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DETERMINATION OF AVERAGE DYNAMIC PROPERTIES FOR VERTICALLY EXCITED SURFACE FOUNDATION: NUMERICAL STUDY

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ABSTRACT

The average dynamic properties for a vertically excited surface foundation on a uniform or a layered medium are estimated using an approximated solution of Verbic (1972) and the finite element analysis with transmitting boundary of Kausel (1974). In the system with a uniform medium, the effect of the shear wave velocity of the medium is significant on the frequencies of the system but it does not affect the damping ratio. The effect of the density of the medium has effects both on the frequencies and damping ratio. The radius and the mass of the disk have effects both on the frequencies and the damping of the system while the Poisson's ratio of the medium has little effect on the dynamic properties. The duration of an impact has a limited effect within the range of values considered herein. In the layered system the bottom layer was assigned a different shear wave velocity from the top layer, and the thickness of the latter was also changed. In this case, the damping ratio, which was not affected in the uniform case, also changes.

Keywords: dynamic properties, surface foundation, rigid disk, vertical excitation

INTRODUCTION

The identification of the behavior of a circular foundation is an important subject in the seismic analysis of soil-structure interaction problems. Once the steady state response of the disk under harmonic excitation is identified, it can be inserted in the equation of motion for the soil-structure system formulated using an appropriate model for the structure (Parmelee, 1967). The transient response of the soil-structure system under an impact or any arbitrary loading can be also evaluated by Fourier analysis assuming the system remains linear. A number of studies have focused on the dynamic behavior of foundations assuming the foundation as a harmonically excited massless disk on an elastic medium (Shah, 1968; Kashio, 1970; Luco and Westmann, 1971; Veletsos and Wei, 1971; Meek, 1972; Veletsos and Verbic, 1974; Kausel, 1974).

In this study, a set of parametric studies was performed to investigate the effect of the soil and foundation material properties on the frequencies and the damping ratio of the system. The shear wave velocity, density and Poisson's ratio of the soil, modeled as an elastic material, and the radius and mass of a rigid disk, representing the foundation, are the basic properties of the system which were varied in the parametric analysis. The frequency response functions were calculated using the approximated solution formulated by Verbic (1972), and the natural undamped, damped and resonant frequencies and the damping ratio are estimated assuming the soil-foundation structure as a SDOF (single degree

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of freedom) system. Finite element analyses using a consistent transmitting boundary by Kausel (1974) were used to model the uniform case, as well as a simple layered profile. The effect of different combinations of shear wave velocities in the two layers on the dynamic properties of the system was also evaluated.

RIGID DISK ON ELASTIC HALFSPACE

A system of vertically excited foundation is introduced in Figure 1. The system includes a rigid, massless circular disk placed on the surface of a non-dissipative, homogeneous, linearly elastic halfspace. The disk is subjected to a harmonic vertical force $P_z e^{i\omega t}$ with an excitation amplitude P_z and a circular frequency ω, resulting in an amplitude of vertical displacement w_0. The amplitude of excitation and displacement are related as

$$P_z = Q_z w_0 \tag{1}$$

where Q_z is the frequency response function of a given system.

The frequency response function can be represented by real and imaginary terms, K_{real} and K_{imag} as

$$Q_z = K_{real} + iK_{imag} \tag{2}$$

where the imaginary part K_{imag} is the damping coefficient c multiplied by the circular loading frequency ω ($K_{imag} = \omega c$). Equation 2 is also expressed in dimensionless terms as

$$Q_z = K_z(k_z + ia_0 c_z) \tag{3}$$

where k_z and c_z are the dimensionless dynamic coefficients, function of the Poisson's ratio ν of the elastic halfspace, and the dimensionless frequency parameter $a_0 = \omega R / c_s$. The properties R and c_s represent the radius of the disk, and the propagation velocity of shear waves in the elastic halfspace, respectively. The quantity K_z is the vertical static stiffness of the system defined as $K_z = 4GR/(1-\nu)$ where G is the shear modulus of the elastic halfspace. The shear modulus can be evaluated from the shear wave velocity using the equation, $G = \rho c_s^2$, with ρ being the density of the halfspace. Values for the coefficients k_z and c_z are available in the work of Shah (1968). The response was estimated assuming the tangential components of the contact pressure are zero at the interface between the rigid disk and the elastic halfspace. No load was applied to the surface beyond the disk footprint.

