

## **CENTRIFUGE MODELLING OF ROCKING BEHAVIOUR OF BRIDGES ON SHALLOW FOUNDATIONS**

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### **ABSTRACT**

The capacity, energy dissipation, and self centering characteristics of shallow foundations have potentially important effects on the performance of a soil-foundation-bridge deck system. As the foundation width decreases, the energy dissipated by the foundation increases while permanent settlement and rotation increase and construction costs decrease. Centrifuge tests on rocking of bridge foundations show that square shallow foundations with dimensions significantly smaller than those designed according to current procedures performed well. The rocking foundations displayed excellent ductility, relatively small permanent deformations, and they provide a self-centering mechanism. Furthermore, the moment capacity of rocking foundations can in most cases be accurately predicted. Considering these benefits, rocking foundations should be considered to be a practical tool for reducing demands on structural elements of a bridge and as a potential tool for dissipating energy through plastic deformation of the soil.

**Keywords:** shallow foundations, bridge, soil-structure interaction, centrifuge, earthquake

### **INTRODUCTION**

A schematic diagram of a rocking bridge bent is shown in Figure 1(a). The important parameters are the masses of the deck and footing, the stiffness of the column, length of the footing ( $L=B$ ), critical contact length ( $L_c$ ) which is the length of foundation in contact with the soil when the bearing capacity is mobilized, the height to center of gravity of the deck mass ( $H_{cg,deck}$ ), and the ground motion.

Housner (1963) pointed out that structures allowed to rock survived in several earthquakes while more modern structures were severely damaged. Housner (1963) showed that the dynamic characteristics of rocking systems on an elastic half space are much different than those of elastic fixed base systems. In practice there are currently two approaches used by the California Department of Transportation (Caltrans) to characterize rocking of bridge foundations. The first approach, typically used in new construction, allows for no rocking or uplift of the footing. This ensures that the failure mechanism is plastic hinging in the column (Alameddine and Imbsen 2002). Column yielding may be preferred because soil response is considered to be unreliable and column damage may be easily inspected following an earthquake.

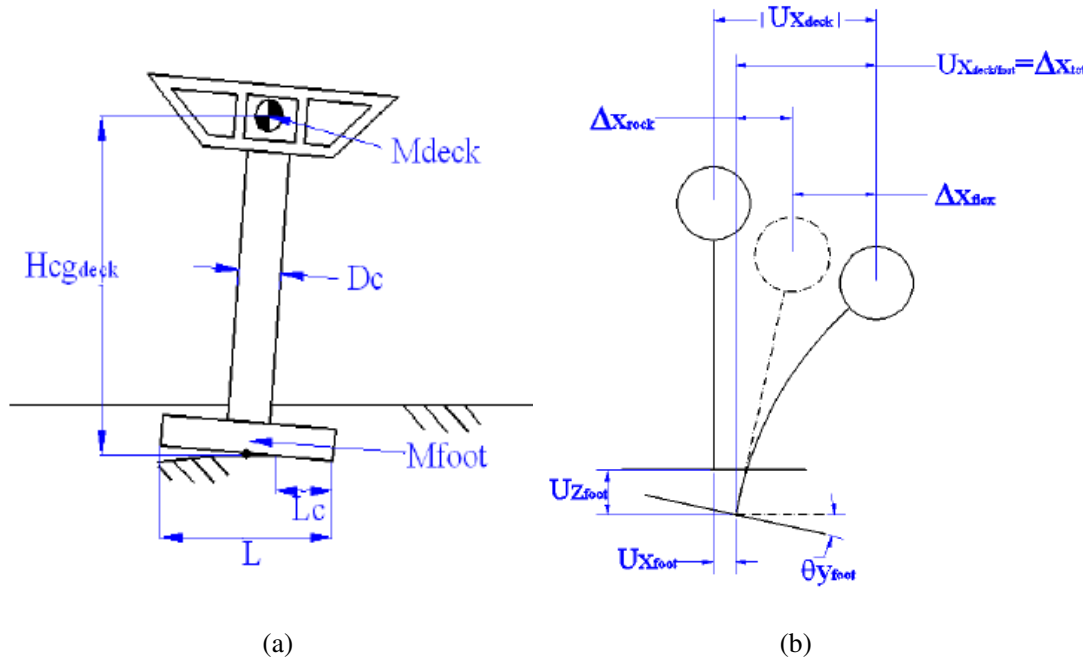
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**Figure 1(a). Schematic of problem and definition of some system parameters.**  
**(b) Definition of coordinate system and displacements; x is horizontal and z is downward.**

The second approach is used in retrofit evaluations, in which case rocking of the footing is permitted. Estimates of the displacements of the foundation are calculated using a procedure based on the concepts of Housner (1963) implemented in an algorithm formulated by Priestly et al (1996) and packaged by Caltrans in a program called Winrock (Alameddine and Imbsen 2002). The Winrock procedure assumes no permanent vertical or horizontal translation of the foundation and no plastic deformation of the soil and damping is estimated based on radiation damping only. Also, it does not provide a means to estimate settlements, sliding, or permanent rotation.

