

## A SIMPLIFIED NON LINEAR MODEL TO ANALYZE SITE EFFECTS IN ALLUVIAL DEPOSITS

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### ABSTRACT

In this work, we consider a non linear viscoelastic constitutive model: it involves both non linear elasticity as well as non linear viscous behaviour. The main objective of this model is to reproduce the  $G(\gamma)$  and  $D(\gamma)$  curves synthesizing the dependence of both the shear modulus and damping on the excitation level. To do so, the non linear elastic part of the model is described by a hyperbolic law. The non linear viscous part combines a NCQ for linear damping (generalized Maxwell body, Moczo et Kristek, 2005) and a non linear viscous contribution described by a hyperbolic variation with the strain level. Furthermore, this model complies with the thermodynamic principles of continuum mechanics (e.g. derivation from potentials and dissipation function).

Starting from this mechanical formulation, the analysis of 1D strong motion amplification is then performed thanks to a discretization by the finite element method. Various numerical issues have been carefully considered to describe the whole algorithmic procedure (for both elastic and viscous non linear components of the constitutive law). Detailed validations of the model have shown its ability to recover low amplitude ground motion response. For larger excitation levels, since the model includes the main features of soil non linear behaviour, the analysis of seismic wave propagation in soil layers leads to interesting results: at the free-surface the amplification peaks are shifted to lower frequency values (when compared to the input motion); higher frequency components are not overdamped as for the equivalent linear model; the amplification level is generally lower.

Keywords: site effects, amplification, wave propagation, non linear behaviour, viscoelasticity

### INTRODUCTION

The analysis of seismic wave propagation in alluvial basins is a difficult task since various phenomena are involved at different scales: resonance at the scale of the whole basin (Semblat & al., 2003), surface waves generation at the basin edges (Delépine & Semblat, 2005), soil non linear behaviour at the geotechnical scale (Bonilla et al., 2006), strong motion... It is not easy to handle all these different features of seismic wave propagation at the same time although the interaction between, for instance, surface wave generation and shear modulus decrease could be strong. The impact on the amplification process could then be very large and the classical interpretation of the phenomena may be distorted.

Non linear soil behaviour is very important in case of strong ground motion since the mechanical features of many soils depend on the excitation level, the loading history... Various approaches are possible to model this dependence: equivalent linear model, non linear cyclic constitutive laws, elastic-plastic laws.

The first model suggested to model the non-linearity is the equivalent linear model (Schnabel et al., 1972). This approach attempted to simplify the non-linear problem by solving equations in the

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frequency domain. This model is the most used because of its simplicity (e.g. SHAKE software). Recent researches have tried to attenuate the main drawback of this model, that is the over-damping of higher frequencies (cf. Figure 9). They have introduced frequency and pressure dependences of sediment layer parameters (Sugito & al., 1996, Kausel & Assimaki, 2002). This extension may induce some convergence problems in the algorithm (Bonilla, pers. com.).

Concerning the nonlinear models, another family of models considers both the hyperbolic law and the Masing criteria (Masing, 1926). Different approaches are based on these two rules, including variation and extension of the method. For example, Matasovic (1993) and Matasovic and Vucetic (1995) suggest a model with a modified hyperbolic law. Other models include dependence of confining pressure (Hashash & Park, 2001 & Park & Hashash, 2004) and pore pressure (Bonilla et al., 2005).

To fill the gap between these 2 family of models (equivalent linear and nonlinear), we propose herein a model of intermediate complexity: it involves both non linear elasticity and non linear viscosity. This viscoelastic nonlinear model is designed to approximate the nonlinear hysteretic behaviour of surficial soils, with only one nonlinear input parameter.

## MECHANICAL FORMULATION

### Features of the non linear behaviour considered

The non linear behaviour of the soil is supposed to be characterized by the following properties of elasticity and viscosity:

A/ **Elastic part**: We consider that the shear modulus depends on the shear strain and decreases following the hyperbolic law :

$$G(\gamma) = \frac{G_0}{1 + \alpha|\gamma|} \quad (1)$$

Such a law is presented in the papers of Hardin & Drnevich (1972).