An approximation of the frequency response functions was introduced by Verbic (1972):

$$k_z = 1 - b_1 \frac{(b_2 a_0)^2}{1 + (b_2 a_0)^2} - b_3 a_0^2 \tag{4}$$

$$c_z = b_4 + b_1 b_2 \frac{(b_2 a_0)^2}{1 + (b_2 a_0)^2} \tag{5}$$

$$Q_z = K_z[1 + ib_4a_0 - b_1\frac{(b_2a_0)^2}{1+ib_2a_0} - b_3a_0^2]. \qquad (6)$$

The coefficients b_1, b_2, b_3, and b_4 depend on Poisson's ratio as shown in Table 1.

Table 1. Coefficients b_i for frequency response function (after Verbic, 1972)

υ	0	1/3	0.45	0.5
b_1	0.25	0.35	-	0
b_2	1.0	0.8	-	0
b_3	0	0	-	0.17
b_4	0.85	0.75	-	0.85

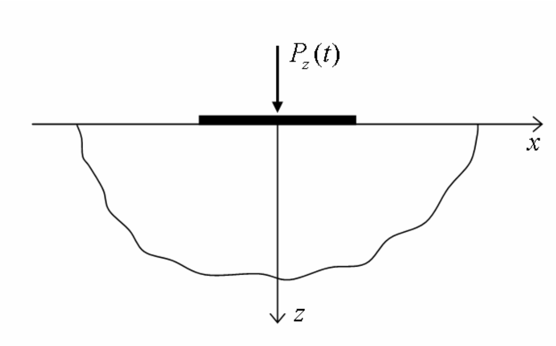


Figure 1. Vertically excited rigid disk on elastic halfspace

The frequency response function of the system including the mass of the rigid disk m can be derived through equilibrium considerations as

$$\frac{P(\omega)}{W(\omega)} = Q_z(a_0) - m\omega^2 \qquad (7)$$

Equation 7, together with the approximated frequency response function Q_z, was used to analyze the vertically excited system in this study.

Analytical Approaches

Three different approaches to extract the natural undamped, damped and resonant frequencies and the damping ratio from the frequency response functions are presented in this section. In all approaches the system consisting of the rigid disk and the elastic halfspace is simplified to a SDOF system considering the disk as a point mass, and the elastic medium as a single spring and damper.

Static spring constant approach

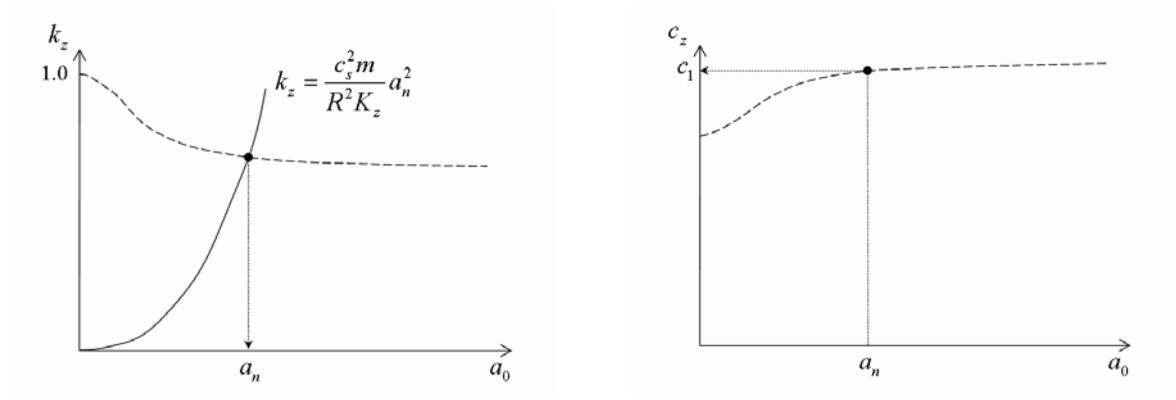
The undamped natural frequency and the damping ratio of a system can be evaluated from the real and imaginary dynamic coefficients k_z and c_z of the frequency response function Q_z. The procedure is illustrated schematically in Figure 2. The dimensionless undamped natural frequency a_n of the soil-structure system is estimated at the intersection of the curve of k_z and $\omega_n = \sqrt{K_{real}/m}$ as shown in Figure 2(a). The dimensionless coefficient c_1 is the value of c_z at the dimensionless undamped natural frequency a_n. Then, a_n and c_1 are used to obtain the undamped natural circular frequency

ω_n, and the damping ratio ξ. The damped natural circular frequency ω_D and the resonant circular frequency ω_r are estimated from

$$\omega_D = \omega_n \sqrt{1-\xi^2} \tag{8}$$

$$\omega_r = \omega_n \sqrt{1-2\xi^2} \tag{9}$$

which are valid for damping ratio smaller than $1/\sqrt{2}$.