The model tests described herein and elsewhere have shown that the soil around the foundation does not remain elastic and a significant amount of energy is dissipated by foundation rocking through moment-rotation, shear-sliding, and vertical load-settlement hysteresis (Gajan et al. 2005, Taylor et al. 1981, and Faccioli et al. 2001).

Some of the known effects of rocking are summarized below:

- Peak moment demand in the column is limited by moment capacity of the foundation. In this respect the foundation can act like a mechanical fuse.
- Uplift of a foundation stores gravitational potential energy. Closure of the gap upon unloading restores the footing toward its initial position.
- Local bearing pressures increase causing plastic deformation of soil around the footing which is a source of hysteretic damping.
- Rocking results in lengthening of the natural period which tends to reduce acceleration and force demands and increase displacement demands on the superstructure.
- The magnitude of settlement caused by rocking depends on the number of cycles and amplitude of loading as well as the bearing capacity.

One goal of this paper is to help engineers to quantify the above factors so that rocking may be accounted for in the design process. If properly quantified, the benefits of rocking may be used to reduce construction costs without unduly sacrificing performance.

## CENTRIFUGE EXPERIMENTS

To explore the response of bridges on shallow foundations, centrifuge experiments were performed on the geotechnical centrifuge at UC Davis. As described by Schofield (1980) and Kutter (1995), scaling factors for centrifuge modeling are well established in the literature. The basic centrifuge scaling laws can be derived by defining the length scale factor as  $L^* = L_m/L_p$ , by assuming that identical materials are used in model and prototype (hence material densities scale according to  $\rho^* = \rho_m/\rho_m = 1$ ), and requiring that the stresses in the model should scale to be identical to those in the prototype; i.e.,  $\sigma^* = \sigma_m/\sigma_p = 1$ . Because soil has nonlinear mechanical properties that are a function of confining stress, it is important that stresses scale one to one. Simple dimensional analysis shows that  $\sigma^* = 1$  may be accomplished by increasing accelerations (including gravity) by a factor  $a^* = 1/L^*$  and scaling time by  $t^* = L^*$ . In the present paper,  $L^* = 1/42.9$  and  $a^* = 42.9$ .

Testing was performed on many test structures which were placed into one model soil container. The structures were spaced so that they were an adequate distance from each other and the walls of the container. Each structure location was given a station name, Station A through Station G. Slow cyclic tests at Stations A and B were directly loaded with a hydraulic actuator and Stations C through G (Fig 2 and 3) were excited by ground motions applied to the base of the soil container.

The model tests were scaled from typical bridge configurations used by Caltrans. The prototype footings were square with widths of 3, 4, or 5 times the diameter of the column ( $D_c = 1.8$  m). The prototype scale mass and width of the footings at dynamic stations C, D, E, and F are (336 Mg, 8.9 m), (246 Mg, 7.1 m), (173 Mg, 5.4 m), and (246 Mg, 7.1 m) respectively. The mass of the deck at stations C, D, E, and F is 926 Mg. The fixed base natural period of these structures is 1.6 seconds. The total structure mass and footing width of structures at stations A and B are (1250 Mg, 8.9m) and (1080 Mg, 5.4m) respectively. Nevada Sand (mean grain size of 0.15 mm) was placed by dry pluviation in air to a uniform relative density of about 80% to create a 210 mm deep soil deposit in the 1.76 x 0.9 m (75.6 x 38.9 m prototype scale) model container. For this density, and for pressures appropriate to the footing loads, the friction angle is about 40° to 42° (Gajan 2006). The foundation soil contained accelerometers for measuring both vertical and horizontal accelerations.

The vertical bearing capacity of a shallow foundation on sand for bridge structures turns out to be quite large ( $F = 30$  to 70 for the experiments described here). This large factor of safety with respect to bearing capacity is reasonable because the governing criteria for large footings on sand tend to be the allowable settlement and the required moment capacity; bearing capacity does not determine their size. The footings are sized by allowable bearing pressures. The footings described herein had bearing pressures that ranged between 80% and 150% of the pressure that would be expected to cause 25 mm of settlement under the static vertical loads.

The prototype structure was a typical reinforced concrete single column bridge bent modeled as a “lollipop” structure with a deck mass and column connected to a shallow spread footing. Figures 2 and 3(a) depict the system modeled in the centrifuge tests carried out at UC Davis. The deck was modeled by a steel block, the reinforced concrete column was modeled by an aluminum tube that had a bending stiffness,  $EI$  closely scaled to the calculated  $EI$  of the cracked section of the prototype concrete column. The footings were constructed of aluminum plates with sand glued to their bases to provide a rough concrete-like interface with the soil.

