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EXPERIMENTAL AND NUMERICAL STUDIES OF THE WAVE FIELD GENERATED BY THE OSCILLATION OF A MODEL STRUCTURE

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ABSTRACT

A series of release tests were performed at an instrumented bridge pier model in scale 1:3, which is located at the EUROSEITEST site (http://euroseis.civi.auth.gr). The work presented herein is mainly focused on the study of the wave field generated in the surrounding soil, the characteristics of soil structure interaction and the accuracy of the numerical modeling of the experiment. The analysis of experimental results took place on the time and frequency space as well and indicates the complicated nature of soil-structure interaction. Various themes are studied such as the amplitude of the free field movement, the frequency content, the damping factor as well as the spatial damping of wave propagation and finally the type of the propagating waves by the investigation of the orbits of motion. The finite element model is constructed in 3D space and contains the linear elastic pier model and the first two upper linear elastic layers of the soil profile. The numerical analysis took place in the time domain whereas the results are studied also in the frequency domain by the use of FFT. The comparison between the experimental and numerical results is satisfactory enough. Therefore, the validation of a finite element model with the experimental results enables a most thorough and versatile study of the composite nature of the phenomenon, which is related to the dynamic soil-foundation-structure.

Keywords: soil-structure interaction, wave propagation, experiment, finite element analysis.

INTRODUCTION

Soil-structure interaction as well as waves produced by the oscillation of the structure founded in the soil is a very interesting theoretical and experimental subject. The main advantages of in situ experiments are the real soil conditions, the existence of natural boundaries and real forces sources such as ambient or seismic vibration.Various experiments have been conducted during the past years in real scale structures as well as in model structures. An extensive literature has been published referring to the experiments conducted at the Milikan Library Building by Foutch A.D. and Jennings C.P., 1978 and Luco J.E. et al., 1987, 1988. The research group of Stewart P.J. et al, 1999 has studied empirically and theoretical the contribution of soil-structure interaction effects on the dynamic response of 57 buildings under strong motion events. Mucciarelli M. et al., 2003 studied the wave field generated by a force vibration of a 3storey seismic isolated building in Rapolla, Italy. Considering experiments referring to model structures the most characteristic and well known case is the Hualien and Lotung project. The containment models have been excited by forced vibration and strong motion events as well (Graves H.L. et al., 1996). Regarding the EUROSEITEST site pull out tests have been conducted at a 5storey reinforced concrete building by Manos G.C. et al., 1994, 2005 and Guegeun P. et al., 2000 at the 5storey reinforced concrete building.

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In the present paper the experimental results as well as the numerical results by the use of the finite element method are presented considering pull out tests that have been performed at a bridge pier model constructed and instrumented at the EUROSEITEST site. The target of the study is twofold and refers to the understanding of the complicated dynamic soil-structure phenomenon under controlled force conditions and the validation of the finite element model created.

STRUCTURE AND SOIL FIELD

The reinforced concrete bridge pier model was erected in 2004 under scale 1:3 and is similar to bridge pier models that were tested at the ELSA, European Joint Research Center (Pinto A.V., 1996). The structure consists of a rectangular foundation $2.5x2.5m^2$, a pier-column $0.20\ x0.5m^2$ and a deck $3.9\ x2.0m^2$. The total height of the structure is 4.15m. During the experimental process an additional mass was placed on the deck in order to change the dynamic characteristics of the structure. At the distance of approximately 14.5m from the pier, another model structure exists which refers to 5storey concrete building and was tested during previous projects (Figure 1). The soil profile characteristics of the Volvi Test Site (Figure 1) were investigated during past experimental projects by various geotechnical and and geophysical surveys (Pitilakis K. et al., 1999). Due to the soft soil layers the soil-structure interaction effect is expected to be quite significant.

Depth [m]	Description	Vs [m/s]	ρ [t/m³]
0-4	Silty Clay Sand	135	2.05
4-20	Silty Sand and Sandy Clay Gray-Black	225	2.15
20-70	Silt-Silty Sand, Sandy Clay Light Brown	325	2.10
70-100	Clayey Silt and Marly Caly Gray-Green	425	2.15
100-130	Silty Clay with sand and gravels Brown to Green	525	2.15
130-184	Silty-clayey Sand, Sandy Clay or Silty Clay with gravels, Clayey-silty Sand	835	2.20
184-196	Weathered Schist	1350	2.50
196-200	Gneiss	2600	2.60

Figure 1. Model structures and soil profile characteristics at the Volvi Test Site.

SOIL-STRUCTURE INTERACTION EXPERIMENTS

The thorough and systematical experimental study of dynamic soil-structure interaction under real soil conditions requires the proper instrumentation of the oscillating structure and the surrounding soil. The aim of the free vibration release tests performed herein is twofold: (a) to investigate the characteristics of the wave field in the ground due to the oscillation of the structure and (b) to study the soil-foundation-structure effects.

Description of the experiment

In the frame of the preliminary experiments, a “pull out” release test took place on the 06/04/2004. The force was applied at the deck of the model by the use of a tendon. One edge of the tendon was attached to the deck of the pier model while the other end on the roof of the auxiliary house, which is located at a distance of 7,8m from the pier. Once the applied force reached the desired level, the tendon was released and the structure was free to vibrate. Figure 2 presents the instrumentation of the free field. Since the direction of the applied force refers to XX axis, the majority of the instruments was placed on the free ground surface between the pier and the 5storey-building model. Additional instruments were place on the YY axis and on a diagonal axis. Laboratory of Strength of Materials (A.U.T.H., LSM) was the responsible of the experiment while various institutes and laboratories assisted the Laboratory of Soil Dynamics and Earthquake Geotechnical Engineering of AUTH in the free field instrumentation (Geophysical Laboratory of AUTH, Institute of Engineering Seismology and Earthquake Engineering , Laboratory of and Seismological Laboratory of Athens).

