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EFFECTS OF PARTICLE SHAPE ON G_{MAX} OF GEOMATERIALS

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ABSTRACT

The small strain shear modulus of geomaterials is an essential soil parameter for benchmarking the non-linear stress-strain behaviour of soil under both monotonic and cyclic loading. Shear modulus is used in solving boundary value problems such as the seismic response of soil during earthquakes. It is also important in modelling the behaviour of soil under foundations where only very small deformation take place e.g. vehicle load, excavation, and tunnelling.

Results from laboratory tests at large strains (e.g. Gilboy, 1928, Clayton et al., 2004) suggest that the stiffness of soil at medium to large strains is greatly affected by particle shape. More recent studies (Cho et al., 2006) have shown that the shear wave velocity, which is related to shear modulus, is dependent on angularity, sphericity, and roundness. Unfortunately, Cho et al.'s dataset did not include any reference to void ratio, which has been found to be an important parameter, and so their conclusions require further investigation.

To investigate the relationship between particle shape and shear modulus, resonant column tests have been performed on dry specimens using granular material having different particle shapes. The materials considered in these tests were glass Ballotini, Leighton Buzzard sand fraction B and E, as well as mixtures of Leighton Buzzard sand and an addition of 0.1-mm mica, which consists of fine and platy particles. The results show that particle shape considerably affects the elastic threshold strain, modulus degradation as a function of strain, and damping ratio. The addition of a small proportion of fine mica considerably reduces the small strain shear modulus, increases shear-strain susceptibility, and increases the damping ratio of the mixture.

Keywords: particle shape, small strain shear modulus, resonant column test, damping

INTRODUCTION

In modern geotechnical design the response of the soil under static or dynamic loading is required to calculate the displacement of the structure, and ensure that such displacements are within acceptable limits. The modelling of such soil structure interactions are routinely solved using a variety of numerical techniques using computers. With all these techniques the shear modulus of the soil is a key parameter required to analyse the response. It has been shown that for a given soil the magnitude of shear modulus is dependent on a number of factors such as stress state, material characteristics, which includes such attributes of the soil as void ratio, particle size, particle shape, gradation, fabric, cementation, etc., and strain level. At very small strains ($<10^{-4}$%) the maximum shear modulus (G_{max}) can be measured and is considered a fundamental soil property.

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Some researchers (Barrett, 1980; Sukumaran and Ashmawy, 2001; Bowman et al., 2001, Santamarina and Cho, 2004; Cho et al., 2006) suggest that three independent properties, namely, form, roundness (cf. angularity), and roughness, are necessary to characterise particle shape. Form is the measure of the overall shape or the uniformity of the three orthogonal dimensions (i.e. length, width and height) of a particle. Roundness and roughness are the measures of surface irregularities at large and small scales respectively.