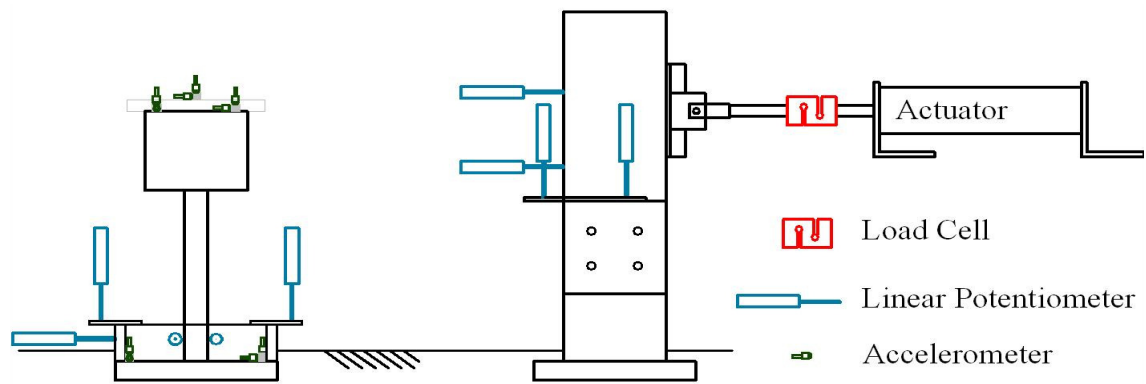
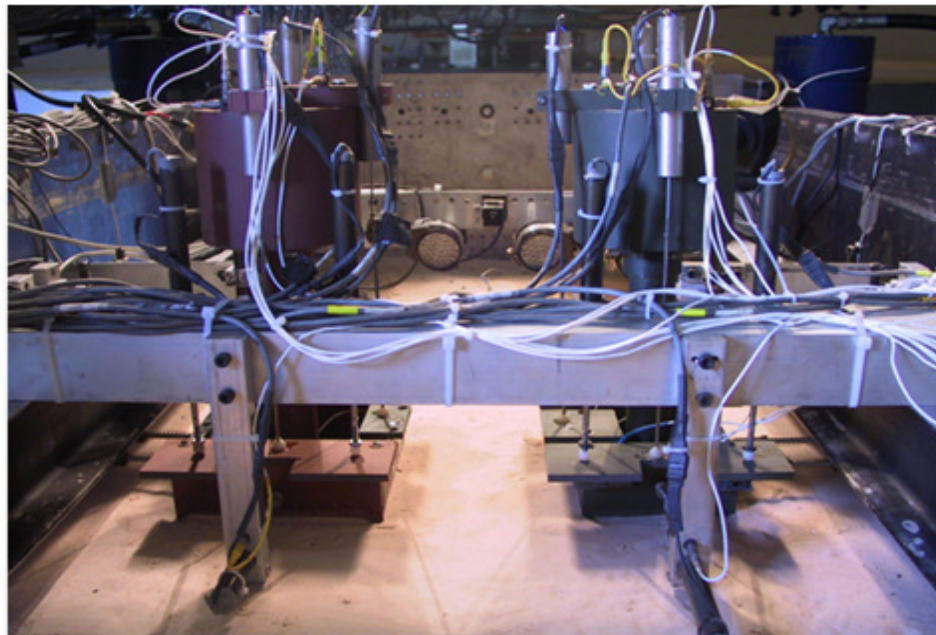


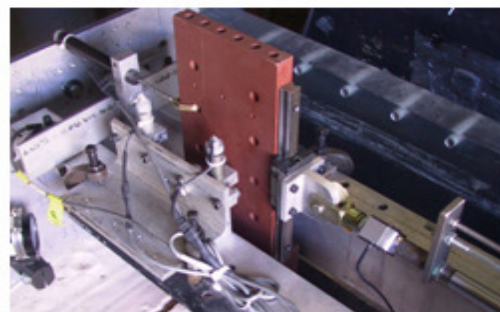
Figure 2. Side view of structures and instrumentation for a dynamic test and a slow cyclic test.



(a)



(b)



(c)

Figure 3. (a) Two dynamic stations fully instrumented. (b) Gapping around perimeter of footing after slow cyclic test. (c) Connection between top of rigid wall with bearing rail attachment to load cell and actuator in a slow cyclic test.

For slow cyclic tests, the vertical load on the footing was scaled, but the distribution of the mass and the stiffness of the structure were not considered important parameters. Therefore, essentially rigid steel plates were used to provide the desired mass and the wall acted as a vertical cantilever upon which lateral loads were applied by a hydraulic actuator acting horizontally at a height approximately equal to the elevation of the effective height of center of gravity of the prototype deck-footing system.

### **Loading and test setup sequence**

At the time that the sand was placed, all seven model foundations were embedded to a depth of 40 mm (1.7 m prototype) at seven stations (A-G). Structures at one or two stations were tested during a given spin; the structures were bolted to their embedded foundation, then the centrifuge was spun and the loading events were applied. After stopping the centrifuge, the model structures were removed and new structure(s) were placed at other station(s) for testing in the next spin.