B/ **Viscous part**: the soil damping depends also on the strain, with the following law :

$$\beta(\gamma) = \beta_0 + (\beta_{\max} - \beta_0) \cdot \left(1 - \frac{G(\gamma)}{G_0}\right) \quad (2)$$

with  $\beta_0 = \frac{1}{2 \cdot Q_0}$ ,  $Q_0$ , the quality factor for low strain values &  $\beta_{\max}$  the damping at the maximum

strain values. This equation has already been suggested by different authors (Heitz, 1989). It is an extension of the equation proposed by Hardin & Drnevich (1972).

C/ **Principle of causality**: in this viscoelastic model, we use a quality factor  $Q$  depending of the frequency. We consider the generalized Maxwell body (Semblat, 1997) and follow the implementation suggested by Emmerich et Korn (1987).

The main hypothesis is that the relaxation function only consists in  $n$  single peaks of amplitudes  $l_j$  at discrete relaxation frequencies  $\omega_j$ . We are looking for a nearly constant quality factor for a chosen frequency range (Near Constant Q model).

In the linear viscoelastic case, the stress-strain relationship for 1D wave shear wave propagation is written as follows:

$$\tau(t) = G_0(\gamma - \sum_{j=1}^n \zeta_j(t)) \quad (3)$$

$\zeta_j$  are the coefficients of the relaxation function, solution of the following first order equation

$$(Emmerich \& Korn, 1987): \dot{\zeta}_j(t) + \omega_j \cdot \zeta_j(t) = \omega_j \frac{y_{j,0}}{1 + \sum_{j=1}^n y_{j,0}} \gamma(t)$$

where ‘j’ corresponds to the Maxwell cell considered. We have chosen ‘n’ Maxwell cells, to obtain a quasi-constant  $Q$  value for a chosen frequency range. The  $y_{j,0}$  values are functions of the unrelaxed and relaxed modulus of each Maxwell cell. So we also have to solve ‘n’ differential equations.

To obtain a nonlinear viscosity (B), we introduce the function  $y_j(\gamma)$  depending on strain (Equation

$$(2)) \text{ as follows: } y_j(\gamma) = \frac{\beta(\gamma)}{\beta_0} y_{j,0}.$$

Then the complete stress-strain equation (3) becomes:

$$\tau(t) = G(\gamma) \cdot (\gamma - \sum_{j=1}^n \zeta_j(t, y_j(t, \gamma))) \quad (4)$$

The main advantages of the nonlinear viscoelastic behaviour constituted by equations (1) to (4) can be derived from an elastic potential and a dissipation function and can be easily extended to 2D or 3D wave propagation

### Equation to solve

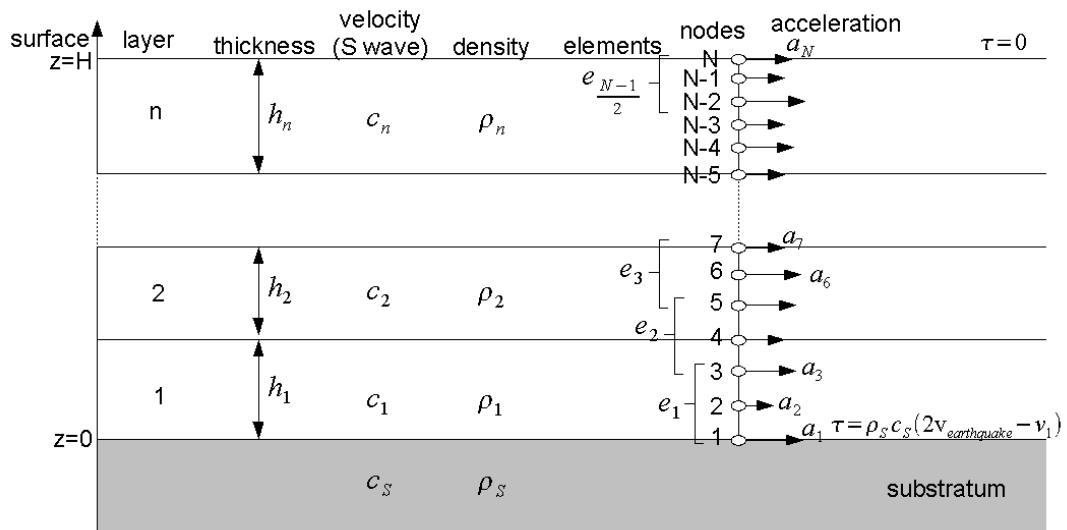
Our aim is to solve the following equation:

$$\frac{\partial}{\partial z} \tau - \rho \frac{\partial^2 u}{\partial t^2} = 0 \quad (5)$$

with  $\tau$ , the horizontal stress.