(a) (b)

Figure 2. Static spring constant approach

Maximum response approach

The resonant frequency and the damping ratio can be evaluated from the dynamic response factor of the SDOF system. The resonant frequency is defined as the forcing frequency at which the largest response occurs (Chopra, 2001). Practically, in most common buildings, the difference between the resonant and natural frequencies is not significant. In the system investigated here, the difference is far more than that for a normal building, as the energy dissipation in the halfspace is very large. For a damping ratio ξ smaller than $1/\sqrt{2}$, the resonant circular frequencies for displacement $\omega_{r,d}$, for velocity $\omega_{r,v}$, and for acceleration $\omega_{r,a}$ are derived as

$$\omega_{r,d} = \omega_n \sqrt{1-2\xi^2} \tag{10}$$

$$\omega_{r,v} = \omega_n \tag{11}$$

$$\omega_{r,a} = \omega_n / \sqrt{1-2\xi^2}\,. \tag{12}$$

The maximum of the dynamic response factors for displacement R_d, for velocity R_v, for acceleration R_a corresponding to their respective resonant frequencies are

$$R_{d,\max} = \frac{1}{2\xi\sqrt{1-\xi^2}} \tag{13}$$

$$R_{v,\max} = \frac{1}{2\xi} \tag{14}$$

$$R_{a,\max} = \frac{1}{2\xi\sqrt{1-\xi^2}} \tag{15}$$

Since the dynamic response factor for displacement, R_d, is the ratio of the amplitude of the dynamic displacement to the static displacement, it can be evaluated from the frequency response function H_z which is the inverse of Q_z as

$$R_d = K_z \left| H_z(\omega) \right|. \tag{16}$$

The resonant frequency for displacement $\omega_{r,d}$ and the damping ratio ξ were estimated from the maximum value of the dynamic response factor for displacement in this study.

Velocity time history approach

The damped natural frequency and the damping ratio can be estimated from the free vibration response history of the system. A triangular impact with the duration T_d is transformed to the frequency domain and multiplied by the frequency response function Q_z. After transforming the results back to the time domain using the inverse Fourier transform, the displacement time history of the system is obtained. Although, this is not the true free vibration response of the system, the two are quite similar for small impact durations. Then, the damped natural frequency and damping ratio are evaluated from the ratio of two successive maxima or minima. The displacement $u(t)$ of a viscously damped SDOF system in free vibration at time t is

$$u(t) = e^{-\xi\omega_n t}[A\cos\omega_D t + B\sin\omega_D t] \tag{17}$$

where the coefficients A, and B can be obtained from the initial conditions as $A = u(0)$, $B = [\dot{u}(0) + \xi\omega_n u(0)] / \omega_D$. The ratio of the displacement at time t to the displacement at $T_D / 2$, a half vibration period later, is

$$\frac{u(t)}{u(t + T_D / 2)} = -e^{\xi\omega_n T_D / 2}. \tag{18}$$

Then, the damping ratio can be evaluated from

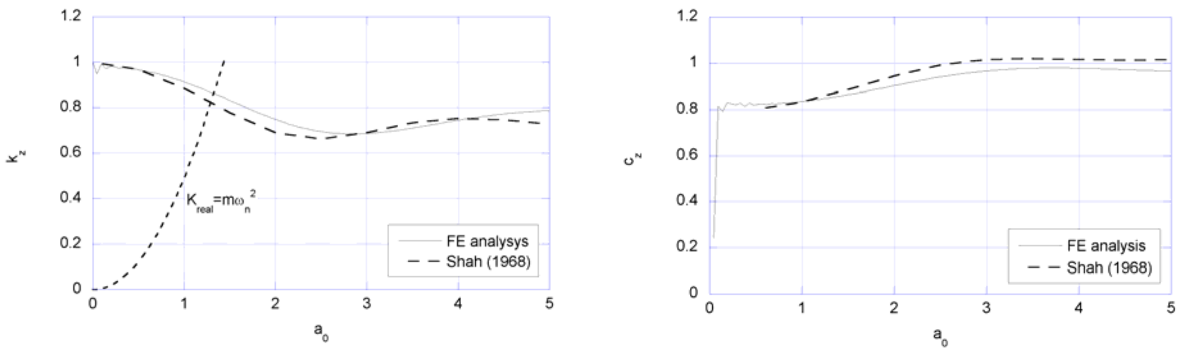
$$\xi = \frac{\delta}{\sqrt{\delta^2 + \pi^2}} \tag{19}$$

where $\delta = \ln[-u_i / u_{i+1}]$. The quantities u_i and u_{i+1} represent two successive maxima or minima. The damped natural frequency is estimated from the half of the damped period $T_D / 2$. This approach is also valid for the velocity or the acceleration time histories. The velocity time history was used to estimate the dynamic properties in this study.