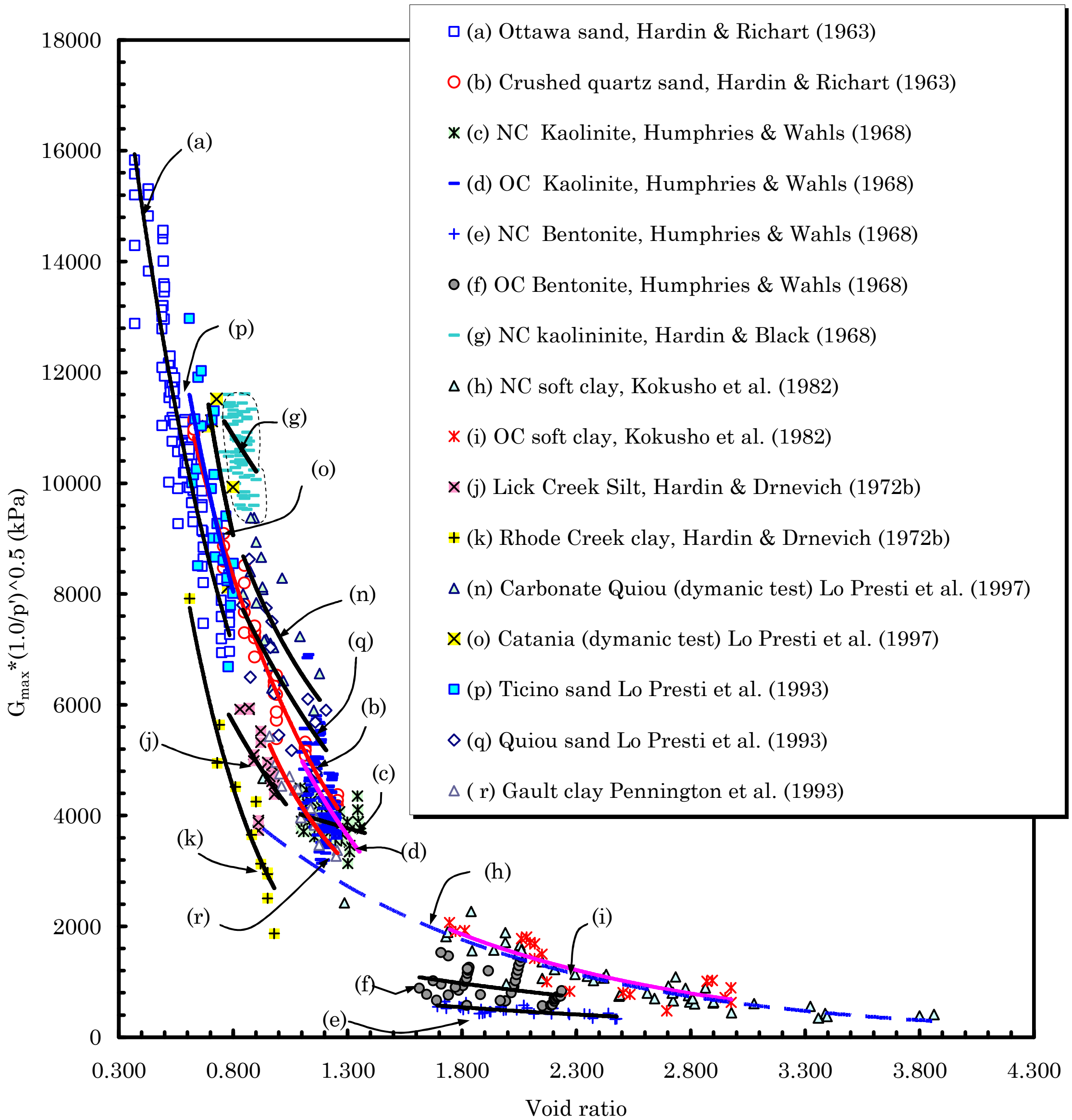

Figure 1. Normalised small strain shear modulus of various materials

The influence of void ratio on G_{max} could have been over-emphasized and the influence of particle shape on G_{max} had been underestimated. Hardin (1961) found that V_s of Ottawa sand and crushed quartz sand varied linearly with void ratio, and observed that V_s in dense crushed quartz at 95.8 kPa was similar to that of loose Ottawa sand. The grain size of both materials was the same i.e. fraction No 20 - No 30. The result led to his conclusion that V_s at D_r = 100% might be quite different for the two sands, however their V_s at the same void ratio were the same. He concluded that grain shape affected V_s through void ratio; the void ratio of the crushed quartz sand (extremely angular grains), was greater than that of Ottawa sand (rotund grains), causing V_s to be lower in the angular material. The work of Hardin and Richart (1963), and Hardin and Drnevich (1972a) suggested that for uncemented soils, void ratio was the controlling factor and that particle shape were of minor importance and in essence only contributed to changes in void ratio and effective strength envelope.

Figure 1 shows the small strain shear modulus of various geomaterials (both granular and cohesive soils) against void ratio. The shear modulus values given in Fig. 1 have been normalised ($G_{max}/\sigma'_0{}^{0.5}$, where σ'_0 is mean effective stress) to remove the effect of stress state on the soil behaviour. The results clearly show that G_{max} decreases with an increase in void ratio. However, where the void ratios of different soils overlap (e.g. at e = 0.8), the shear modulus apparently varies over a wide range. It has been suggested that G_{max} is affected by particle characteristics e.g. relative density (Seed et al., 1986; Seed and Idriss, 1970), coefficient of uniformity (Iwasaki and Tatsuoka, 1977; Lo Presti et al., 1997; Saldago and Bandini, 2000; Menq and Stokoe, 2003), mineralogy and grain angularity (Lo Presti et al., 1997), and D_{50} (Menq and Stokoe, 2003).

