 11.3 m/s
- V_r = 10.8 m/s
- V_p = 1000 m/s
- $\nu \approx 0.5$
- Damping ratio (ξ) = 0.031
- ρ = 0.98 g/cm^3

The model footings were constructed of acrylic resin, with dimensions as shown in table 2. Nii (1987) reports that even though the acrylic footings were smooth, the surface of the half-space rubber was slightly adhesive, and the footings and half-space behaved as though they were bonded. The circular and rectangular footings were forced into vertical vibration via a mechanical shaker. The experiments were conducted at frequencies where the effects of reflected waves from the boundaries of the model half-space were negligible or diminished (Nii [1987]), and results of the experiments are reported for dimensionless frequencies up to 8.

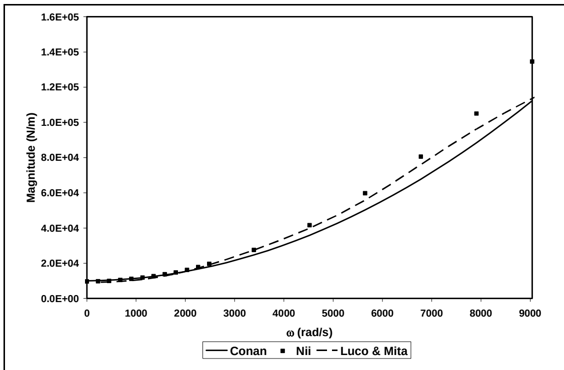
Table 2. Model Footings of Nii (1987)

Shape					
Circular			Rectangular on Surface		
Location	Radius (cm)	Thickness (cm)	Length/Width	½ Width (cm)	Thickness (cm)
Surface	1*, 2, 3, and 4	1	1	1*, 2, 3, and 4	1
Fully Embedded	2*	2	1.5	1*, 2, and 3	1
			2	1* and 2	1

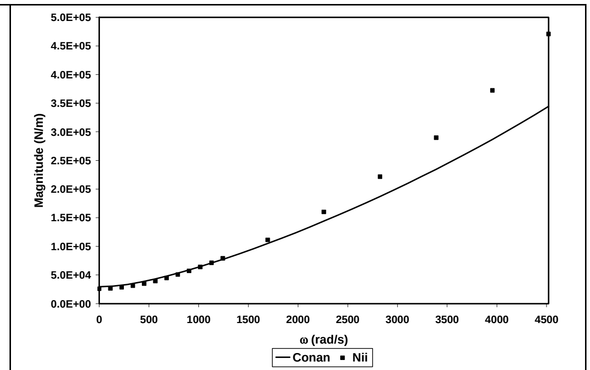
*Note: Analyzed via cone model herein.

Vertical impedances were computed with cone models for five of the experiment footings shown in table 2: two circular footings, one surface and one embedded, and three rectangular footings with aspect ratios of 1, 1.5, and 2. As with the comparisons presented above, cone models for the rectangular footings were based upon equivalent circular disks, in this case based upon contact area for the vertical mode of vibration.

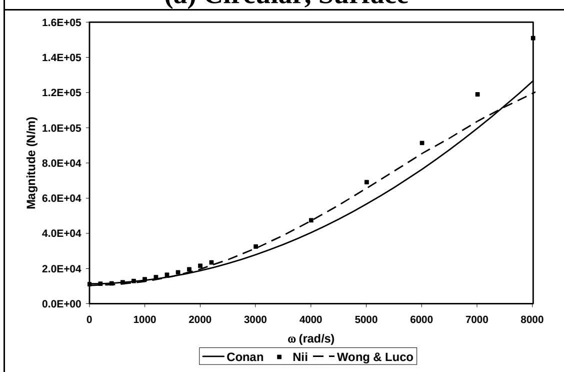
Cone model vertical impedance solutions are compared with the experimental results of Nii (1987) in figure 8 for each of the five footings considered. As in the previous sections, plotted in each graph is the magnitude of the impedance function in dimensional form versus circular frequency. It is observed that agreement between cone model predictions and the Nii (1987) experimental results is quite good at low frequencies for all five cases. There is a consistent under-prediction of the magnitude at higher frequencies for all five cases. Evaluation of the real (stiffness) and imaginary (damping) components of the impedance functions (not shown), also reveals a consistent pattern for all five cases: the real component is over-predicted, while the imaginary component is under-predicted, with a net under-prediction of the overall magnitude as noted in the figures. However, the ±20% engineering accuracy documented by Wolf and Deeks (2004) appears to be maintained, except for maybe the embedded, circular footing (figure 8b).