Parametric Studies

A series of parametric studies was performed to evaluate the effect of the properties of the foundation, represented by a rigid disk, and the soil, as an elastic halfspace, on the frequency response and the damping ratio of the vertically excited system. The effects of the shear wave velocity c_s, the density ρ and Poisson's ratio ν of the halfspace, the radius R and the mass m of a disk, and the duration of an impact T_D are presented in this section. After performing an analysis with a baseline set of properties, each property was modified independently to evaluate changes in the results. The baseline set of properties used is: $c_s = 180$ m/s, $\rho = 2000$ kg/m^3, $\nu = 0.25$, $R = 0.457$ m, $m = 500$ kg, and $T_d = 1.0 \times 10^{-2}$ s.

The undamped natural cyclic frequency f_n, the damped natural cyclic frequency f_D, the resonant cyclic frequency f_r, and the damping ratio ξ for the different sets of input properties were estimated using the three different approaches described in the previous section (Table 2). Although the results show small differences due to the different interpretations, they still display the same trends. The shear wave velocity c_s of an elastic halfspace has the largest effect on the frequencies of the system, as the modulus depends on the wave velocity following a quadratic relationship. Since the wave velocity shows no effect on the damping ratio ξ, it affects all the frequencies in the same way. Thus, all the frequencies - the undamped natural frequency f_n, the damped natural frequency f_D, and the resonant frequency f_r - increased when the shear wave velocity c_s increased. The damping ratio ξ increases when the density of the medium, ρ, increases. The effects of ρ on the frequencies are not straightforward. The undamped natural frequency f_n and the damped natural frequency f_D increase, while the resonant frequency f_r decreases with increasing density. The effect of Poisson's ratio ν is not significant, considering the wide range of values tried herein. The radius R of the disk has a large effect on the damping ratio and a limited one on the frequencies (since the damping in the system is 'geometric' damping). The undamped natural frequency f_n increases with increasing radius, however the changes in the damped natural frequency f_D and the resonant frequency f_r are not straightforward as they are also influenced by the damping ratio ξ, which increases when the radius increases. The mass m of a disk also has significant effect on the damping ratio and a limited one on the frequencies, but in a reversed trend to that of the radius R. The undamped natural frequency f_n and the damping ratio ξ tend to decrease when the mass m increases. The frequencies and the damping ratio estimated with three different durations of the impact T_d for the velocity time history approach are presented at the bottom of Table 2. The effect of the impact duration T_d is not critical within the range of values used here, which may not be true for larger values of T_d.



(a) (b)

Figure 3. Frequency response functions from FE analysis and exact solution

Table 2. Frequencies and damping ratio from approximatred analytical solution

Modified property		Static spring constant approach				Maximum response approach				Velocity time history approach			
		f_n (Hz)	f_D (Hz)	f_r (Hz)	ξ	f_n (Hz)	f_D (Hz)	f_r (Hz)	ξ	f_n (Hz)	f_D (Hz)	f_r (Hz)	ξ
c_s (m/s)	141	63.5	50.9	33.7	0.60	53.1	45.0	35.2	0.53	57.5	46.5	32.0	0.59
	162	73.0	58.4	38.7	0.60	61.0	51.7	40.4	0.53	64.8	52.6	36.6	0.58
	180	81.1	64.9	43.0	0.60	67.8	57.5	44.9	0.53	73.2	59.5	41.6	0.58
	198	89.2	71.4	47.3	0.60	74.5	63.2	49.4	0.53	81.8	66.7	46.9	0.58
	216	97.3	77.9	51.6	0.60	81.3	69.0	53.9	0.53	87.4	71.4	50.6	0.58
ρ (kg/m^3)	1600	73.2	61.9	48.0	0.53	65.0	56.7	47.0	0.49	65.5	55.6	43.4	0.53
	1800	77.3	63.6	46.1	0.57	66.5	57.2	46.0	0.51	70.8	58.8	43.7	0.56
	2000	81.1	64.9	43.0	0.60	67.8	57.5	44.9	0.53	73.2	59.5	41.6	0.58
	2200	84.6	65.7	38.3	0.63	68.7	57.6	43.7	0.55	73.9	58.8	38.3	0.60
	2400	88.1	66.2	31.7	0.66	69.3	57.4	42.3	0.56	75.3	58.8	35.4	0.62
υ	0	71.7	53.0	42.7	0.57	62.1	53.2	42.5	0.52	65.4	54.1	39.6	0.56
	0.25	81.1	64.9	43.0	0.60	67.8	57.5	44.9	0.53	73.2	59.5	41.6	0.58
	0.33	85.1	67.2	42.1	0.61	70.3	59.3	45.8	0.54	72.8	58.8	40.2	0.59
	0.50	88.8	70.8	46.3	0.60	89.4	71.3	46.7	0.60	89.6	71.4	46.7	0.60
R (m)	0.229	61.7	60.4	59.2	0.20	61.6	60.4	59.1	0.20	60.1	58.8	57.6	0.20
	0.305	69.6	66.0	62.2	0.32	68.3	64.9	61.3	0.31	65.9	62.5	58.8	0.32
	0.457	81.1	64.9	43.0	0.60	67.8	57.5	44.9	0.53	73.2	59.5	41.6	0.58
m (kg)	500	64.9	64.9	43.0	0.60	67.8	57.5	44.9	0.53	73.2	59.5	41.6	0.58
	750	67.4	58.9	49.0	0.49	61.8	55.1	47.4	0.45	63.6	55.6	46.1	0.49
	1000	59.1	53.7	47.8	0.42	56.1	51.5	46.3	0.40	56.5	51.3	45.5	0.42
	1250	53.3	49.5	45.4	0.37	51.6	48.2	44.4	0.36	51.3	47.6	43.6	0.37
T_d (s)	2.0×10^{-3}									71.6	57.5	38.5	0.60
	4.0×10^{-3}									72.3	58.1	39.1	0.59
	1.0×10^{-2}									73.2	59.5	41.6	0.58