The sequence of testing involved 5 different spins. Prior to the first spin, a rigid wall structure was attached to the square footing. Then an actuator was attached to the wall as depicted in Figure 2. Restraint on the sides of the wall was provided to prevent out of plane movement as described in detail by Gajan (2006). After spinning up to 42.9 g, slow cyclic lateral loading was applied by an actuator. The actuator was typically commanded to apply packets of 3 cycles of a sinusoidal displacement wave. For Stations A and B, 8 to 12 packets of sine waves with amplitudes varying between 0.14% to 5.4% of the effective height were applied to the structure. Results from the last three packets of sine waves with amplitudes 5.4%, 2.2%, and then 0.54% are shown in Figure 4.

Structures at Stations C-G were subject to dynamic loading using the shaking table mounted on the centrifuge to shake the entire model container. The ground motions imposed on the model container were scaled and filtered motions from recordings in the Tabas 1978 earthquake and a Los Gatos recording of the 1989 Loma Prieta earthquake. These motions come from the near field records posted at the SAC Steel Project (2006) website. Twelve to fifteen scaled motions were applied to each structure. The testing sequence for dynamic stations started with low amplitude step waves, followed by scaled down earthquake ground motions, then large amplitude earthquake ground motions and finally step waves similar to those applied before strong shaking. The peak ground accelerations ranged between 0.1 g and 0.8 g.

The instrumentation is depicted in the drawings in Figure 2 as well as the photos in Figure 3. For slow cyclic testing, each structure was instrumented with a load cell to measure the horizontal actuator load and four displacement transducers; any three of the four displacement sensors was sufficient to determine the rotation, settlement and sliding of the footing.

During dynamic loading, six accelerometers were placed on the foundation and six on the deck in order to resolve all six rotational and translational degrees of freedom for these relatively rigid bodies. Six displacement transducers were also used to determine the six displacement degrees of freedom of the footing. A plastic frame, shown in Figures 2 and 3(a) was attached to the embedded footings to provide accurate surfaces on which to mount the displacement transducer probes.

## **EXPERIMENTAL RESULTS**

The coordinate system as well as definition of displacements is indicated in Figure 1(b). The data from all of the accelerometers and displacement transducers mounted at various points on the foundation and deck were combined and processed to provide the measurements of the translational and rotational accelerations and displacements at the center of gravity of the foundation and deck mass. The deck mass included attached instrumentation and half the column mass. The footing mass included the attached instrumentation, half of the column mass, as well as the plastic frame fixed to the foundation.

The rocking moment,  $M_y$ , and the horizontal sliding force,  $F_x$ , computed at the base center point of the footing are calculated as follows:

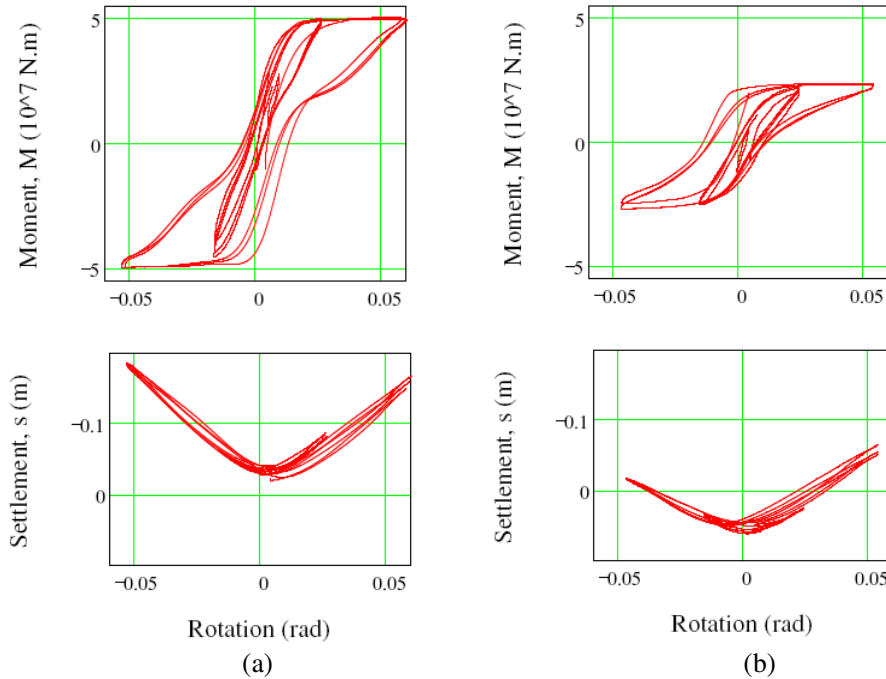
$$M_y = -(m * a_x * H_{cg})_{deckcg} + (I * \alpha_y)_{deckcg} - (m * a_x * H_{cg})_{footingcg} + (I * \alpha_y)_{footingcg} + (m * (1g + a_y) * U_{x_{deck/foot}})_{deckcg} \quad (1)$$

$$F_x = -(m * a_x)_{deckcg} - (m * a_x)_{footingcg} \quad (2)$$

The five terms in equation (1) represent the moments due to lateral acceleration of the deck, the rotational acceleration of the deck, the lateral acceleration of the footing, the rotational acceleration of the footing, and the static and dynamic P-Δ moments. The greatest contributor to moment is the first term, the inertial force from the deck mass. At large rotations the P-Δ term becomes significant. The  $I*\alpha$  terms of the deck mass and foundation are relatively small. But inclusion of these terms in the equation resulted in significantly improved data quality. Some “noise” that appeared in the data was eliminated by accounting for the moments due to the  $I*\alpha$  terms. The inertia forces associated with added mass of the soil adjacent to the footing was neglected.