It has been shown that at large strains (more than 0.1%) particle characteristics have significant impact on soil strength and deformation (Terzaghi, 1925; Gilboy, 1928; Kolbuszewski, 1963; Koerner, 1970; Holubec and D'Appolonia, 1973; Vermeulen, 2001; Clayton et al., 2004; Santamarina and Cho, 2004; Cho et al., 2006). The addition of small amount of platy particles to rotund sands is sufficient to change the soil behaviour from sand-like (relatively high strength, low compressible, and dilative) to clay-like (relatively low strength, high compressible, and contractive) (Terzaghi, 1925; Gilboy, 1928; Terzaghi, 1948; Mundegar, 1997; Clayton et al., 2004).

At small strains Santamarina and Cascante (1998) undertook tests using specimens prepared with steel balls having different degrees of surface roughness. Their RC test results indicated that G_{max} reduced with the surface roughness. Clayton et al. (2004) observed a considerable reduction in the undrained Young's modulus (at strains ranging from 0.01% to 1%) for a coarse rotund granular material mixed with a small quantity of fine, platy particles, even though the void ratio of the mixture was lower than that of the coarse material alone. Recently, Cho et al. (2006) have established relationships between the particle shape (i.e. roundness, sphericity, and regularity) and shear wave velocity, V_s, of some natural and crushed sands by expressing V_s as:

$$V_S = \alpha \left(\frac{\sigma'_0}{1kPa} \right)^{\beta} \quad (1)$$

Where α is the shear wave velocity, V_s at 1.0 kPa and β is the stress exponent reflecting the sensitivity of V_s to σ'_0 in the polarisation plane. Their data suggest that there are correlations between α, β and particle shape, leading to the conclusion that G_{max} decreases with irregularity. However, their observed differences in α and β may be attributable to variations in void ratio, since no mention is made in their paper of accounting for changes in void ratio. Hardin and Richart (1963) suggested that α was a linear function of void ratio. It has also been shown that the stress exponent β is also a function of void ratio (Clayton et al., 2005).

In summary, there is some evidence of the effects of particle shape on small strain shear modulus in the literature, but it is limited. This paper presents initial laboratory results, obtained using a resonant column, using dry specimens containing granular material having different particle shapes. The test results show that particle shape does in fact affect soil properties at small strain.

TEST METHODOLOGY

To investigate the effect of particle shape, dry specimens containing granular material with distinct, differing particle shapes were tested using a resonant column apparatus. The test materials selected were smooth spherical, 1.00mm in diameter, glass Ballotini (GBB), a round to sub rounded, nominally 1-mm diameter, Leighton Buzzard sand (LBSB) and an angular, nominally 0.1 mm diameter, Leighton Buzzard sand E (LBSE). The nominal diameter for the sands were determined by the fraction passing the 1.16 mm sieve and retaining by the 0.6 mm sieve and the fraction passing the 0.15 mm sieve and retained by 63 μm sieve for the LBSB and LBSE, respectively. In addition, a platy, 0.1 mm diameter (material passing the 0.15 mm sieve and retained by the 63 μm sieve), mica was mixed with LBSB. Fig. 2 shows scanning electron microscope images for the range of material used which highlights the particle characteristics of the materials. Table 1 summarises the material properties of the specimens used in this research such as maximum and minimum dry densities, void ratio, resonant frequencies of the specimens and their respective values of G_{max}.