RIGID DISK ON LAYERED MEDIUM

A subsurface profile can be generally described by a series of horizontal layers with different material properties. This causes wave reflections and refractions on the interfaces between layers and prevents the system of interest from acting as a uniform ideal system. In this section, the effect of layers on the frequencies and the damping ratio of the system consisting of a vertically excited rigid circular footing on a layered medium were observed using a finite element program with a consistent transmitting boundary based on Kausel (1974).

The core region under the footing is modeled with toroidal finite elements, expanding the solution in a Fourier series in the circumferential direction, and the outside region is modelled using the consistent boundary for the same number of the Fourier expansion. For the case of a vertical load, the problem is axisymmetric and only the $n = 0$ term of the expansion is needed. This model requires discretization of the soil into thin layers, assuming a linear variation of the displacements in the vertical direction within each layer. In addition, the core region is also discretized in the radial direction. The model assumes the existence of much stiffer, essentially rigid, rock at depth.

The real and imaginary parts of the frequency response function Q_z of the rigid footing on the layered medium can be evaluated using finite element analysis. The frequencies and the damping ratio of the system will be evaluated by the three approaches presented above considering the system as SDOF system. The frequency response function Q_z is expressed in terms of dimensionless terms as presented in Equation 3. If $a_0 = 0$, the frequency response function becomes the static stiffness of the system. In the parametric study, the real part of the frequency response function from the finite element analysis when $a_0 = 0$ is considered as the static stiffness.

Definition of finite element model

The estimated dimensionless coefficients k_z and c_z from finite element analysis were compared with those from the elastic halfspace solution. The finite element analysis was performed with $c_s = 180$ m/s, $\rho = 2000$ kg/m^3, $\nu = 0.25$, $R = 0.457$ m. The rigid bottom boundary was at $48R$ away from the surface. A mesh with square elements of size $\Delta L = R/4$ was used for whole finite element region and $5\,\%$ internal hysteric damping was applied to prevent excessive fluctuation of the frequency response functions. This damping effect was then subtracted from the results after calculation of the frequency response functions. Figure 3 shows the results of the finite element analysis compared to the elastic halfspace solution by Shah (1968). For the purpose of comparison, and only in this case, the analytical static stiffness for the elastic halfspace, $K_z = 4GR/(1-\nu)$, was used for the interpretation of the finite element analysis results. The coefficients of the frequency response function from both methods are in good agreement, which indicates the finite element model can replicate the analytical elastic halfspace solution.