### Slow Cyclic Tests

As seen in Figure 4, both the smallest ( $B = 3$  Dc) and the largest ( $B = 5$  Dc) foundations show large moment-rotation loops indicating significant energy dissipation. The moment capacity shows negligible degradation with the amplitude of rotation.



**Figure 4. Slow cyclic load-deformation behavior of footing (a)  $B = 5$  Dc. (b)  $B = 3$  Dc.**

The footings show mobilization of their moment capacity at about 2% rotation. The smaller footing ( $B = 3$  Dc) has about 3/5 of the moment capacity of the largest footing ( $B = 5$  Dc), consistent with expectations for rotation about one edge of the footing. The theoretical moment capacity may be expressed as:

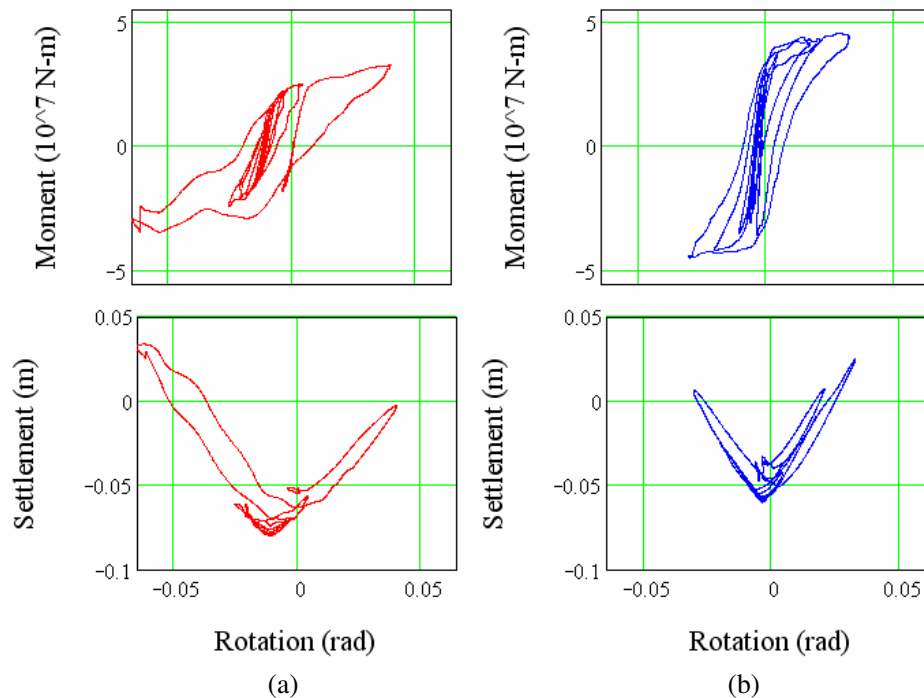
$$M_{capacity} = V(B - Bc)/2 \quad (3)$$

Where  $V$  is the vertical load,  $B$  is the footing width, and  $B_c$  is the width of the footing required to support the vertical load  $V$ . The fact that the moment capacity for the small footing is slightly less than  $3/5$  of the capacity of the large footing is expected because the mass of the small footing is slightly less than that for the large footing (vertical load  $V$  is slightly smaller) and furthermore, for the small footing,  $B_c$  is a larger fraction of the total width,  $B$ .

The shape of the moment-rotation loops for the large foundation differs from that for the small foundation in that a larger percentage of the plastic rotation is recovered by the large foundation. Figure 4 shows a sudden recovery of rotation while unloading from about  $2.5$  to  $1.5 \times 10^7$  Nm. The flag shape type hysteresis loop seen in the larger foundation is hypothesized to be caused by soil falling underneath the foundation during uplift. The gap around the edge of the footing, shown in Figure 3 appeared to develop by soil sloughing under the footing to fill gaps formed during rocking. For a given amplitude of rotation, the size of the gap is proportional to the footing size; because the gap is larger for a large footing, the sloughing of particles under the large footing may be more significant than it is for the small footing. Settlement rotation plots show that the larger foundation has a net uplift after the series of tests, while for the smaller footing, there is a net settlement. This is also consistent with the hypothesis that soil was falling into the gap under the large footing.

