

NONLINEAR SOIL BEHAVIOUR UNDER STATIONARY AND TRANSIENT EXCITATION

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ABSTRACT

The simulation of wave propagation through the ground due to strong earthquake loading requires the usage of nonlinear constitutive equations. Considering transient excitation (e.g. strong earthquakes) the effects resulting from excitation and change of state of the soil are time varying and in general not separable. Under stationary excitation not only changes of state are directly observable also asymptotic behaviour can be achieved. In a first step specimens of sand with different initial states (void ratio, mean effective stress) were subjected to monotonous and cyclic loading in numerical element tests. The results show the link between static and cyclic/dynamic state-dependent behaviour. In a second step a column of the same sand (1D-model-FEM, identical material parameters) was subjected to stationary harmonic excitation (constant amplitude and frequency). The state of the soil along the column changes with increasing number of cycles, resulting in a transition of modes. A periodic response after a sufficient number of cycles was always found. Depending on excitation amplitude and frequency, as well as on initial state, a transient amplification or attenuation from the bottom to top of the column is observed. Additionally the development of liquefaction is considered. The same column was also subjected to earthquake excitation in a third step. The analysis of the results is rather complex in detail due to the loss of separability between variation of state and excitation.

Keywords: soil liquefaction, nonlinear dynamic soil modelling, numerical methods, strong ground motion, hypoplasticity, 1D-simulation

INTRODUCTION

In order to get reliable estimates for the effects of earthquakes on structures an accurate modelling of the wave propagation in the uppermost soil layers (the so-called subsoil) is required. Additionally the interaction between soil and foundation and between foundation and structure has to be included. A complete system of soil-foundation-structure-interaction (SFSI) is in general highly non-linear (Bühler et. al, 2006). Considering real earthquake excitation leads to problems in separating the effects from the source signal and change in state of the upper soil. This contribution presents numerical simulations with a hypoplastic constitutive law evaluating effects in the subsoil by parametric studies.

MECHANISMS AND REQUIREMENTS

Figure 1 sketches wave propagations from a seismic energy source to the surface. Only the subsoil is of interest here. For cases where shear-waves are dominating near the surface, it is justified to consider 1-dimensional wave propagation only. More precisely speaking, a 1D-model is sufficient if the ground consists of horizontal layers and other wave types are of minor importance. For simulations linear or equivalent linear models (e.g. 'SHAKE' Schnabel et al., 1972) or similar programs are commonly

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used. If the state of soil is changing during an earthquake by a decrease of effective stress up to liquefaction, nonlinear and transient models including the stress/strain behaviour of the soil are required (Kramer, 1996; Iai and Tobita, 2006). Using nonlinear constitutive laws for 1-D wave propagation, measured and predicted accelerations for free-field conditions showed good agreement (Cudmani et al., 2003).

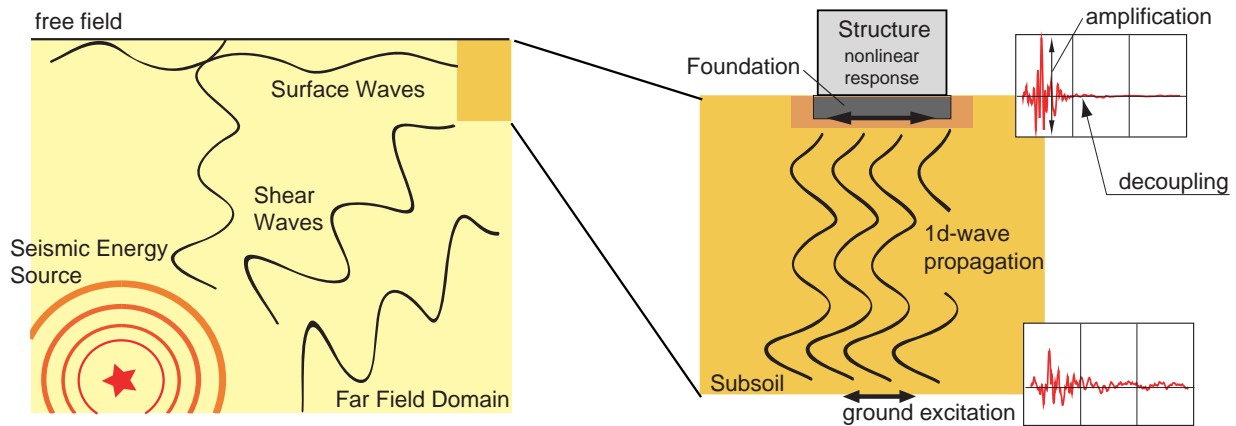


Figure 1. Principal scheme of a dynamic SFSI far-field- and subsoil including foundation and structure

In order to model the dynamic behaviour in soft and liquefiable soils realistically, it is necessary to take the following essential aspects into account:

1. Non-linearity of the soil, development of excess pore water pressure and the associated reduction of effective pressure for undrained conditions,
2. hysteretic nature of soil damping,
3. radiation of wave-energy through the model boundaries (geometric damping), and
4. the state of soil as well as the base excitation signal applied to the subsoil.

The relevant mechanisms, the constitutive law being used and influence of state of soil and base excitation will be considered in the following sections.