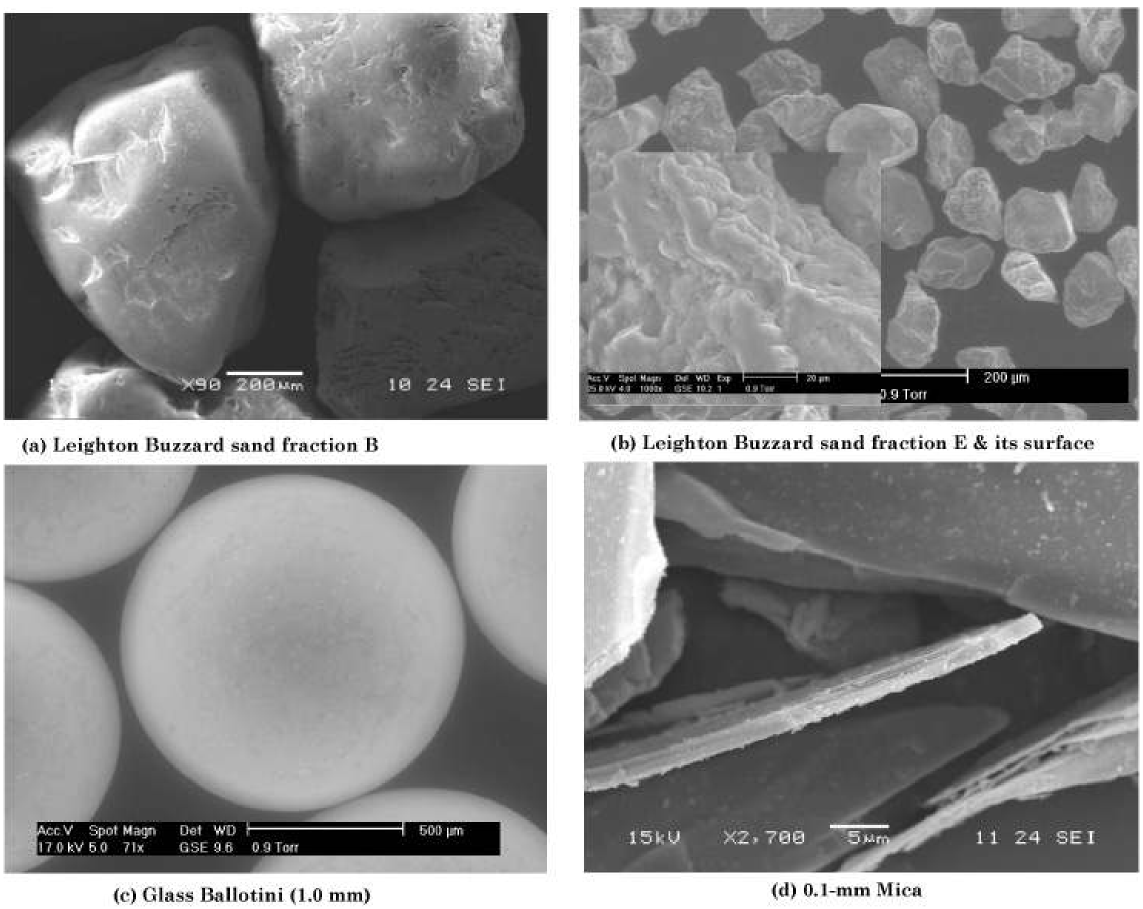

Figure 2. Scanning electron microscope images of test materials

Specimens GB, LBSB and LBSE were formed in a split mould using the dry pluviation method described in detail by Cresswell et al. (1999). The advantage of pluviation over tamping and vibratory compaction is that it can attain uniform density without particle crushing (Lo Presti et al., 1992). The specimen containing the LBSB and mica mixture however could not be pluviated due to the possible segregation of the heavier sand particles from the light mica particles. Therefore for this specimen a thin layer of sand (10 grams) was placed in the mould before 1 gram of mica was rained over top. This mixture was tamped slightly using a rubber tamper and slight tapping on the outside of the mould. Tamping and tapping helped to increase the density with the mica partially moving downward into the sand. This procedure was repeated until the required specimen height was formed. The overall mixture gave a mica content of 10% by mass. The overall distribution of mica along the entire specimen was reasonably uniform since the thickness of each layer was very thin. Great care was taken to prepare a right cylindrical specimen with uniform diameter. Before removing the mould, an effective stress σ'_0 of up to 70 kPa was applied to hold the specimen, via suction. Void ratio of the specimen was measured at this point taking the thickness of membrane into account.

All of the specimens were tested under a range of isotropic air confining pressures from 100 kPa to 400 kPa. At each pressure step, as given in Table 1, the specimen was subject to increasing shear strain from 10^{-5}% to 10^{-2}% with the corresponding resonant frequency at each applied strain measured. This was repeated for each pressure step. To prevent air migration, which could change the effective stress in the specimen, silicon oil fluid bath was placed around the specimen.

Table 1. Material properties of specimens tested

N^O	materials	roundness (Angularity)	G_s	γ_{max} (kg/m^3)	γ_{min} (kg/m^3)	e	σ'_0 (kPa)	f_r (Hz)	G_{max} (MPa)
1a	LBSB1a	rounded to sub-rounded	2.65	1800.5	1438.8	0.532	100	167.8	215
1	LBSB1		2.65	1800.5	1438.8	0.511	100	177.6	246
							200	201.7	334
							300	215.1	392
							400	223.7	433
2	GB1	sphere	2.50	1623.4	1470.6	0.578	100	154.1	181
							200	174.2	240
							300	185.0	277
							400	193.5	309
3	LSBE1a	angular	2.65	1638.6	1309.5	0.741	100	124.7	112
4	LBSE 1		2.65	1638.6	1309.5	0.694	100	127.8	116
							200	144.2	151
							300	156.7	183
							400	165.8	209
5	Mixture LBS B &10% mica 0.1mm		--	--	--	0.447	100	173.9	235
6	LBSB2	rounded to sub-rounded	2.65	1800.5	1438.8	0.610	100	158.6	201
							200	181.3	276
							300	194.2	326
							400	203.2	364
7	GBB2	sphere	2.50	1623.4	1470.6	0.601	100	139.8	146
							200	161.1	202
							300	173.9	242
							400	183.3	274