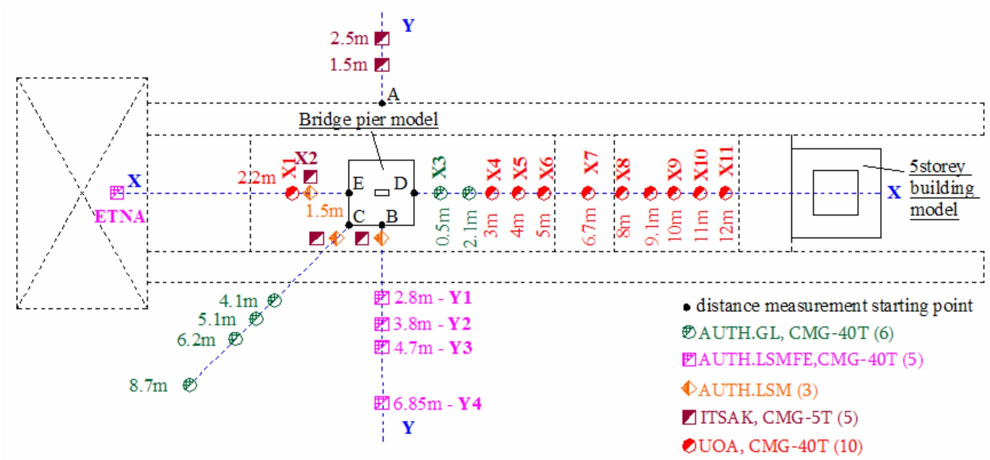


Figure 2. Free field instrumentation, 06/04/2004.

Experimental results

The wave field generated by the structure oscillation and propagating in the surrounding soil can be studied thoroughly in the time and frequency domain. Spatial ground motion decay and orbits of motion contribute also to further understanding of the physics of the problem.

Time Domain

The study of the recordings in the time domain is restricted to the first 15 seconds of the motion. The vertical component is referred as vertical, the horizontal on axis XX as in plane and the other horizontal on axis YY as out of plane. In Figure 3 the free field displacements are depicted for the three components of motion. The vertical and the in plane displacements follow almost the same pattern of time decay, which is similar to the expected free vibration scheme. The amplitude of vertical displacement is almost equal to the in plane due to the fact that the horizontal force, which is applied at the deck of the pier model, is transferred to the foundation-soil interface as a horizontal force and a moment around the out of plane axis. Another characteristic observation is that the instruments that are placed at distances larger than twice the foundation's dimension record vertical displacement which follows the exponentially decay pattern only for the first 5 seconds of motion. Later on the motion is dominated by other source contributions that can be attributed to the existence of nearby structures such as a surface concrete mass and the 5storey-building model. On the other hand in plane displacement is not at all affected by the additional contributions and the exponential decay scheme lasts 15 seconds even in the case of the most distant instrument. The horizontal motion recorded in the out of plane direction follows a more complicated pattern than the other two components. In particular, the out of plane displacements decay exponentially with time only for the first 5 seconds of the motion whereas continue to maintain almost the same amplitude possibly by contributions of nearby structures. The limited but obvious amplitude of out of plane displacements is attributed to a possible eccentricity in force application and the three dimensional nature of the phenomenon as well.

Axis XX			axis YY		
vertical [m]	out of plane [m]	in plane [m]	vertical [m]	out of plane [m]	in plane [m]