Mechanisms

There are four mechanisms which have to be especially taken into account for a dynamic soil model: densification of soil, amplification of waves, decoupling of wave propagation in saturated soil, and liquefaction of soil. The phenomenon of soil liquefaction is one of the non-linear soil effects, which is most difficult to consider in the simulation. During strong earthquakes, liquefaction can cause large ground displacements which lead to devastating effects on structures as we know from many structural failures in past earthquakes. Most constitutive models available nowadays have not achieved yet the stage of development required to predict soil liquefaction reliably. The mentioned phenomena can be explained with the time history diagrams in Figure 1 (right). First, one can see an enlargement of amplitude between the signal at depth and at the surface, i.e. amplification. This results from a decay in shear stiffness G due to a reduction of the effective mean pressure p' . With ongoing excitation a decoupling occurs, which means that the movement of the surface is no longer coupled to the excitation. Due to the cyclic shearing in the soil there is a gradual densification, also depending on the initial density, whereas under undrained conditions an increase in pore water pressure leads to a reduction of effective mean pressure p' . If the pore water pressure reaches the total mean pressure the skeleton decays into a suspension, then we are talking about liquefaction. A liquefied zone is not able to transmit shear waves anymore, this leads to described effect of decoupling.

Constitutive laws for soil

Nonlinear material laws are required for realistically evaluating the behaviour of the subsoil. The presented model allows examination of liquefaction and cyclic mobility for saturated soils.

The subsoil behaviour is modelled with hypoplastic (Gudehus, 1996) and visco-hypoplastic constitutive equations (Niemunis, 2003) which are able to simulate the response of cohesionless and

cohesive soils under monotonous loading. The extension of the constitutive relation by the concept of intergranular strain enables a realistic alternating loading.

In contrast to elastoplastic models the used hypoplastic description distinguishes between material parameters and state variables. The material parameters are: the so-called granulate hardness h_s and the exponent n describing the compressibility of the grain skeleton, the critical state friction angle φ_c , the characteristic void ratios e_{d0} (lowest), e_{c0} (critical) and e_{i0} (highest) at zero pressure (Index 0) and the exponents α and β controlling the density influence on peak friction angle and compressibility.

These models are used to model plastic deformations of grain skeletons incorporating the critical state concept of soil mechanics. Strength and stiffness depend on current stress state σ , density (via void ratio e), deformation rate $\dot{\epsilon}$ and deformation history δ . Subsequently commonly used parameters such as shear modulus G , damping D and velocity of propagation c are state dependent, and do not have to be known in advance as with other constitutive relations.

The total mean stress p is separated in the constitutive law into effective stress p' and pore water pressure p_w : $p = p' + p_w$. The mechanical behaviour of soil is estimated with effective stress p' . In case of undrained deformation, the evolution of the pore water pressure p_w depends on volumetric deformation taking into account the bulk modulus of the water k_w .

In the simulations following hypoplastic soil parameters for quartz sand were used (Table 1).

Table 1. Hypoplastic soil parameters (quartz sand)

Soil	h_s [MPa]	n	φ_c [°]	e_{d0}	e_{c0}	e_{i0}	α	β
sandy fill	4000	0.25	32.1	0.55	0.95	1.05	0.07	1.0

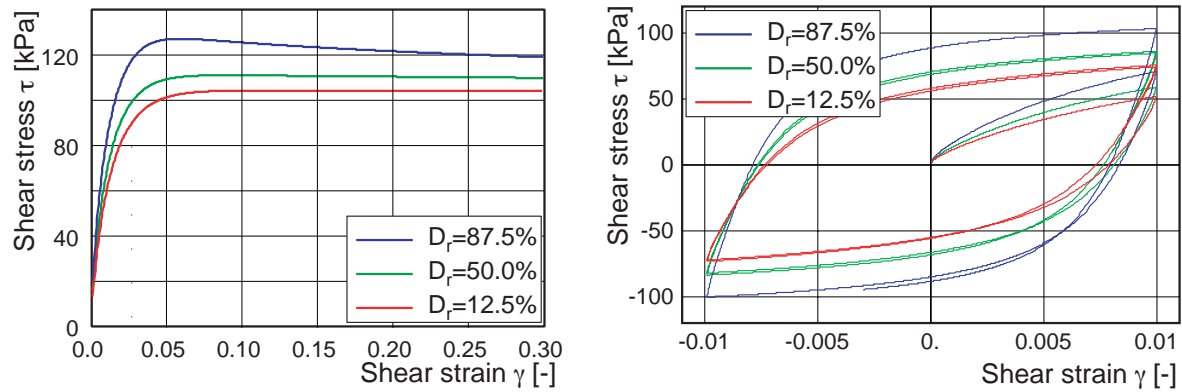


Figure 2. Calculated simple-shear test: monotonic (left) and cyclic (right); shear-stress τ vs. shear-strain γ for different relative density (quartz sand)

Stationary results for a simulated monotonous and cyclic simple-shear-test are shown in Figure 2 and 3 for K_0 -state with mean pressure $p' = 100$ kPa. The cyclic tests were performed with a shear-strain amplitude γ up to 1%. The influence of relative density D_r is visible for dense ($D_r = 87.5\%$; $e_0 = 0.60$), medium ($D_r = 50\%$, $e_0 = 0.75$) and loose ($D_r = 12.5\%$, $e_0 = 0.90$) sand. The relative densities are calculated after the common relation

$$D_r = \frac{e_{\max} - e_0}{e_{\max} - e_{\min}}, \quad (1)$$

with the initial void ratio e_0 and the limit void ratios e_{\max} ($e_{c0} \approx e_{\max} = 0.95$) and e_{\min} ($e_{d0} \approx e_{\min} = 0.55$) according to the German standard DIN 18126. The decay of equivalent shear-modulus G and simultaneous increase of damping ratio D for shear-strain amplitude $\gamma > 10^{-4}$ arises similarly to measurements of resonant-column tests.