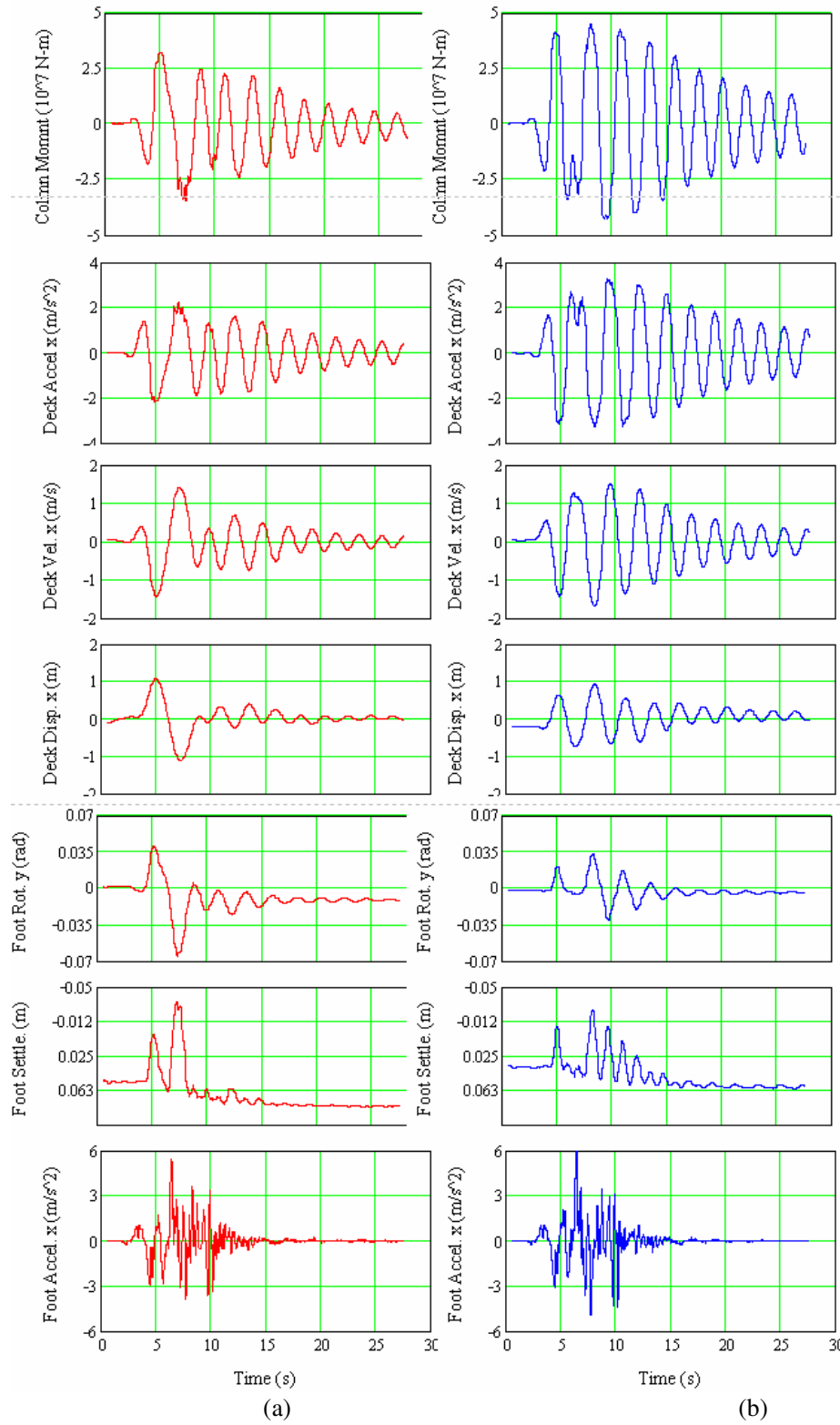
### Dynamic Tests

Figure 5 shows the moment-rotation and settlement-rotation response of the two structures simultaneously loaded during a scaled version of the Los Gatos ground motion. These structures are identical except for their foundation widths. During dynamic shaking both footings show a net settlement. The larger footing has a larger bearing capacity and smaller rotations. The shapes of the moment-rotation curves are similar to those observed in the slow cyclic tests shown in Figure 4.



**Figure 5. Moment rotation and settlement rotation loops for Los Gatos event scaled to pga of 0.55 g . (a) Station E ( $B=3 D_c$ ). (b) Station F ( $B=4 D_c$ ).**