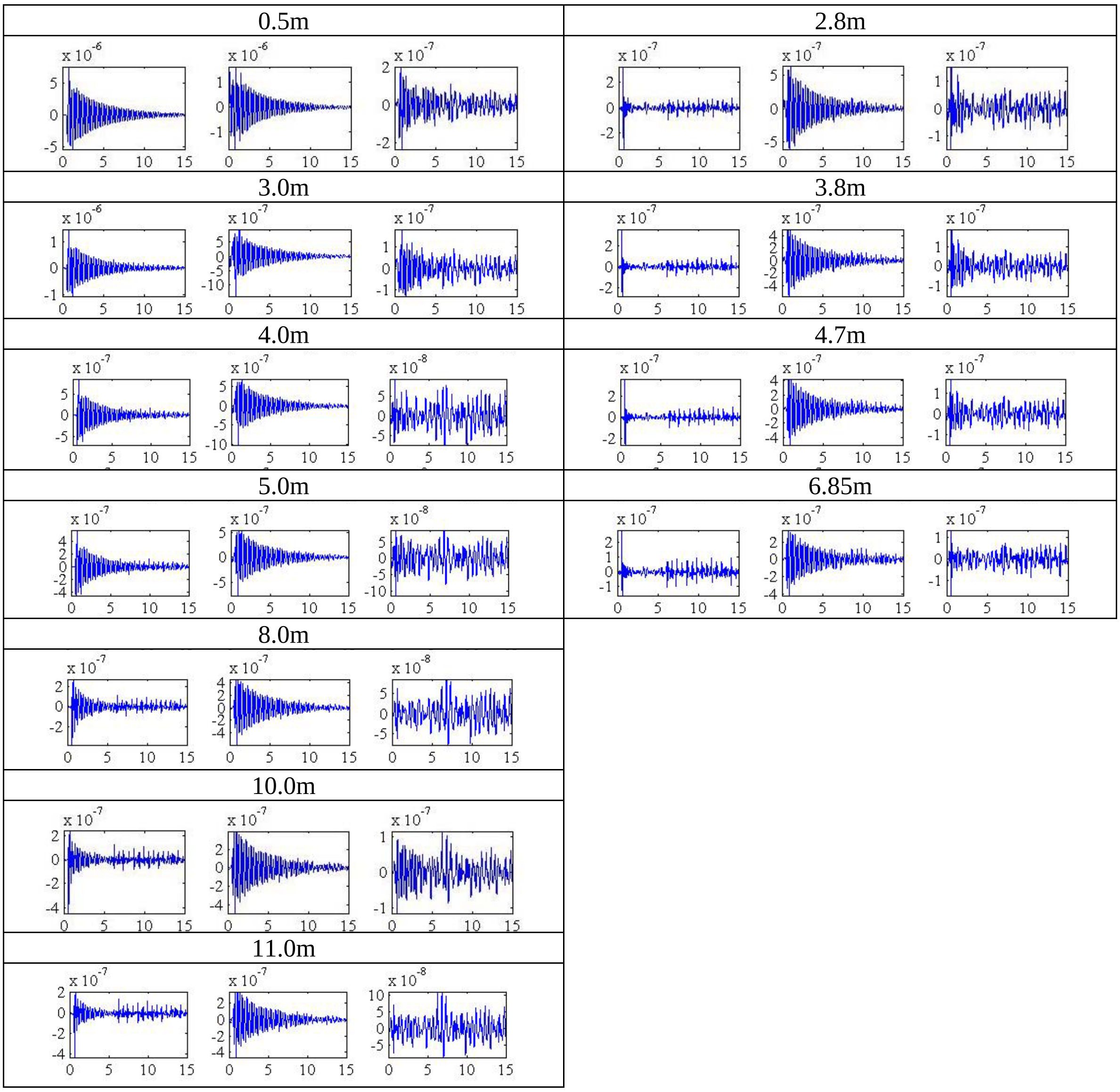


Figure 3. Free field displacements on axis XX and YY.

In Figure 3 the free field displacements on the out of plane axis are also depicted. The largest amplitude is observed in the in plane displacements that decay exponentially with time as expected. The observed amplitude is almost half of the amplitude of in plane displacements on the in plane axis. On the other hand the vertical displacements on the out of plane axis follow a completely different evolution pattern in comparison to the ones on the in plane axis. In particular, the exponential decay lasts only for the 2.5 first seconds of motion whereas the contribution of other sources is obvious even in the closest to the foundation instrument. Furthermore the amplitude of vertical displacements on the out of plane axis is much lower than on the in plane axis. The aforementioned can be attributed to the fact that the moment at the base of the structure due to the applied horizontal force at the top of the structure is developed around the out of plane axis. The exponential decay of the out of plane displacements lasts for the 5 first seconds of motion whereas the motion continues to maintain almost the same amplitude due to the contribution of the motion of nearby structures.

Frequency Domain

The study of the recordings in the frequency domain is achieved by the application of Fast Fourier Transform. In Figure 4 the amplitude of the Fast Fourier Transform of the three components of free field displacements on in plane axis is depicted. The most characteristic shape of FFT is the one of the in plane component. In particular, only one frequency dominates the motion, which is equal to 3.3Hz. Therefore, the aforementioned value is equal to the natural frequency of the soil-pier system for a horizontal excitation of the superstructure on the in plane axis or a combination of an applied moment and horizontal force on the foundation level.