Since the damping ratio of the specimens was less than 5%, the following relationship could be used for estimating the stiffness of the specimens.

$$2\pi\, f_r = \sqrt{\frac{k}{I_0 + I_a}} \quad (2)$$

Where k is the stiffness of the specimen, I_0 and I_a are the moment of inertia of the drive system and top disc respectively and f_r is the measured resonant frequency of the specimen. During calibration of the resonant column apparatus it was found that I_0 increased with resonant frequency (Clayton et al. 2005). Therefore, based on regressive analysis of aluminium samples of known stiffness, the following relationship was used to eliminate the dependency of I_0 on resonant frequency.

$$I_0 = 2.324 \times 10^{-8} f_r^{\,2} - 8.404 \times 10^{-7} f_r + 2.747 \times 10^{-3} \quad (3)$$

The values of G_{max} were then calculated using simple theory of torsion:

$$G_{max} = \frac{K \times L}{I_P} \quad (4)$$

Where L is the height of the specimen and I_p is the area polar moment of inertia of the specimen.

TEST RESULTS

The small strain shear modulus of all materials is summarised in Table 1. Figure 3a shows the change in shear modulus with shear strain for all specimens tested at σ'_0 = 100 kPa. It can be seen that G of glass Ballotini sample No2 (GB2, e = 0.578) is significantly lower than that for Leighton Buzzard sand B sample N^{o}2 (LBSB2, e = 0.601) even though the ratios of LBSB2 and GB2 were reasonably similar. In addition, the G of the mixture of LBSB and mica was lower than that of LBSB1 even though the void ratio of the mixture (e = 0.447) was lower than that of LBSB1 (e = 0.511).

The normalised shear modulus as a function of shear strain at σ'_0 = 100 kPa is presented in Fig. 3b. It can be seen that for all specimens there is a range where the normalised shear modulus is independent of strain, as shown by the stiffness plateau. It is adopted here that the elastic limit, or elastic threshold strain, is the strain at which G/G_0 = 98%. The elastic threshold strain at σ'_0 = 100 kPa for LBSE was the highest (0.004%, e = 0.694); those of LBSB1 and the mixture were almost equivalent, about 0.001-0.0015%; and that of GB2 was very low (0.0008%). The rate of stiffness degradation with respect to strain for LBSE was notably slow, the lowest among the materials, although its void ratio was the highest. At a strain of 0.01%, G/G_0 of LBSE was 94%. In contrast, the rate of degradation of stiffness for GB was significantly faster. At a strain of 0.01%, G/G_0 of GB2 was only 74%. When the strain was lower than the elastic threshold strain, the rate of stiffness degradation of LBSB1 was slightly higher than that of the mixture. However, when the strain exceeded the elastic threshold strain, the stiffness degradation of the mixture was considerably faster

The effect of confining pressure (σ'_0) on shear modulus is presented in Fig. 4. The results show that the shear modulus of all materials increased in an exponential manner with confining pressure. The stress exponent β (obtained from the equation for line of best fit to the data points for each specimen) varied from 0.39 to 0.46. Considering the values obtained for each specimen it can be seen that the stress exponent increases with void ratio (decrease with relative density).