**Figure 6. Time histories for 0.55 g Los Gatos event. (a) ( $B = 3$  Dc), (b) ( $B = 4$  Dc).**



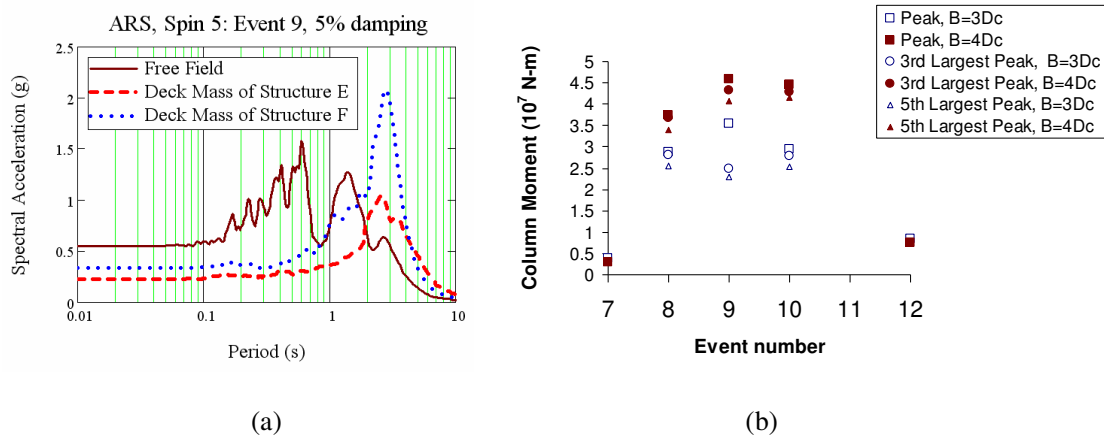
Figure 6 shows time histories of column moment, deck acceleration, velocity and displacement, and the footing rotation, settlement and horizontal acceleration for Stations E and F. The column moment time histories show that the structure with  $B = 3$  Dc has about 20% smaller moment demand than the structure with  $B = 4$  Dc. The acceleration along the shaking direction at the center of gravity of the deck mass is proportional to the largest term contributing to the moment (first term in Equation 1). As such, it makes sense that the deck acceleration time history is very similar to the column moment time history. (The sign of the moment and acceleration time histories are opposite due to the sign convention.) The deck velocity for  $B = 3$  Dc has only two large peaks while that for  $B = 4$  Dc has a slightly larger peak velocity followed by many large amplitude velocity cycles.

The peak deck displacements for the small footings are approximately 15% higher than for the large footing. Larger displacement demands are expected for the small footing because the rotational stiffness of the foundation decreases as the footing width becomes smaller. It must be noted, however that the dynamic response of a system depends upon the relationship between the system natural period and the predominant period of the shaking, and that the rocking natural period depends on footing size and the amplitude of the rocking. The footing rotation time histories show that  $B = 3$  Dc footing experienced two large amplitude rotation pulses followed by many smaller cycles, while the  $B = 4$  Dc footing has more cycles, but they have a smaller amplitude. Both foundations experience approximately a 0.5% change in permanent rotation. A strong correlation is observed between footing rotation and settlement indicating that most of the settlement is caused by moment and not by shear force or vertical loading. The settlement time history shows the position of the center of the base of the footing. Positive spikes correspond to uplift, which occurs at twice the frequency of rocking; an uplift cycle occurs when the foundation rocks to the right and another when the foundation rocks to the left. The last time series in Figure 6 shows the horizontal acceleration of the center of gravity of the foundation along the direction of shaking. The footing response is quite similar for both footings.

Figure 7(a) shows the acceleration response spectrum of the ground surface free field motion and the motion of the center of gravity of the deck mass for  $B = 3$  Dc and  $B = 4$  Dc during the 0.55 g Los Gatos event. Both rocking structures attenuate the short period motions and amplify the long period motions. Note that the fixed base period of both structures is approximately 1.6 seconds, and the natural period for the rocking systems (based on the deck accelerations in Figure 6) was observed to be approximately 2.2 to about 3 s, depending on the amplitude of rocking.

The dynamic testing showed that as footing size decreased, the permanent deformations increased and the moment demand on the column decreased. The  $B = 3$  Dc footing settled about 30 mm per large shaking event while the structures on the larger foundations ( $B = 4$  Dc or 5 Dc) settled about 15 mm per strong shaking event.

Figure 7(b) shows the largest amplitude peak, the 3<sup>rd</sup> largest amplitude peak, and the 5<sup>th</sup> largest amplitude peak column moments for a sequence of shaking events imposed on the structures with  $B = 3$  Dc and  $B = 4$  Dc, subject to the same shaking event. From this it is easily seen that during large shaking events the capacity of the foundation limits the moment demand on the column. The peak column moments are consistently larger for the larger footing.



**Figure 7(a). Acceleration response spectra for ground surface motion and motions of the decks for Stations E and F during 0.55g Los Gatos motion (Event 9). Figure 7(b). Peak moments in column during the strong shaking events in Spin 5.**

## DISCUSSION

### Dynamic vs. Slow Cyclic

There were some differences in the load deformation behavior of the foundation under slow cyclic testing and dynamic testing. The amplitude of foundation rotations during dynamic testing tended to be smaller than those imposed during slow cyclic testing. The amplitude of rotation depends on the dynamics of the system and frequency content of the ground motion hence the amplitude of dynamic deck response also varied with footing size. Despite the fact that the amplitude of rotation tended to be smaller during the selected ground motions for dynamic tests, settlements were larger during dynamic loading than during slow cyclic loading. This was due partially to the settlement of the ground surface during shaking, but also may be associated with a reduction in bearing capacity caused by dynamic shaking of the soil combined with dynamic loading from the footing. In all cases, the magnitude of settlements were small enough that the performance may be judged to be satisfactory. For the smallest footing in the largest shaking event, the settlements were about 30 mm per shaking event.