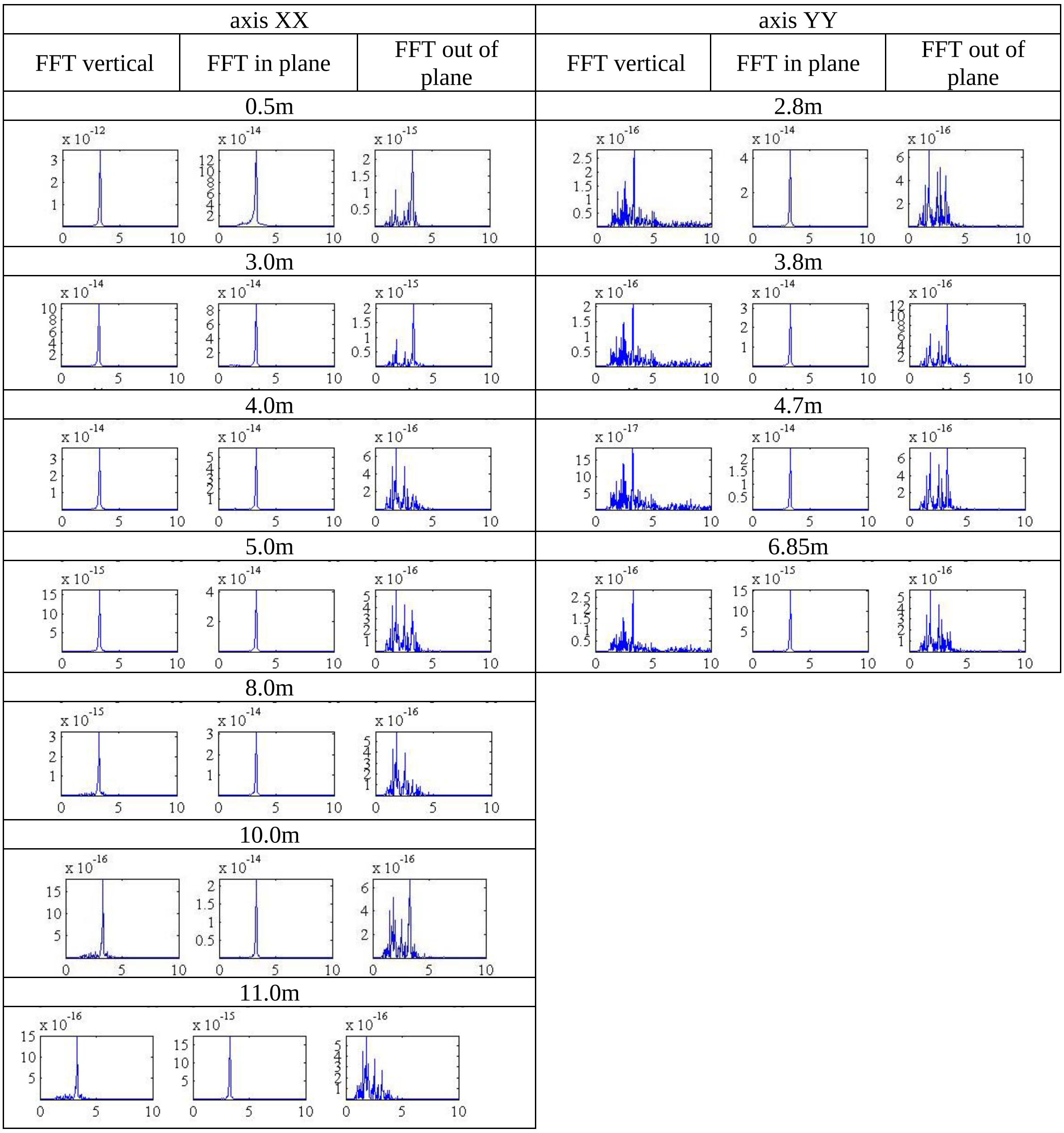


Figure 4. FFT of free field displacements on axis XX and YY.

The FFT of the vertical component reveals also the fundamental frequency of the soil-pier system whereas the FFT of the out of plane component follows a more complicated pattern. The frequency of 3.3Hz is obvious but the contribution of additional harmonics is in certain cases stronger. In particular, even in the instrument closest to the foundation the contribution of a harmonic of 1.833Hz frequency is observed. According to additional pull out tests for a horizontal forced applied at the deck of the pier on the out of plane axis, the aforementioned frequency is attributed to the translational motion of the soil-pier system on the out of plane axis. Therefore, the existence of the out of plane displacement is not only attributed to the three-dimensional nature of the physical phenomenon but to a possible eccentricity on the application of the force as well.

In Figure 4 the amplitude of the Fast Fourier Transform of the three components of free field displacements on the out of plane axis is also depicted. The clearest shape of the FFT amplitude shape is the one that corresponds to in plane displacements. In particular, it reveals that the motion in the corresponding component is dominated by the free vibration of the soil-pier system and is controlled by the frequency of 3.3Hz. In contrast to the FFT amplitude of the vertical displacement on the in plane axis, the one corresponding on the out of plane axis is completely different. Although the natural frequency of the soil-pier system is also evident, the total motion includes additional harmonics such as the one of 1.833Hz. The visibility of the range of frequencies contributing to the total time history can be attributed not only to the small amplitude of the vertical displacement but to the existence of the linear nearby concrete railways which divert the soil medium from the absolute free field conditions. Similar to the pattern of the FFT amplitude of vertical displacements is the one of the out of plane. Apart from the two stronger frequencies (1.833Hz, 3.3Hz) additional harmonics influence the motion indicating the complexity of the physical phenomenon.

Damping estimation

The estimation of the damping coefficient cannot be estimated analytically due to the complexity of the phenomenon. Therefore, the experimental process is considered to be a necessity. According to Chopra K.A., 1995, the damping coefficient can be estimated by the exponential line that encloses the free vibration time history. According to Figures 3 and 4 the only components of motion that are indicative for the calculation are the ones that follow the exponential decay pattern. In particular, the vertical and in plane component on the in plane axis and the in plane component on the out of plane axis. The following equation is used for the calculations.

$$\zeta = \frac{1}{2 \cdot \pi \cdot j} \cdot In\left(\frac{u_i}{u_{i+j}}\right) \quad (1)$$

where u_i is the amplitude of displacement and j the number of cycles between the peak values used for the calculation. In Table 1 the values of damping coefficient for each instrument are provided. The characteristic observation is that the damping coefficient slightly differs between different positions. Furthermore, the mean value of the damping coefficient for the vertical component on the in plane axis equals to 1.4945E-2 and is larger than the value corresponding to in plane component which equals to 1.0803E-2. The aforementioned can be attributed to the fact the duration of the motion in the in plane direction is larger than in the vertical direction. Furthermore, the mean value of damping coefficient in the in plane direction on the out of plane axis equals to 1.1745E-2, which is slightly larger than the corresponding in the in plane axis.

Table 1. Damping coefficient.