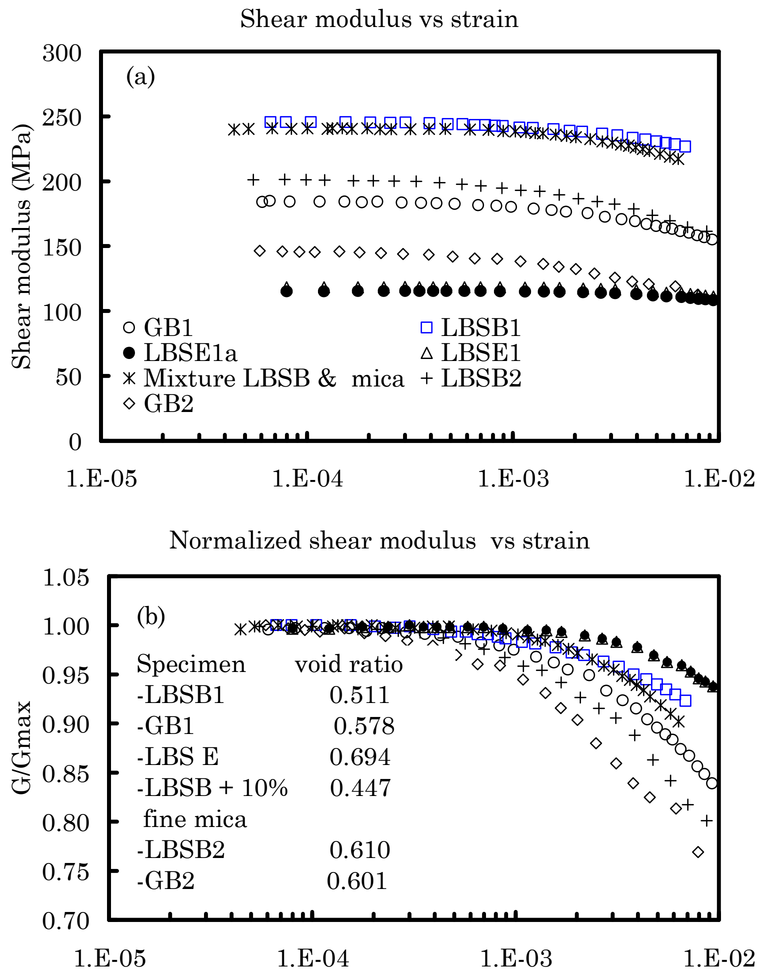

Figure 3. Shear modulus as a function of shear strain at 100 kPa

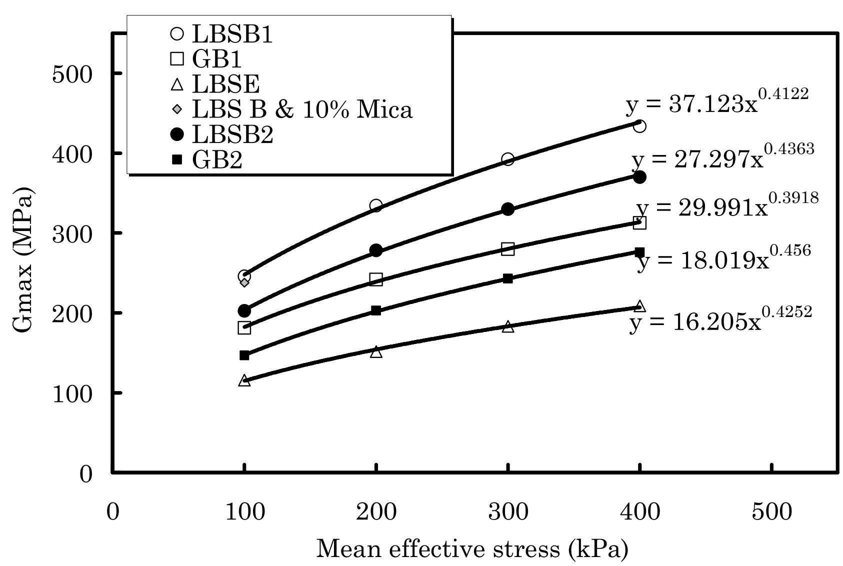

Figure 4. Effect of mean effective stress on G_{max}

The variation in shear modulus with void ratio at different confining pressure is presented in Fig. 5 In Fig. 5b, where the shear modulus was normalised by $\sigma'_0{}^{0.46}$, it is clear that as the void ratio increased, the shear modulus decreased.

System damping, including material and equipment-generated damping, was evaluated by both free decay vibration and half power point (HPP) methods. Damping ratios, D, estimated by HPP method at σ'_0 = 100 kPa, of all specimens are presented in Fig. 6. When shear strains were below 0.0001%, due to effects of background noise, the damping ratios of all the specimens were rather scattered. It is presumed here that the minimum damping ratios, D_{min}, are taken at 0.0001%. Figure 6 show that damping ratio generally increases with void ratio. Damping ratio of GB2 (e = 0.601) was higher than that of GB1 (e = 0.578), and damping ratio of LBSB2 (e = 0.610) was higher than that of LBSB1 (e = 0.511). However, at the same void ratio, damping ratio of GB2 was significantly higher that of LBSB2. The minimum damping D_{min} of LBSB1 and the mixture were 0.36% and 0.64% respectively, so that D_{min} of the mixture was 1.8 times higher than that of specimen LBSB1.