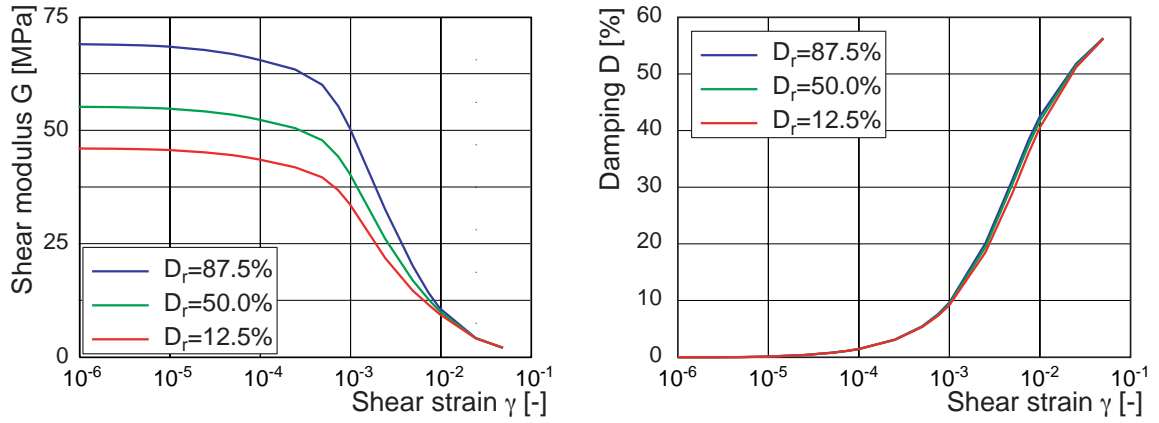


Figure 3. Calculated equivalent shear-modulus G and damping ratio D vs. shear-strain amplitude γ for different relative density with soil parameters for a quartz sand

The equivalent shear-modulus G_0 for small shear-strain amplitudes γ_0 in dependence of effective mean pressure p' is shown in Figure 4 (left). With known density of saturated soil ρ_{soil} ($\approx 2.1 \text{ to/m}^3$) velocity of propagation of shear-waves c_s can be easily calculated in dependence of depth, cf. Figure 4 (right):

$$c_s = \sqrt{G / \rho_{soil}} \quad (2)$$

with

$$p' = \frac{1 + 2 \cdot K_0}{3} \cdot (\rho_{soil} - \rho_{water}) \cdot g \cdot z. \quad (3)$$

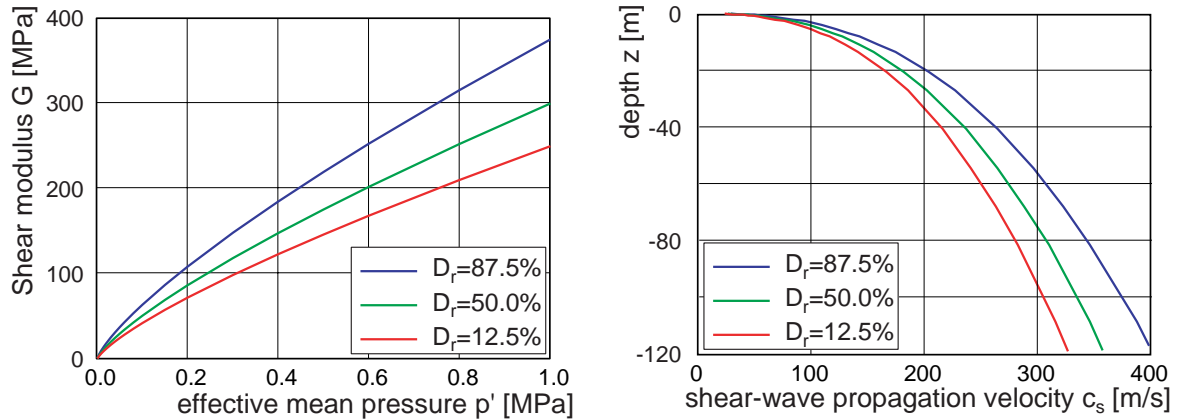


Figure 4. Calculated equivalent shear-modulus G_0 vs. effective mean pressure p' for small shear-strain amplitude γ (left) and shear-wave propagation velocity vs. depth (right) for different relative density with soil parameters for a quartz sand

Excitation

For any scheme of a dynamic SFSI like in Figure 1 an excitation at the lower boundary of the subsoil region has to be applied. If the subsoil is modelled down to bedrock or ends up higher measured earthquake data from deep stations or synthesized signals for specific sites can be used (Gottschämmmer et al., 2002). If both are lacking a signal from a surface rock station can be used as bedrock excitation instead. These signals contain fast changing velocity amplitudes and a wide frequency content. For systematic investigations of nonlinear effects due to subsoil behaviour sinusoidal signals are much more favourable. The range of frequency and amplitude for the tests can be adapted to real earthquake signals. As an upper boundary of the amplitude and frequency content of earthquake signals measured at the surface during the 1989 Loma Prieta earthquake ($M_w=7.1$) at two sites have been taken.