## NUMERICAL FORMULATION

We implemented the model in the framework of the finite element method. We considered a 1D model, composed with quadratic finite elements under vertical SH waves. As shown in Figure 1, we have 1 degree of freedom per node. In the following, matrices are denoted with square brackets ‘[.]’ and vectors with braces ‘{...}’.



**Figure 1. 1D model for a layered soil deposit**

The equation of motion of our model, with the nonlinear viscosity and elasticity parts, is written as follows:

$$[M] \cdot \{a_n\} + [C_f] \cdot \{v_n\} + \{P_e(u_n)\} - \{P_v(u_n)\} = \{F_n\} \quad (6)$$

with  $M$ ,  $P_e$  &  $P_v$  the mass, stiffness and viscous damping matrices of the system.

The matrix  $C_f$  corresponds to the radiation condition at the substratum/sediments interface.

We also have the differential equation system corresponding to the linear viscosity:

$$\dot{\zeta}_j + \omega_j \cdot \zeta_j = \omega_j \frac{y_j(\gamma)}{1 + \sum_{j=1}^n y_j(\gamma)} \gamma \quad (7)$$

### Algorithm considered

#### -IMPLICIT CASE

##### Time integration method

For the time domain integration, we choose the method called,  $\alpha$ -HHT-method (Hughes, 1987). The method is an extension of the Newmark formulation, considering an attenuation of the high frequencies. The method is based on the two following equations. For the velocity and displacement, we have respectively:

$$v_{n+1} = v_n + \Delta t \cdot [(1 - \beta_1)a_n + \beta_1 a_{n+1}] \quad (8)$$

$$u_{n+1} = u_n + \Delta t \cdot v_n + \frac{\Delta t^2}{2} \cdot [(1 - \beta_2)a_n + \beta_2 a_{n+1}] \quad (9)$$

with  $\beta_1 = \frac{(1 - 2\alpha_{HHT})}{2}$  and  $\beta_2 = \frac{(1 - \alpha_{HHT})^2}{2}$  are parameters governing the stability and numerical dissipation of the algorithm. According to Hughes (1987), for  $\alpha_{HHT} = [-1/3; 0]$ , the method is unconditionally stable.

We consider that, at time  $n \cdot dt$ , we know the expressions of the displacement, velocity and acceleration at all nodes. The aim is to find at time  $(n+1) \cdot dt$  the acceleration  $a_{n+1}$ . The equation of motion in our case becomes with the previous equations for  $u$  and  $v$ :

$$([M] + (1 + \alpha_{HHT})\beta_1 \Delta t [C_f]) \cdot \{a_{n+1}\} + (1 + \alpha_{HHT})(\{P_e(u_{n+1})\} - \{P_v(u_{n+1})\}) - \alpha_{HHT}(\{P_e(u_n)\} - \{P_v(u_n)\}) + [C_f] \cdot (\{v_n\} + (1 + \alpha_{HHT})\Delta t(1 - \beta_1) \cdot \{a_n\}) = \{F_{n+\alpha_{HHT}}\} \quad (10)$$

To find the acceleration value  $a_{n+1}$ , we combine the Newmark method with the Newton-Raphson algorithm explained in the following.