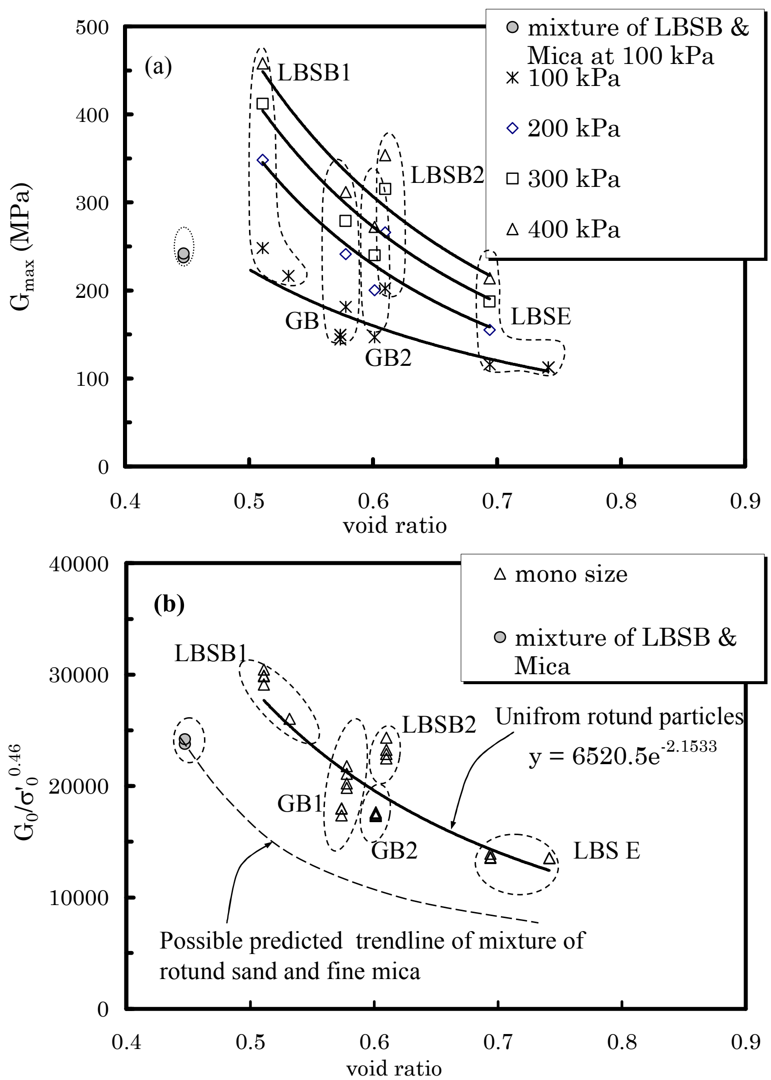

Figure 5. Effect of void ratio on G_{max}

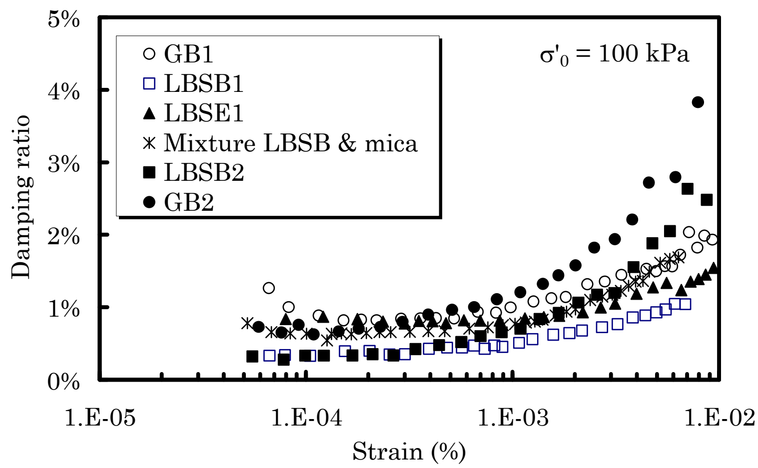

Figure 6. Damping ratio as a function of shear strain at 100 kPa

EFFECT OF PARTICLE SHAPE ON SMALL STRAIN PROPERTIES

It can be seen from the results presented in Fig. 3b that LBSE had the lowest stiffness degradation while GB had the highest. It is suggested that this behaviour is dependent on the surface roughness of the individual particles. It can be seen from Fig. 2b and Fig 2c that LBSE had a higher degree of surface roughness compared to that of GB. Surface roughness increases friction angle between individual particles, and modulus degradation is due to slippage at grain contacts. Therefore the high surface roughness and angularity of LBSE influences its rate of stiffness degradation with respect to strain. This leads to a high elastic threshold strain (0.004%) and a small reduction in modulus at high strains (6% reduction in G at 0.01% strain). When considering the damping ratio for the specimens, it was shown that LBSE was less susceptible to strain, and almost strain independent at higher confining pressures (Fig. 7), although the void ratio of LBSE was the highest. This strain independency is thought to be due to the high surface roughness and high angularity of LBSE, which locks the particles, preventing slippage and rotation.

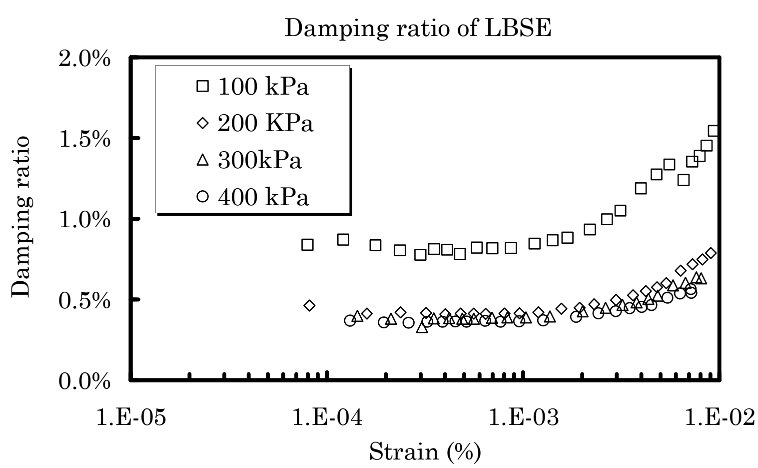

Figure 7. Damping ratio of LBSE estimated by HPP method as a function of shear strain at different confining pressure

In contrast, the dry smooth surface and spherical shape of GB gives a low friction angle and small regular contact area at grain contacts leading to an increased likelihood of sliding compared to the LBSE. In addition, the rolling is a possible particle motion for GB. Therefore as strain increases, rotation combined with a low resistance to sliding makes the stiffness of GB more susceptible to cyclic strain. This was evidenced by the elastic threshold strain of GB occurring at the lowest strain for all materials tested and with higher modulus degradation with strain. Damping in particulate materials for strains above elastic strain threshold is related to loss of energy due to particle movement and slippage at grain contacts. Damping for GB showed the highest damping values at higher strain reinforcing the hypothesis that the GB had lower resistance to sliding and that the particles were able to rotate about each other.