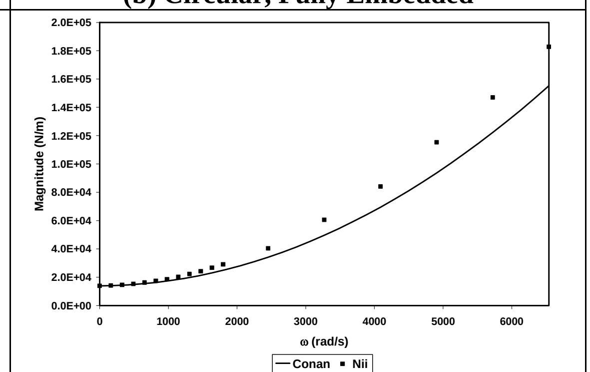
(a) Circular, Surface



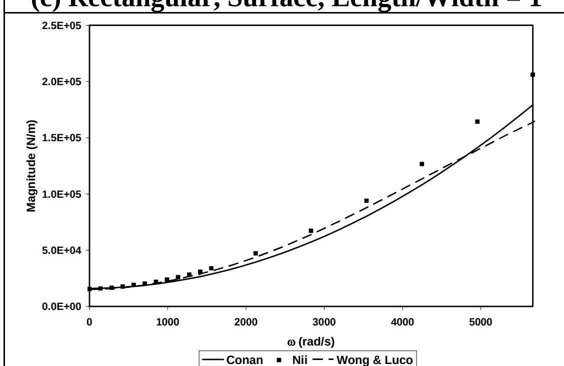
(b) Circular, Fully Embedded



(c) Rectangular, Surface, Length/Width = 1



(d) Rectangular, Surface, Length/Width = 1.5



(e) Rectangular, Surface, Length/Width = 2

Figure 8. Vertical Impedance Functions for Five Nii (1987) Model Footings

Also shown in figure 8 for three of the five cases are impedance functions from the literature that were determined via more rigorous, three-dimensional, elastodynamic solution techniques. The solution for the surface case (figure 8a) was obtained from Luco and Mita (1987). The two solutions for the rectangular footings (figures 8c and 8e) were obtained from Wong and Luco (1978). Solutions for the two remaining cases (circular, fully embedded and rectangular, surface, length/width = 1.5) were not readily available for Poisson's ratio near 0.5.

First, it is noted that the three impedance functions (experimental, rigorous, and cone) are in good accord for the three cases presented (circular, square, and rectangular). This is encouraging. On the other hand, as one might expect, there is closer agreement between the "rigorous" impedance function solutions from the literature and the experimental results, than between the "approximate" cone model solutions and the experimental results. However, it is interesting to note that the relative agreement between impedances for all three cases presented is very similar. This might suggest that the "error" in the cone model impedances is not due to approximating a square or rectangular footing with an equivalent disk.

CONCLUSIONS

Cone model solutions for surface and embedded round, square, and rectangular footings have been compared with results from rigorous theoretical methodologies and from experimental measurements. For the vast majority of cases presented, the ±20% engineering accuracy documented by Wolf and Deeks (2004) can typically be obtained via cone models, even for foundations that significantly deviate from an axi-symmetric shape.

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