Parametric Studies

A set of parametric studies was performed using finite element analysis to estimate the effect of a varying shear wave velocity in the layered soil deposits on the evaluation of frequencies and damping ratio of the system. The schematic representation of the uniform and the layered systems used in the analysis is shown in Figure 4. The shear wave velocities and thickness of the layers are presented in Table 3. The first digit in the case number represents the height of the top layer h_t, which means the number 1 and 2 denote that $h_t = R$ and $2R$, respectively, while zero means the profile is uniform. The second digit indicates the selection of shear wave velocity for the bottom layer, c_{sb}, which means the numbers 1, 2, 3, 4, and 5 represent $c_{sb} = 141$ m/s, 162 m/s, 180 m/s, 198 m/s, and 216 m/s, respectively. The results from the three different approaches in the case of $h_t = R$ (Figures 5, 6, and 7) are slightly different, but show the same trend with shear wave velocity as the uniform case. The undamped natural frequency f_n of the system - the frequency at the intersection of K_{real} and $K_{real} = m\omega^2$ - increases when the wave velocity of bottom layer c_{sb} increases (Figure 5(a)). The damping ratio ξ evaluated from Figure 5(b) decreases when c_{sb} increases. The numerical values of the frequencies and the damping ratio are presented in Table 4. The numerical results for the layered profile are presented only for the higher shear wave velocity at the bottom (L14, L15, L24, and L25). For lower shear wave velocity of the bottom layer, the damping ratio is larger than $1/\sqrt{2}$ and Equations (8) and (9) cannot be used. The damped natural frequency f_D increases with increasing c_{sb} as shown in Figure 7 (the distance between the second and the third peaks gets smaller) and the resonant frequency f_r also increases when c_{sb} increases, as can be seen in Figure 6 (the peak of the curve moves to higher frequencies). The damping ratio ξ decreases with increasing c_{sb} in two figures (the particle velocity time history decays slower and the peak of the dynamic response factor R_d increases).

The results for $h_t = 2R$ are presented in Figures 8 to 10 and Table 4. In this case, the change in the shear wave velocity at bottom c_{sb} has less influence on the dynamic properties of the system since only the deeper part of the profile is affected. In Figure 8(a) the undamped natural frequency f_n decreases with increasing shear wave velocity of the bottom layer c_{sb}, because the K_{real} curves for the layered medium intersect at a frequency of approximately 50 Hz and are reversed in order compared to the cases of the uniform medium or $h_t = R$. The damped natural frequency f_D remains approximately the same and the resonant frequency f_r increases with increasing c_{sb} as shown in Figures 10 and 9, respectively. In both figures, a decreasing damping ratio ξ with increasing c_{sb} is observed. These trends can also be found in the numerical data in Table 4. It is important to remember, however, the trends observed hold only for the cases studied herein. The response may be different if soil properties or the thickness of the layers change.

Table 3. Cases of analysis

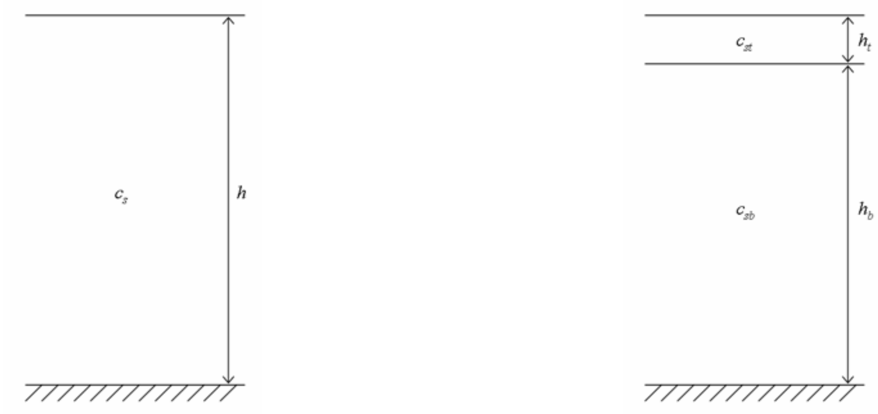
Case number	H		c_s (m/s)	
L01	48R		141	
L02	48R		162	
L03	48R		180	
L04	48R		198	
L05	48R		216	
Case number	h_t	h_b	c_{st} (m/s)	c_{sb} (m/s)
L11	R	47R	180	141
L12	R	47R	180	162
L14	R	47R	180	198
L15	R	47R	180	216
L21	2R	46R	180	141
L22	2R	46R	180	162
L24	2R	46R	180	198
L25	2R	46R	180	216