X [m]	vertical	out of plane	Y [m]	out of plane
0.5	1.1090E-2	1.1728E-2	2.8	1.1360^{E}-2
3	1.3860E-2	1.0880E-2	3.8	1.2124E-2
4	1.4835E-2	1.0955E-2	4.7	1.1783E-2
5	1.4214E-2	1.1856E-2	6.85	1.1712E-2
8	1.5988E-2	1.0350E-2		
10	1.6673E-2	1.0009E-2		
11	1.7955E-2	9.8410E-3		

Orbits of motion

Another interesting study refers to the orbits of the motion of the free field. In Figure 7, the orbits of the instrumented free field surface points on the in plane axis are depicted. The horizontal axis corresponds to in plane displacement while the vertical to vertical. The shape of the orbits reveals the existence of Rayleigh surface waves. However the main axis of the elliptic orbit is not the vertical one indicated by the theory but is inclined at an angle of almost 45 degrees. A similar observation concerning the difference between theoretical and experimental results was marked by Barkan D.D., 1962 and Mucciarelli M., et al., 2003. The significant deviations between the experimental and theoretical shape at distances close to the vibrating foundation can be attributed to the contribution of additional waveforms such as shear and compressional waves. The last figure depicts the orbits of all the instrumented points in the same scale. It is obvious that the inclination of the main elliptical axis reduces according to the distance from the surface source, the foundation. The aforementioned is attributed to the fact the amplitudes of the in plane displacement are larger than the vertical. A similar observation is also stated by Barkan D.D., 1962 according to whom the phase angle between the horizontal and vertical component is not maintained stable but varies according to the distance from the source.

The similarity between the experimental and theoretical results is reinforced by the direction of the orbit. In particular, the free field soil points move in a direction, which is opposite to the one that follow the pointers of the clock.

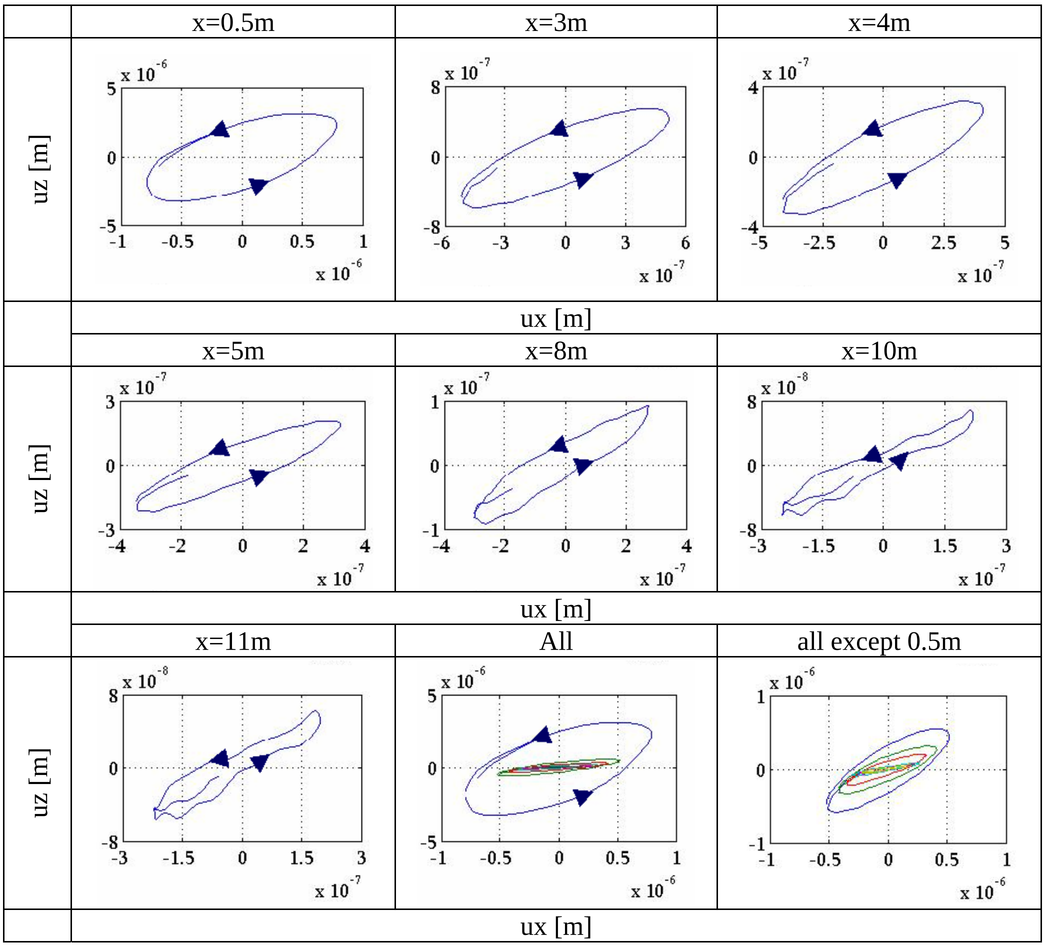


Figure 7. Orbits of motion of free field on axis XX.

Spatial decay of motion

Apart from the hysteretic damping another very important type of amplitude decay is the spatial attenuation or radiation damping especially in the case of an infinite medium such as the soil field. The initial wave energy during the propagation is introduced to a larger volume of medium and therefore the amplitude of the vibration reduces as the distance from the source increases. In Figure 8 the decay of the maximum amplitude of displacement considering the contribution of the three components of motion is depicted having the circle sign. It has to be noted that due to the phase difference not all the components receive the maximum amplitude value simultaneous. However, in this case this detail is ignored. The spatial decay of the amplitude of the surface waves is calculated by the use of equation (2), considering the closest to the foundation instrument as the source due to lack of data of the foundation and is depicted with the rhomb sign. Comparing the two aforementioned curves it is obvious that the experimental spatial decay is more profound than the one according to equation (2).