It is hypothesised that the platy shape of mica influences the G_{max} of the mixture of LBSB and mica. G_{max} of the mixture was lower than that of the LBSB1 alone, even though it's void ratio was lower (suggesting a higher G_{max}). Although only one mixture of LBSB and mica was tested it is suggested that a general trend for normalised shear modulus of this type of mixture would lie below that for the best fit curve of uniform rotund sands (Fig. 5b). When mica particles are added with the LBSB the mica fills the space between the sand particles and consequently reduces the void ratio of the mixture compared to the specimen with just LBSB. The mica particles may also prevent the direct interaction of the sand particles reducing the number point contacts between sand particles. The platy particles can also form flexible bridges between contacts decreasing the stiffness of the material. This leads to a reduction in G_{max} compared to that of LBSB1, and a faster reduction in shear modulus once the strain limit is beyond the elastic strain threshold.

CONCLUSIONS

Preliminary test results for some geomaterials have demonstrated the effect of particle shape on soil properties at small strain, e.g.

- As surface roughness and angularity increased, soil behaved more elastically (elastic threshold strain increased, shear modulus degradation curve shifted to the right)
- Addition of fine, platy mica into rotund sand reduced the stiffness, decreased the elasticity and increased the damping of the mixture, even though the void ratio of the mixture was lower than that of the rotund sand alone.
- As surface roughness and angularity increased, damping ratio decreased and became strain independent

In light of the conclusions drawn from these preliminary tests, a comprehensive laboratory testing programme is being undertaken at the University of Southampton to consider in more depth the effects of particle shape on the small strain behaviour of soil.

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