Table 4. Frequencies and damping ratio from FE analysis

Case number	h		c_s (m/s)		Static spring constant approach				Maximum response approach				Velocity time history approach			
					f_n (Hz)	f_D (Hz)	f_r (Hz)	ξ	f_n (Hz)	f_D (Hz)	f_r (Hz)	ξ	f_n (Hz)	f_D (Hz)	f_r (Hz)	ξ
L01	48R		141		68.0	54.7	36.9	0.59	61.4	50.7	37.0	0.56	63.4	52.6	39.0	0.56
L02	48R		162		78.2	62.9	42.4	0.59	71.3	58.9	43.0	0.56	74.8	62.5	47.1	0.55
L03	48R		180		86.9	69.9	47.1	0.59	78.1	64.5	47.0	0.56	79.7	66.7	50.4	0.55
L04	48R		198		95.5	76.9	51.9	0.59	86.2	71.2	52.0	0.56	92.8	76.9	56.7	0.56
L05	48R		216		104.2	83.9	56.6	0.59	94.6	78.1	57.0	0.56	101.2	83.3	60.4	0.57
Case number	h_t	h_b	c_{st} (m/s)	c_{sb} (m/s)	Static spring constant approach				Maximum response approach				Velocity time history approach			
					f_n (Hz)	f_D (Hz)	f_r (Hz)	ξ	f_n (Hz)	f_D (Hz)	f_r (Hz)	ξ	f_n (Hz)	f_D (Hz)	f_r (Hz)	ξ
L14	R	47R	180	198	89.8	0.51	61.9	0.51	85.7	74.4	61.0	0.50	87.6	76.9	64.5	0.48
L15	R	47R	180	216	92.2	0.45	71.6	0.45	90.0	81.1	71.0	0.43	91.5	83.3	74.3	0.41
L24	2R	46R	180	198	84.4	0.53	56.2	0.53	75.6	66.5	56.0	0.48	78.1	69.0	58.4	0.47
L25	2R	46R	180	216	82.4	0.47	61.3	0.47	71.5	65.1	58.0	0.41	73.1	66.7	59.6	0.41

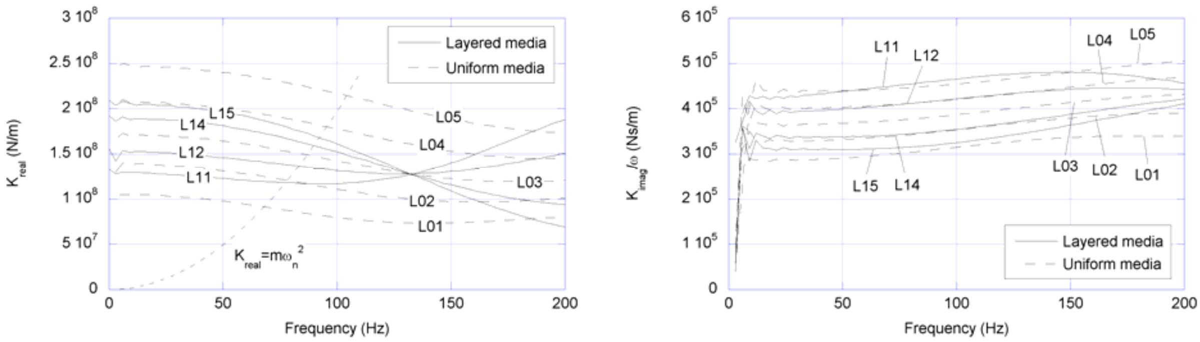
CONCLUSIONS

The effect of various parameters on the natural undamped, damped and resonant frequencies and the damping ratio of a system consisting of a circular rigid disk on a uniform or layered medium was investigated through numerical studies using approximated solutions or finite element analysis with a consistent transmitting boundary.



(a) Uniform medium (b) Layered medium

Figure 4. Uniform and layered media



(a) Real part (b) Imaginary part

Figure 5. Coefficients of frequency response functions (change in shear wave velocity, h_t=R)

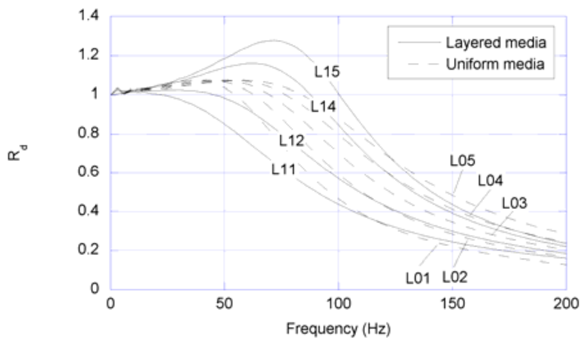


Figure 6. Dynamic response curves (change in shear wave velocity, h_t=R)