$$\frac{A_r}{A_o} = \sqrt{\frac{r_o}{r}} \quad (2)$$

Taking into account the experimental research of Barkan D.D., 1962 an additional spatial decay curve is estimated according to equation (3) and is depicted with the rectangular sign. The aforementioned curve includes the coefficient of energy absorption α which is related to the deviation of the real soil conditions from the totally elastic soil medium. For the value of energy absorption coefficient which equals to 0.2 it is observed that the calculated curve is the same with the experimental one at least for the four instrumented free field soil points. At distances larger than twice the foundation's dimension the calculation overestimates the spatial decay. However it can be observed that the inclination of the experimental curve for the last three points is almost the same with the one of the surface waves spatial decay curve.

$$\frac{A_r}{A_o} = \sqrt{\frac{r_o}{r}} \cdot e^{-\alpha \cdot (r - r_o)} \quad (3)$$

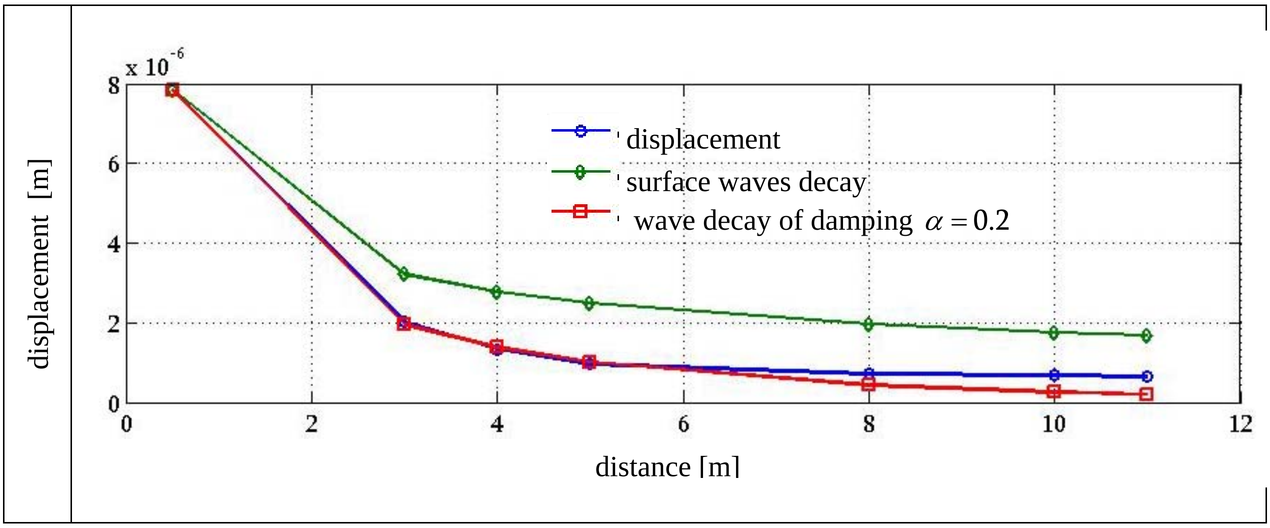


Figure 8. Spatial decay of free field motion on axis XX.

FINITE ELEMENT ANALYSIS

Description of experiment and finite element model

The compatibility between the experimental and the numerical results enables the thorough study of soil-structure interaction effects since it offers the ability to study also the dynamic behavior of the structure where no experimental data was available. Therefore the Finite Element Method (ADINA code) was used. In Figure 9 the free field soil instrumental set up is depicted. The instruments are placed at the indicated distances whereas the pull out test took place at the axis XX from the direction of the auxiliary storage house.

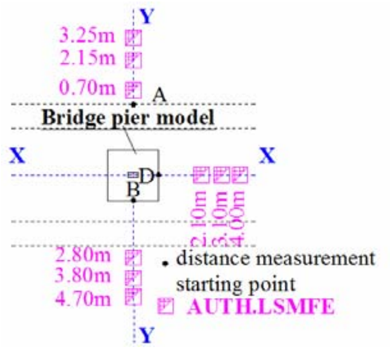
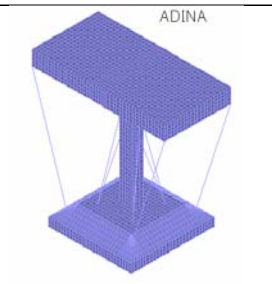


Figure 9. Free field instrumentation, 21/10/2004.

The 3D finite element model is depicted in Figure 10. It includes the bridge pier model, the railways that are used for the movement of the crane, the whole first soil layer and a part of the second soil layer as well. Significant modeling matters such as the proper dimension of the finite elements which is vital for wave propagation, the lateral boundary conditions, the position of the fixed vertical boundary, the Rayleigh damping which refers to the hysteretic damping of the soil layers and of the structure as well, the proper way of loading the structure, the proper time step which is adequate for the time integration have been taken into account. The most characteristic mechanical properties of the soil layers and structural material are also included in Figure 10.



Material	E [KPa]	v	ρ(t/m³)
Reinforced Concrete	32000000	0.3	2.5
1st soil layer [0-4]m	99630	0.33	2.05
2nd soil layer [4-20]m	290250	0.33	2.15

Figure 10. Finite element model and material properties.

Comparison between finite element and experimental results

In Figure 11 the comparison between the experimental and numerical analysis results regarding axis XX are depicted. The word REC states the recordings while the word FEM the results from the Finite Element Method. No recordings were available for the out of plane component from the instrument that was placed at a distance equal to 2.10m from the foundation. In general terms the coincidence between the experimental and numerical time histories regarding the vertical component and the horizontal component parallel to the direction of the applied force is more than satisfying. Furthermore, there are some differences observed regarding the out of plane component. However, the amplitude of the aforementioned displacement can be considered quite satisfying also taking into account the complexity of the three dimensionality of the phenomenon and the contributions of nearby structures.