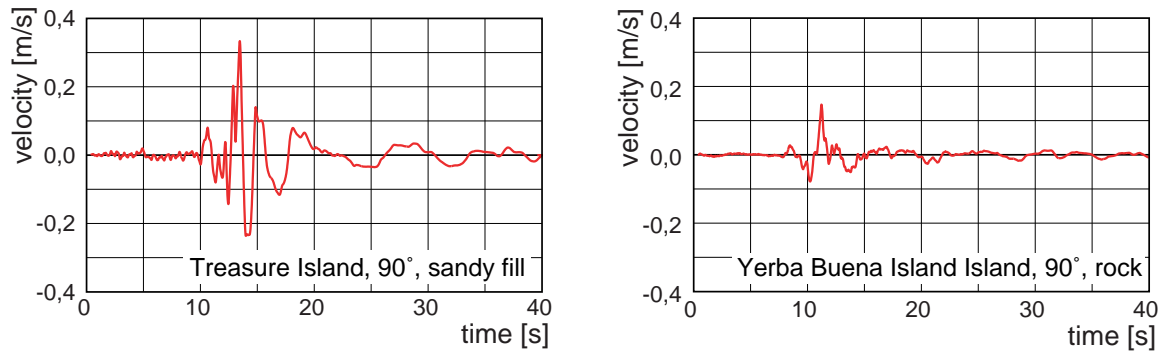


Figure 5. Velocity time history recorded during 1989 Loma Prieta earthquake at two stations in the San Francisco Bay area (Cosmos)

Figure 5 left shows the velocity time history measured at Treasure Island in the San Francisco Bay. This surface station is a so called soil station showing a peak ground velocity (PGV) of about 0.35 m/s. Depending on the type of station and the distance to the epicentre this PGV can be up to 1.16 m/s and more (instrumental intensity X+)(Wald et al., 1999). For the parametric study described below, velocities from 0.05 to 0.80 m/s were therefore considered.

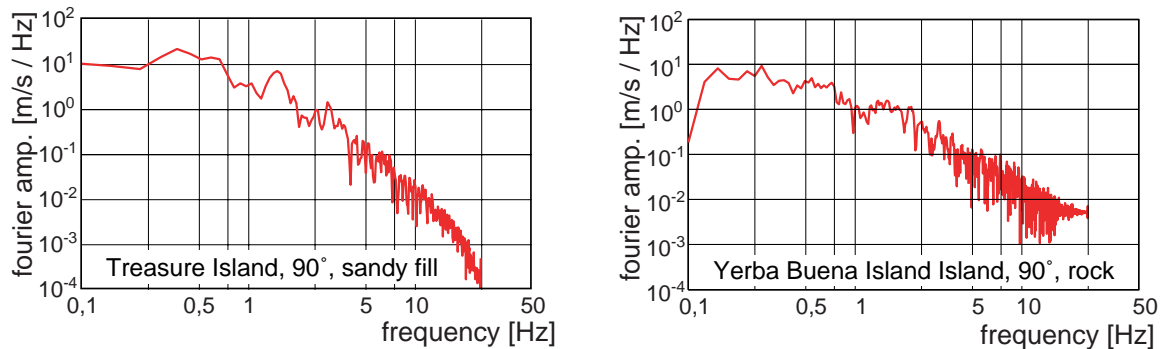


Figure 6. Fourier-spectrum for the Yerba Buena Island velocity time history

Figure 6 (right) shows the frequency content of the Yerba Buena Island velocity time history. The Fourier amplitude declines about 3 orders of magnitude up to a frequency of about 10 Hz. Therefore only frequencies with relevant amplitudes (0.2 to 8 Hz) are considered for further investigations.

WAVE-PROPAGATION FOR FREE-FIELD CONDITION

Nonlinear effects were considered with transient wave propagation from an originally homogeneous soil column for free-field conditions including liquefaction in a quasi one dimensional problem. As earthquake signals contain rapidly changing velocity amplitudes and frequency content (Figure 1), sinusoidal signals are more convenient for systematic investigations. Advantages of stationary excitation are identification of modes of vibration, asymptotic behaviour (attractors) depending on amplitude as well as on frequency. Due to nonlinearity the modes of vibration are also varying with time. In our parametric study the transient behaviour of a saturated soil column under sinusoidal excitation shows amplification and reduction of surface vibrations depending on excitation amplitude, frequency, number of cycles and density of soil. Certainly, these effects also appear with an earthquake signal (cf. Figure 13), but a separation into the mentioned factors (amplitude, frequency, ...) is not possible.

For the simulations a Finite Difference Algorithm including hypoplastic and visco-hypoplastic constitutive relations can be used (Osinov, 2003). The presented calculations were accomplished with FEM in order to check other methods. The full 3D-formulation of the dynamic calculation is a further advantage of the used FE-Code ABAQUS.

FE-Model

The simulations presented were accomplished with the FE-Code ABAQUS 6.5. The FE-model of the soil-column consists of 240 elements (Figure 7 left). An implicit time-integration is used for the dynamic calculations. During the earthquake, water drainage is not permitted, i.e. undrained soil behaviour is assumed. A lateral periodic boundary condition of the soil is imposed by constraining opposite nodes on these boundaries to undergo the same displacement. This boundary condition is reasonable for strong earthquakes since the energy dissipation in the soil due to (hysteretic) damping supersedes the energy radiation from the foundation. The periodic boundary condition provides exact results in the case of level ground with free surface subjected to base shaking without structure (Gudehus et al., 2004). The parameters for variation with sinusoidal bedrock excitation at the bottom nodes of the model are velocity-amplitude and frequency.