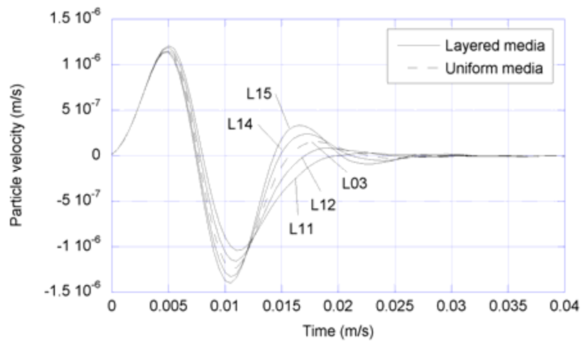
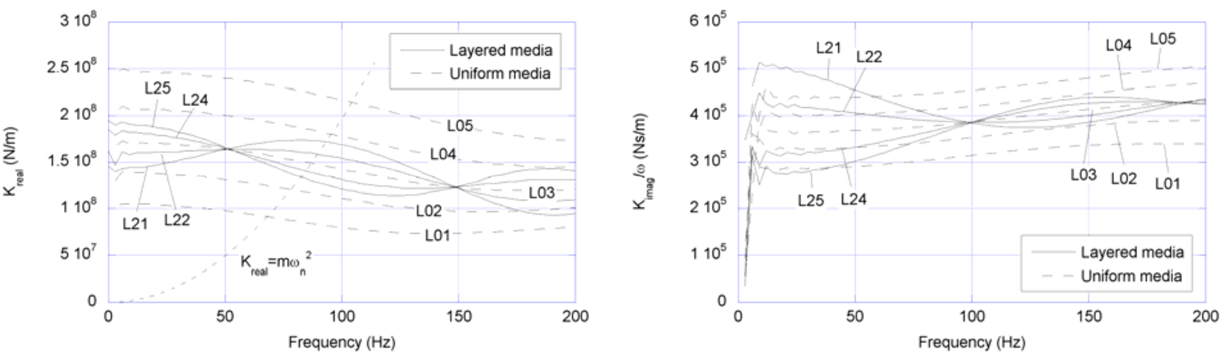


Figure 7. Particle velocity time histories (change in shear wave velocity, h_t=R)



(a) Real part (b) Imaginary part

Figure 8. Coefficients of frequency response functions (change in shear wave velocity, h_t=2R)

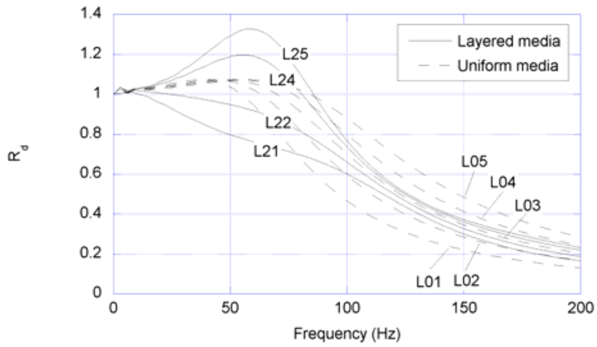


Figure 9. Dynamic response curves (change in shear wave velocity, h_t=2R)

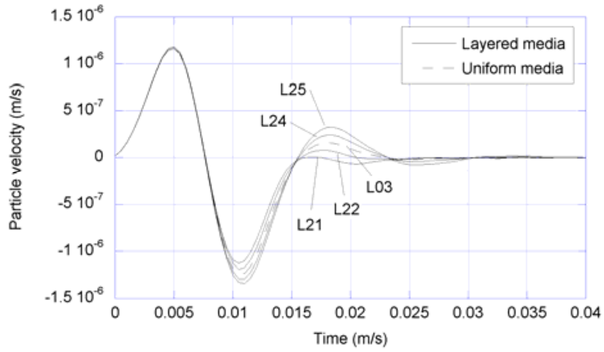


Figure 10. Particle velocity time histories (change in shear wave velocity, h_t=2R)

In the case of the elastic halfspace, the most significant effect on the frequencies is attributed to the shear wave velocity of the medium. The disk radius seems to have a limited influence on frequencies, but the range of R was also quite limited. Both the mass of the disk and the density of the medium affect the natural undamped, damped and resonant frequencies in contrasting trends, due to the influence of these properties on damping ratio. The largest changes on the damping ratio are caused by radius and mass of the disk, with smaller effects due to the density of the medium. The shear wave velocity of the medium does not affect the damping ratio. The Poisson's ratio of the medium has limited effect on the dynamic properties. The duration of the impact had little influence on the results within the range of values considered herein. The three different approaches for the estimate of dynamic properties give approximately the same values for frequencies and damping ratio, and show the same trends. The parametric study on shear wave velocity for a uniform profile was also carried out with finite element analysis, leading to results that are similar to those observed with the elastic halfspace solutions.

In the layered system the bottom layer was assigned a different shear wave velocity from the top layer, and the thickness of the latter was also changed. In this case, the damping ratio, which was not affected in the uniform case, also changes. The natural frequency of the layered soil profile is not the average of the layers' frequencies, but depends on the thickness of the layers. The response of the layered system becomes quite complex and the interpretation of the results is also more difficult to generalize. A simple system consisting of only two layers is examined here. With each additional layer introduced in the system the level of complexity will also rise.

It is important to consider, however, the trends observed hold only for the cases studies herein. The response may be different if the soil properties or the thickness of the layers changes.

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