In Figure 12 the comparison between the experimental and numerical results on axis YY at distances 2.8m, 3.8m and 4.7m is depicted. The agreement regarding the horizontal component at the direction of the applied force is satisfying enough. However, the comparison between the vertical and out of plane is poor. In particular the numerical results lack in amplitude but not in frequencies. The difference can be attributed to the large distance of the instrumented points from the surface source which is the foundation and in the contribution of the role of the nearby railways which act probably as an obstacle to the free filed wave propagation.

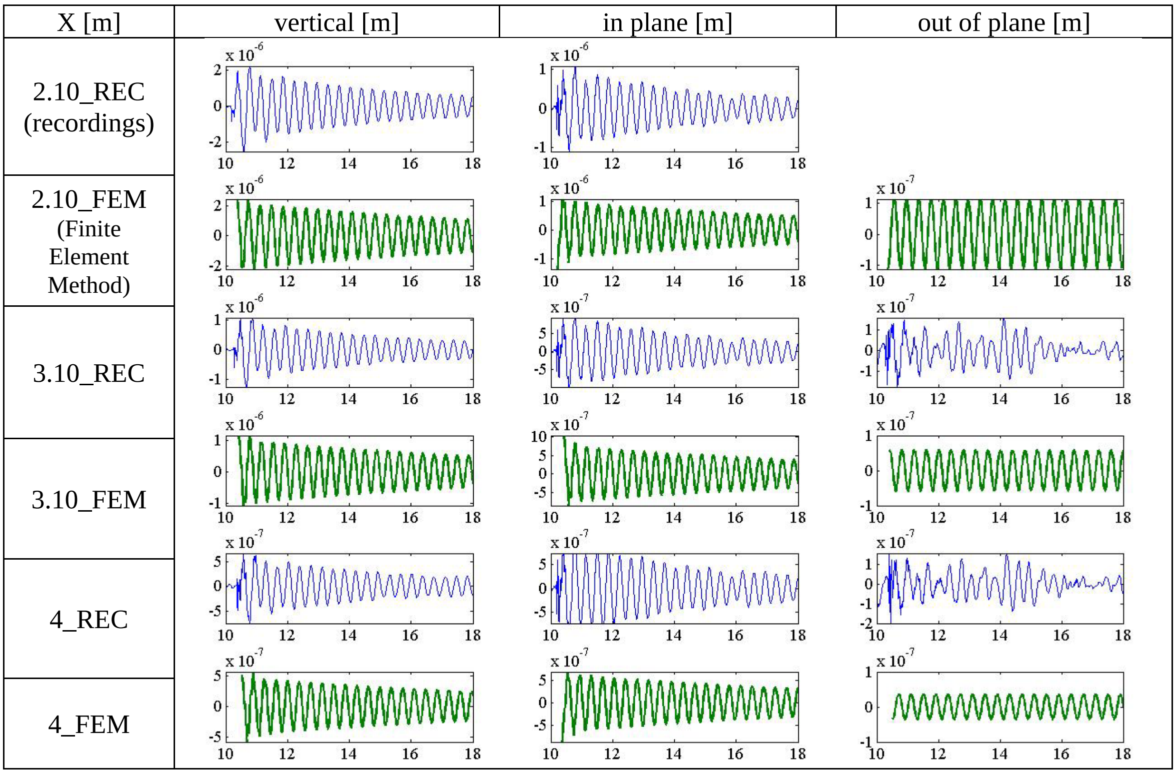


Figure 11. Comparison between recordings and FEM analysis results on axis XX.