Reference Example

As reference simulation the following properties were applied: Frequency $f = 2$ Hz, excitation amplitude $v_{bottom} = 0.4$ m/s, height of soil column $h = 30$ m (Material parameters cf. Table 1) and relative density $Dr = 50\%$ (medium dense, $e_0 = 0.7$). On top of model an extra load of $p_{top} = 10$ kPa is applied because of numerical stability requirements.

Results are evolution of excess pore water pressure and mean effective pressure for different time steps (Figure 7 middle). Beginning at the bottom, p_w increases and simultaneously p' decays rapidly. After about two seconds the effective pressures (Figure 7 right) almost vanish, i.e. the stiffness of the layer is strongly reduced (onset of liquefaction).

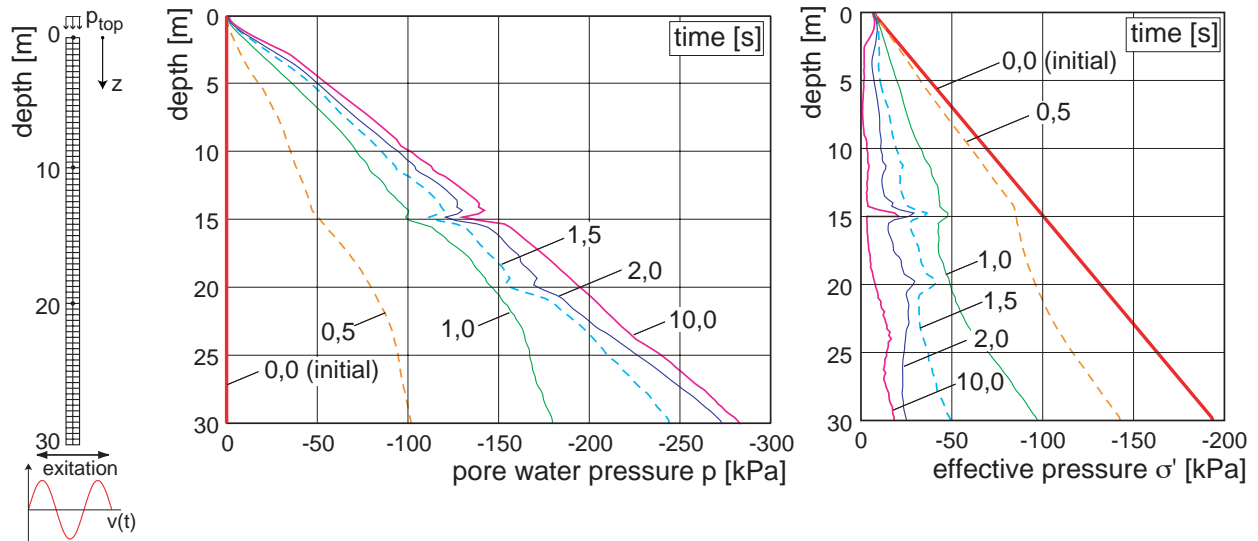


Figure 7. Pore water pressure (left) and effective stress (right) at different times

The wave propagation is shown in Figure 8 (left) with horizontal component of velocity at different depths. Caused by the change of stiffness from bottom to top shear wave velocity decreases and thus amplitude will increase. The transient reduction of the amplitude caused by reduction of stiffness after an initial amplification is clearly shown.

Figure 8 (right) depicts corresponding vertical displacements; the detail shows the higher frequency in the vertical direction due to change of dilatancy and contractancy of soil during shear cycles. As the calculations are performed under undrained condition settlements after consolidation have to be estimated by the change in void ratio.