To estimate the  $\zeta_{n+1}$  coefficients of the relaxation function, solution of the first order differential equation, we use Crank-Nicolson procedure (Zienkiewicz, 2005):

$$\dot{\zeta}_{n+1} = \left(1 + \frac{\Delta t}{2} \omega_j\right)^{-1} \cdot \left( -\omega_j \cdot \left( \dot{\zeta}_n + \frac{\Delta t}{2} \zeta_n \right) + \omega_j \frac{y_j(\gamma)}{1 + \sum_{j=1}^n y_j(\gamma)} \gamma_n \right) \quad (11)$$

The solution  $\zeta_{n+1}$  comes by combining the equation just below in the Taylor series expansion:

$$\zeta_{n+1} = \zeta_n + \frac{\Delta t}{2} \cdot \dot{\zeta}_n + \frac{\Delta t}{2} \dot{\zeta}_{n+1}$$

##### Nonlinear integration

For nonlinear aspect, we use the Newton-Raphson's algorithm. We introduce the variable  $\Psi$ , defined as follows:

$$\{\psi(a_{n+1})\} = ([M] + (1 + \alpha_{HHT})\beta_1 \Delta t \cdot [C_f]) \cdot \{a_{n+1}\} + (1 + \alpha_{HHT})(\{P_e(\tilde{u}_{n+1})\} - \{P_v(\tilde{u}_{n+1})\}) \\ + [C_f] \cdot \{\dot{v}_n\} + (1 + \alpha_{HHT})\Delta t \cdot (1 - \beta_1) \cdot \{a_n\} - \{F_{n+1}\} = 0$$

$$\text{with } \tilde{u}_{n+1} = u_n + \Delta t \cdot v_n + \frac{\Delta t^2}{2} \cdot (1 - \beta_2) a_n$$

The aim of the method is to approximate the solution of the nonlinear equation with successive linear equations using Taylor series expansion. Considering the function  $\psi(a_{n+1}^i)$ , its partial derivative  $\frac{\partial \Psi(a_{n+1}^i)}{\partial a_{n+1}}$ , and an initial value  $a_{n+1}^0$ , we have an iterative process to approach a solution of the equation  $\{\psi(a_{n+1})\} = \{0\}$ :

$$\{a_{n+1}^{i+1}\} = \{a_{n+1}^i\} - \left[ \frac{\partial \Psi(a_{n+1}^i)}{\partial a_{n+1}} \right]^{-1} \cdot \{\Psi(a_{n+1}^i)\} \quad (12)$$

At step 'i', the Newton formulation is written as follows:

$$\{\delta a_{n+1}^i\} = \left( [M] + \beta_1 \cdot \Delta t \cdot [C_f] + \beta_2 \cdot \frac{\Delta t^2}{2} \cdot ([Kt_e(\tilde{u}_{n+1}^i)] - [Kt_v(\tilde{u}_{n+1}^i)]) \right)^{-1} \cdot \{\Psi(a_{n+1}^i)\} \quad (13)$$

#### -EXPLICIT CASE

We also use the Newmark method, with  $\beta_1 = 1/2$  and  $\beta_2 = 0$ . We immediately obtain (Equation 9):

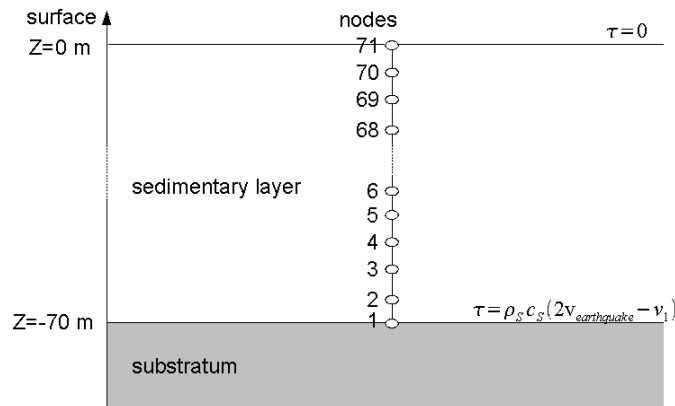
$$u_{n+1} = \tilde{u}_{n+1}.$$

$$\{a_{n+1}\} = ([M] + \beta_1 \cdot \Delta t \cdot [C_f])^{-1} \cdot \{F_{n+1} - (P_e(u_{n+1}) - P_v(u_{n+1}) + [C_f] \cdot \tilde{v}_{n+1})\}.$$