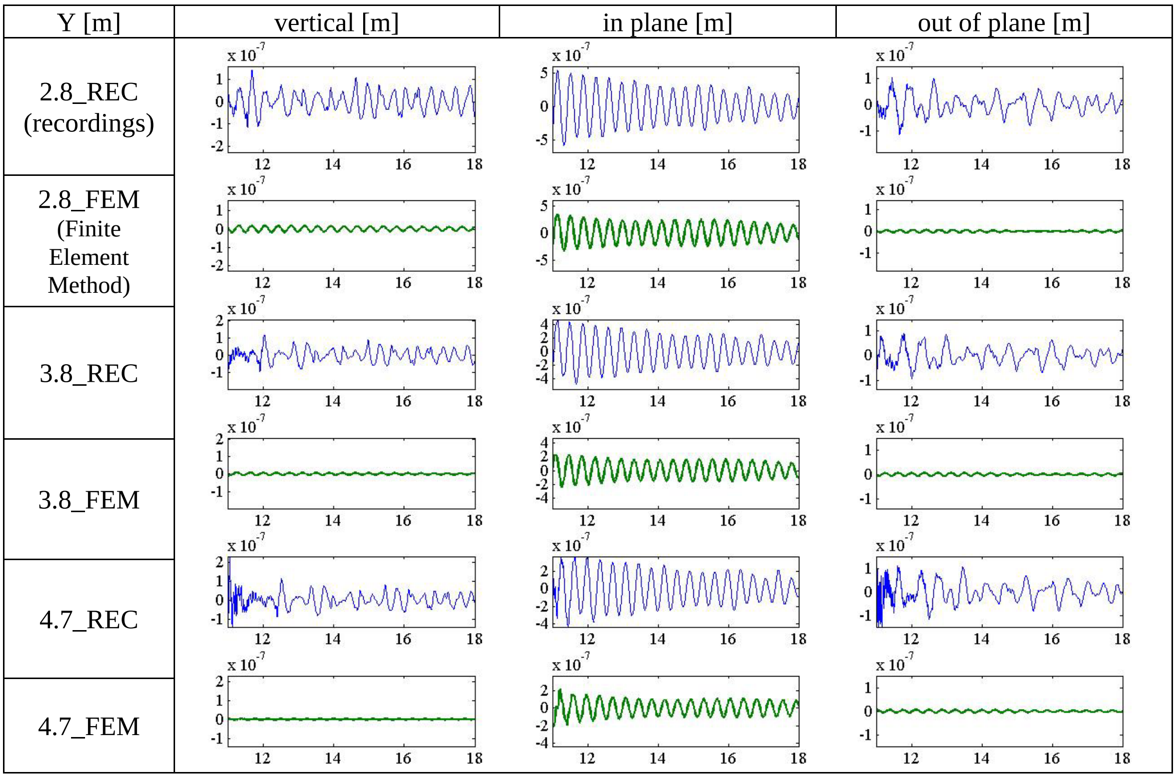


Figure 12. Comparison between recordings and FEM analysis results on axis YY.

However, comparison between the experimental and numerical results on axis YY at distances 0.7m, 2.15m and 3.25m is much better at the opposite site of the axis. Once again at the horizontal component, which is parallel in the direction of the applied force, the finite element model seems to work in the most effective way. Even the agreement between the vertical time histories is quite promising except the difference at the initial time steps. Regarding the out of plane component the difference at the amplitude is still remaining.

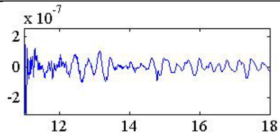
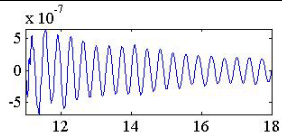
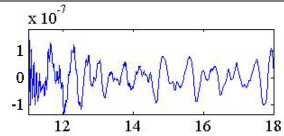
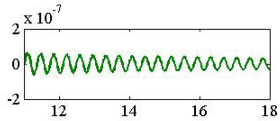
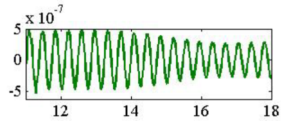
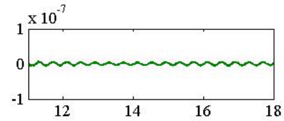
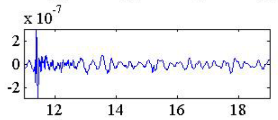
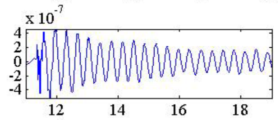
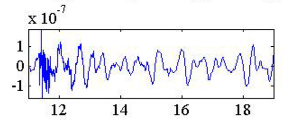
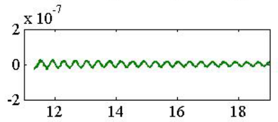
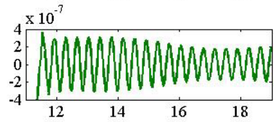
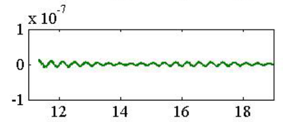
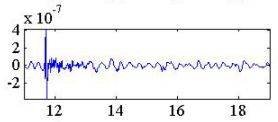
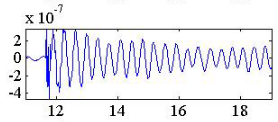
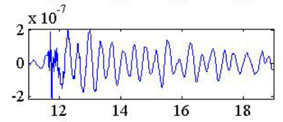
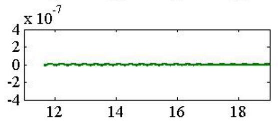
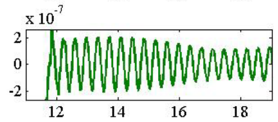
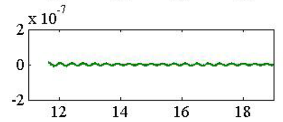
Y [m]	vertical [m]	in plane [m]	out of plane [m]
0.7_REC (recordings)			
0.7_FEM (Finite Element Method)			
2.15_REC			
2.15_FEM			
3.25_REC			
3.25_FEM			

Figure 13. Comparison between recordings and FEM analysis results on axis YY.

In general terms it can be concluded that in small distances near the surface source and in particular at distances which are smaller than twice the foundation's dimension the finite element model produces quite satisfying results regarding the amplitude and the frequency content of the time histories as well. Therefore, this finite element model can be used for a most thorough soil-structure interaction analysis.

Soil-structure interaction study based on numerical analysis

Soil-structure interaction effects

One of the first terms that should be checked in a soil-structure interaction analysis is the effect of the soil on the dynamic characteristics of the structure. Therefore, the most important dynamic characteristic of the structure such as the natural frequency is studied. Two different finite element models were used for this scope. The first one contains only the bridge pier model having all the degrees of freedom at its base fixed. The second model is the one used for the transient analysis. In Figure 14, it can be observed that the contribution of the soil layers underneath the structure reduces the natural frequencies of the bridge pier model at a percentage range of 10%-15%. Therefore, it can be assumed that the effects of soil-structure interaction are important. The aforementioned was one of the first goals of the project considering the stiffness difference between the bridge pier model and the soft surface soil layers.

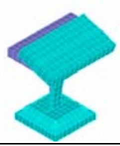
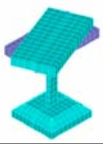
1st	2nd	3rd	natural frequency [Hz]	fixed	soil	Δf
			1st	1.775	1.569	-11.61%
			2nd	2.804	2.522	-10.06%
			3rd	3.200	2.709	-15.34%

Figure 14. Natural frequency modification due to soil-structure interaction.

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