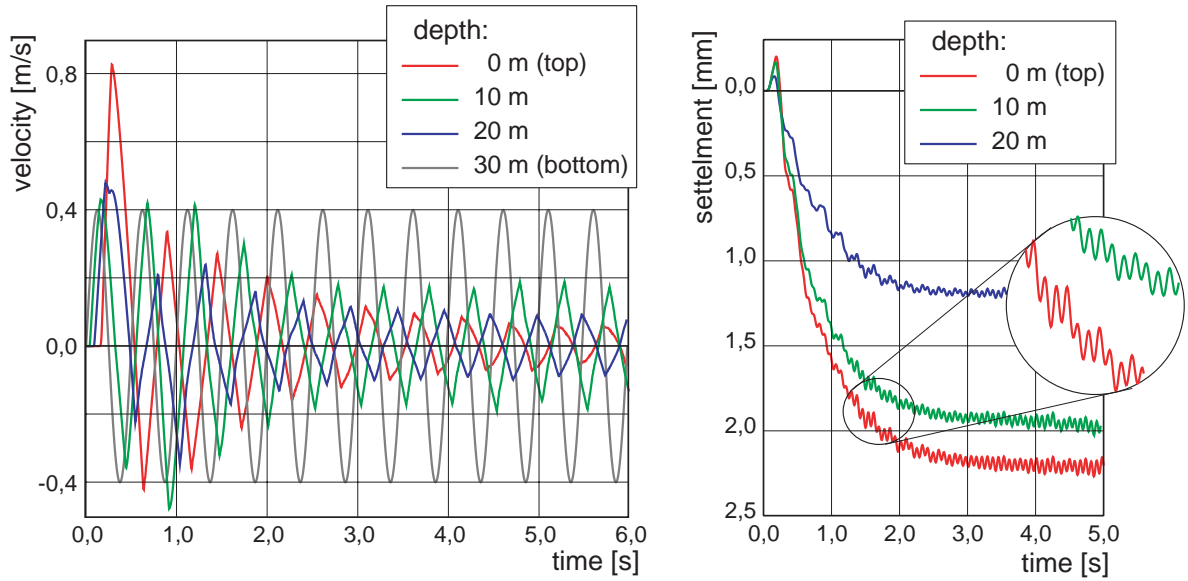


Figure 8. Velocity time history (left) and settlement (right) at different depths

Transient mode of vibration

The amplification is the relation between horizontal velocity amplitude of ground excitation v_0 and horizontal velocity of surface point $v(t)$ in calculation:

$$\text{amplification} = \frac{|v(t)|}{v_0} \quad (4)$$

The maximum value is reached during the first cycles, and after a sufficient number of cycles the residual amplitude arises.

The amplification vs. time for $v = 5 \text{ cm/s}$ is plotted as time-history (dotted) and the envelop (line) in Figure 9. In this case, the change of amplification and mode of oscillation in time is obvious. For particular times t_0 (4.00/4.25 s, 14.00/14.25 s, 20.75/21.00 s) the displacement contour of the column vs. depth is shown. The first mode similar to $\lambda/2$ is visible in a), b) and c) show higher modes (with λ = wavelength for “natural” frequency).

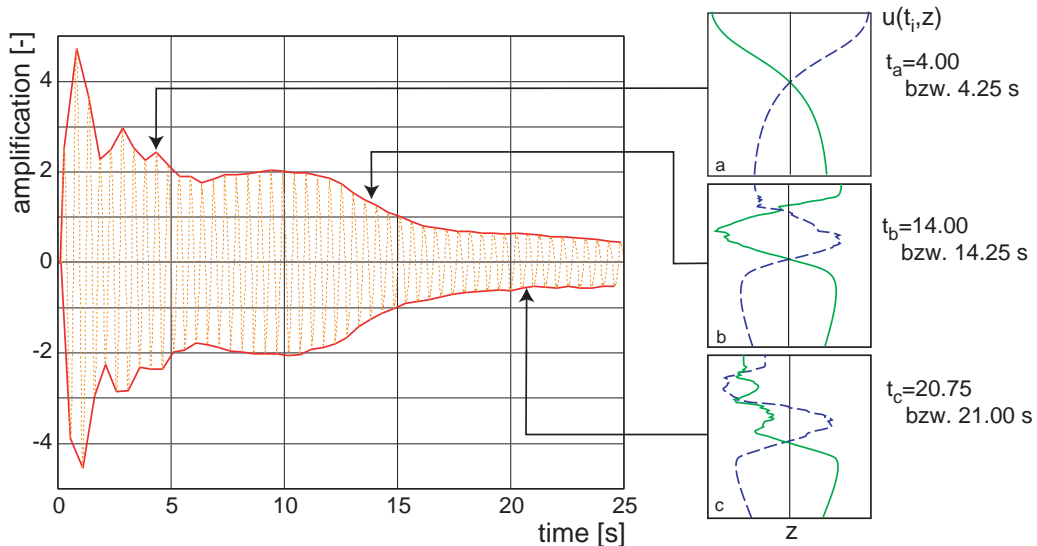


Figure 9. Change in amplification vs. time for $v = 5 \text{ cm/s}$ and $f = 2 \text{ Hz}$; transient change of vibration mode (a, b, c)

Variation of excitation

In a first study the velocity amplitude v of ground excitation was varied between 5 and 80 cm/s. Figure 10 compares the amplification for different excitation amplitudes. The significant amplification

(maximum value) for low velocities in contrast to higher velocities is remarkable Figure 10 (right). Figure 10 (left) shows envelopes of amplification for different excitation velocities. For higher velocities the amplification to a residual value decays even faster. This is caused by the more rapid reduction of stiffness resulting from lower effective pressure and size of shear-strain amplitude caused by dynamic effects.