### Comparison to data from previous tests on building shear walls

Gajan et al. (2005) and others have previously reported data from rocking foundations for shear walls of buildings. The present tests used much of the same equipment that was used by Gajan et al. (2005). The foundations for the bridge structures, however, have a few important differences from the foundations tested by Gajan et al. (2005). First, Gajan et al. (2005) tested relatively rigid walls while the present foundations supported a deck mass attached to the footing by a flexible column.

The shear wall foundations tested by Gajan et al. (2005) had factors of safety with respect to bearing failure of the order of 2 to 15, while the bridge foundations had factors of safety with respect to bearing capacity between about 30 and 70. The size of the bridge foundations was governed by allowable settlement and moment capacity, which led to very large factors of safety with respect to vertical bearing capacity. The bridge foundations were square, while most of the shear wall tests were rectangular, with moment loading in the stiff direction. Due to the above differences, permanent settlements of the rocking bridge foundations on medium dense sand appears to be significantly less than the settlements of building foundations founded on similar soil.

### **Mechanisms for Yielding and Predicting Failure**

In seismic resistant design of bridge structures, the designer ought to make a conscious decision regarding the capacity and demand that will be placed on various elements of the system. One design philosophy is to allow yielding but prevent collapse during extreme shaking events. If yielding is to be allowed, a decision should be made regarding which elements should yield and then to ensure that the yielding elements are ductile and that their capacity should not suffer drastic degradation. Structural engineers tend to recommend that the yielding occur in the column because they can control the ductility and capacity of the column with reinforcing bars and confining steel and because damage to a column can be easily inspected, and evaluated.

Civil Engineers are trained that soil properties are heterogeneous and uncertain, hence they may develop the false impression that the moment capacity of a spread footing has a high uncertainty. On the contrary, with the exception of footings with low factors of safety with respect to bearing capacity, the moment capacity of a spread footing is largely controlled by the size of the footing and the vertical load on the footing and these key factors can be determined with good certainty and hence the moment capacity can be accurately calculated by equation (3). The present study (along with work of many previous researchers) shows that a rocking foundation has very ductile behavior with negligible loss of capacity. Rocking foundations also have significant damping capacity. The uplift of a shallow foundation provides a self-centering mechanism associated with gap closure upon unloading. This self centering upon unloading is not a typical characteristic of yielding reinforced concrete columns.

Assuming that yielding does occur during a large seismic event, the reparability of the yielded element should also be considered. Damage to concrete may be argued to be more dangerous than damage to soil. Damaged concrete columns are likely to crack, spall and crumble under extreme cyclic loading. Soil is already an assembly of tiny pieces of broken rock that are difficult to break into smaller pieces. Soils derive their strength from reliable friction as opposed to concrete cohesion that disappears upon cracking. Practical procedures such as grouting are available that could be used to close up the gaps and restore full contact between the footing and soil. Considering the above factors, yielding of a rocking foundation has potential to serve as a repairable fuse to isolate columns from large seismic demands.

### **CONCLUSIONS**

A series of centrifuge tests is reported that modeled the seismic behavior of a bridge deck mass supported by a flexible column supported on shallow foundations of various sizes. The behavior of rocking foundations was investigated using slow cyclic loading tests as well as loading due to dynamic base shaking. The moment-rotation behavior was similar for slow cyclic and dynamic loading, but the settlements were noticeably larger during dynamic shaking. The performance of a system with smaller footing ( $B = 3 D_c$ ) is in some aspects preferable to the performance of systems supported on larger footings ( $B = 4$  or  $5$  times  $D_c$ ).

As the footing size reduces, moment and curvature ductility demands on the column and acceleration demands on the deck are reduced, but displacement demands on the deck are increased. Smaller footings do suffer greater permanent rotations and settlements than larger footings. The magnitude of settlements, however, appears to be acceptable even for the smallest footings, with permanent settlements being on the order of 30 mm (prototype scale) during large seismic events.

Contrary to what many engineers believe, the moment capacity of rocking shallow foundations is relatively straight forward to calculate. The moment rotation behavior is very ductile with no apparent loss of capacity with large amplitude cyclic loading. Because yielding associated with rocking of shallow foundation is ductile, repairable, and includes self centering due to closure of the gap associated with rocking, foundation rocking may be a preferred mechanism of yielding; engineers should consider the option of allowing shallow foundations to rock as a method of protecting the

columns. The above conclusions are most applicable to shallow foundations on medium dense sandy soils. Additional testing may be required prior to application to other soil types

### ACKNOWLEDGEMENTS

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