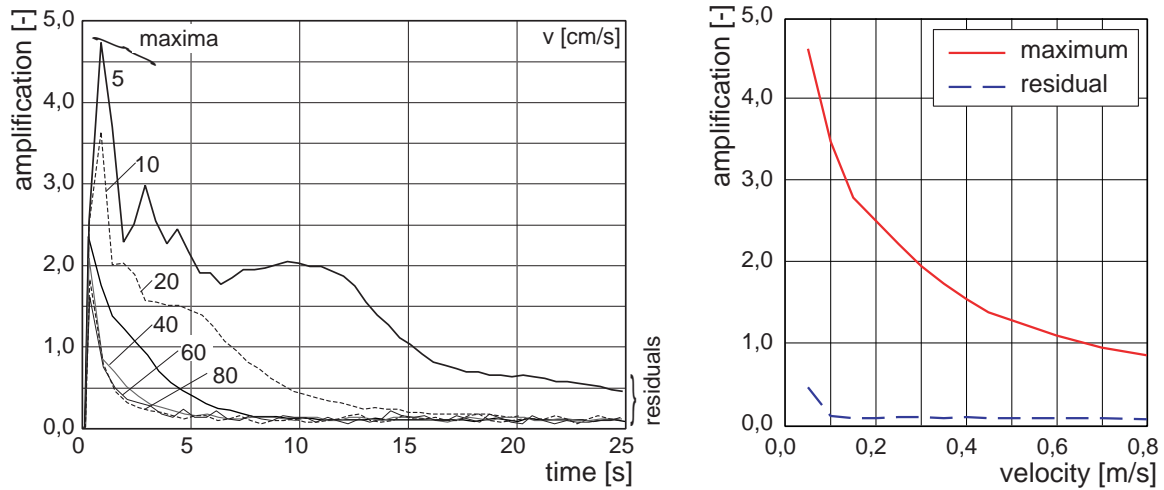


Figure 10. Envelopes for different base velocity amplitudes v (left) and amplification vs. velocity (right) for constant frequency $f = 2$ Hz

As mentioned above, the second varied parameter is frequency. Figure 11 depicts the amplification for two different velocity amplitudes. Starting with rigid body motion at very low frequencies amplification of approximately one occurs. With increasing frequencies a pseudo resonance peak will be achieved for the first few cycles. This amplification suggests a “natural” frequency of about 1.6 Hz for the examined soil column.

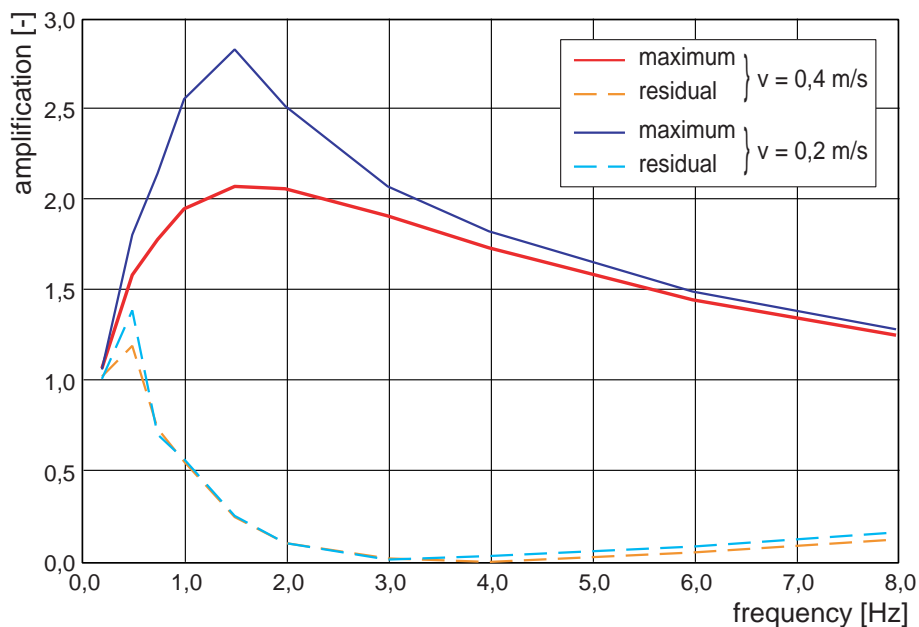


Figure 11. Variation of ground excitation – amplification vs. frequency for two velocities

After a sufficient number of cycles at higher frequencies the residual amplitudes are strongly reduced (amplification $\ll 1$) independent of excitation velocity. This is caused by onset of liquefaction. For lower frequencies there is still a resonance like local maximum for residual amplification at approx. 0.5 Hz due to the elastic range in the constitutive law. In contrast to the maximum curve this curve is shifted to lower frequencies. The change in resonance frequencies and amplifications is due to the

nonlinearity of soil. For small amplitudes and very low frequencies (< 0.25 Hz) the motion of the column is to be assumed as a rigid body (amplification = 1) due to negligible inertia.

Variation of density

Figure 12 and 13 shows the influence of initial relative density. There is a decay of the maximum amplification for decreasing relative density. Medium dense sand has only in first few cycles this amplification. Cyclic shearing leads to densification, and thus to an increase of pore water pressure. After few cycles there is a decoupling for initial relative density lower than 50%. For dense sand negative pore water pressure occurs due to inability of further densification of soil. This results in significant remaining amplification.

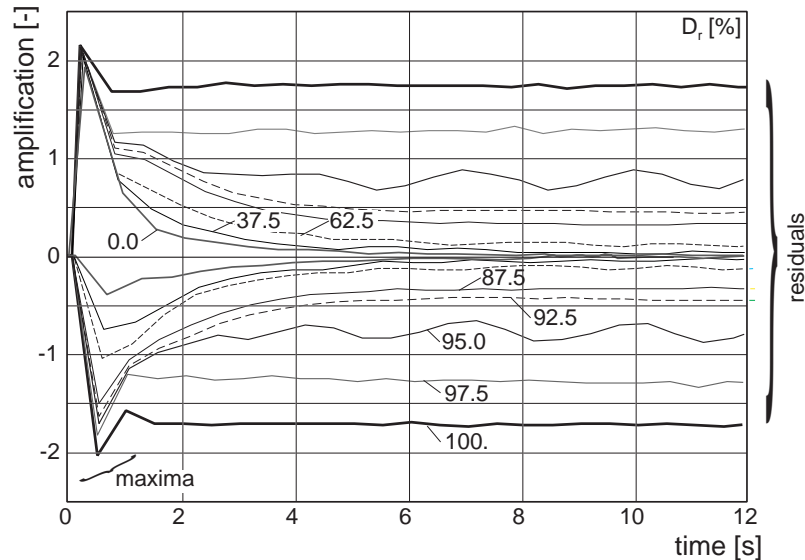


Figure 12. Variation of density; envelopes for different void ratios for $v = 0,4$ m/s and $f = 2$ Hz

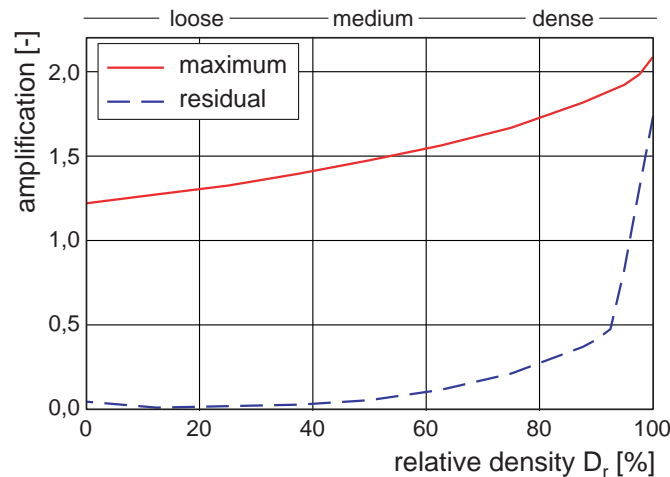


Figure 13. Variation of density; amplification vs. initial void ratio for $v = 0,4$ m/s and $f = 2$ Hz

Remarks on earthquake excitation

The soil modelled with the nonlinear constitutive law shows a transient change in amplification during stationary excitation. Earthquake excitation signals are non stationary and consist in contrast to stationary excitation of substantially fewer cycles of high amplitude. Therefore residual states are usually not achieved during an earthquake. Thus for an evaluation of nonlinear constitutive laws reaching an asymptotic behaviour (attractors) and/or state limits are essential.

Figure 13 shows the velocity history and the evolution of effective stress in a soil column with earthquake excitation; see Fig. 5 (Yerba Buena Island, rock). From bottom to top the velocity

amplitudes increases. The effective pressure decreases with time and vanishes after 12 sec at a depth of about 5 m. Due to the additional load on the top liquefaction near the surface can not occur.

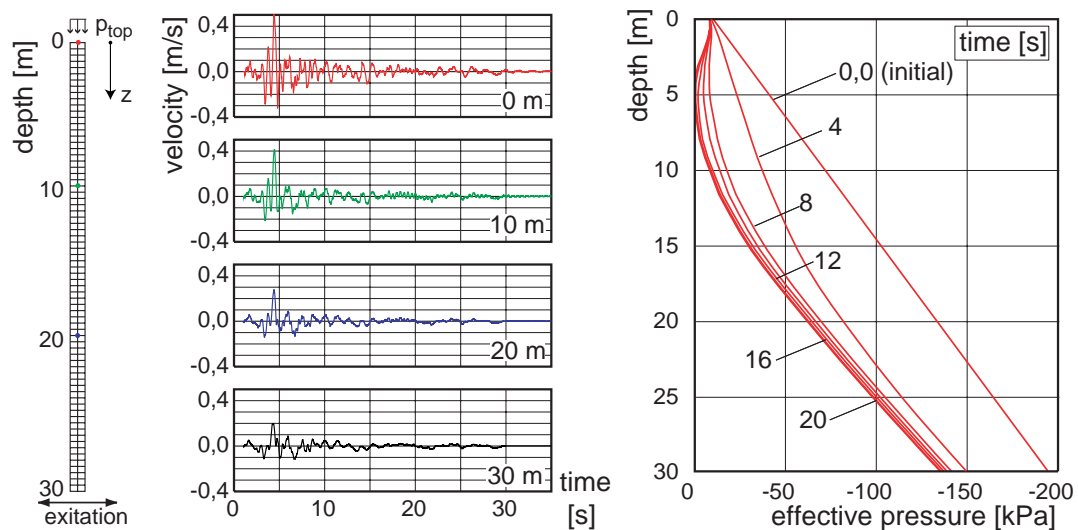


Figure 13. Velocity time history at different depth (left) and reduction of effective pressure at different times (right) for earthquake excitation

CONCLUSIONS

There are several effects in the near surface subsoil observed from experiments and during earthquakes. They lead to a transient change of the state of the soil, increasing pore water pressure thus decreasing effective stress up to liquefaction.

It is possible to model these effects using a hypoplastic nonlinear constitutive relation. Thereby it is not necessary to prescribe a shear strain dependent shear modulus like in linear or semi-linear wave propagation model. Only conventional properties of soil and its state are needed as input data. For dynamic calculation inertia has to be considered additionally.

The soil column can be excited either periodic (e.g. sinusoidal) or earthquake-like. The periodic excitation enables an insight in the transient behaviour of soil. Thus it is possible to identify slowly varying modes of vibration. Additionally the change of amplification with variations of amplitude and frequency is obvious.

Using a homogenous soil column (e.g. without layers) allows a comparably easy interpretation of the results and shows the principal mechanisms of the initial boundary value problem. A soil column with different soil layers (material or density) can be modelled as well. Additional effects due to layering lead inevitably to a lack of a unique interpretation.

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