### VALIDATION OF THE NON LINEAR MODEL

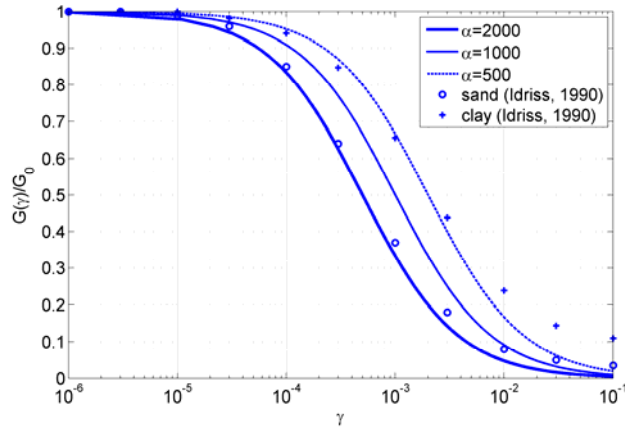
#### Model description

Considering a homogeneous non linear layer, the ground motion is computed with and without non linear viscosity. We chose a sedimentary layer of thickness 70m, with a non linear elastic behaviour (Figure 2). We choose a large thickness for the sedimentary layer to be able to separate the reflected wave from the incident one at the interface. In actual situations non-linear phenomena generally occur in the first 30 meters subsurface heterogeneous layers.



**Figure 2. Profile used to validate the model**

The soil layer is subjected to a 2<sup>nd</sup> order Ricker signal of amplitude 0.1g and frequency 3Hz (Figure 4). We use the hyperbolic law, to describe  $G(\gamma)$  function with a coefficient  $\alpha$  to balance the degree of non-linearity. Comparing to Idriss (1990), we choose  $\alpha = 1000$  (Figure 3). We use the implicit formulation in the following.



**Figure 3. Variations of functions  $G$  vs  $\gamma$  for different  $\alpha$  (500, 1000 & 2000) for clay & sand and comparison with experimental results from Idriss (1990) .**

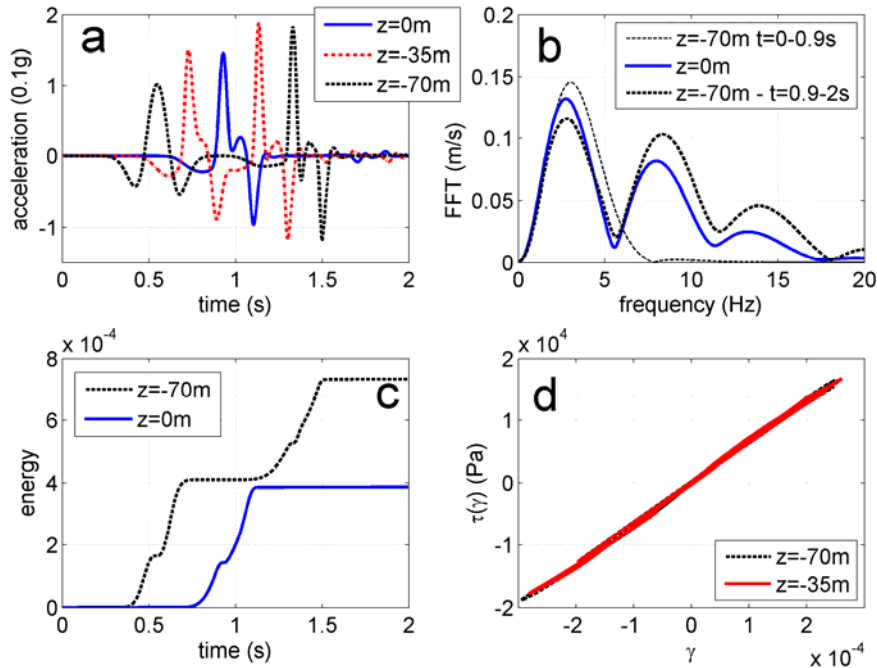
### Soil response for non linear undamped elasticity

Figure 4a: In the case of undamped non linear elasticity, the time domain results (Figure 4) at the bottom, the middle & the top of the layer (free surface) are displayed. We filtered the results between 0.05 and 10Hz. Output signal at the surface is divided by 2. Generally for the output signals, we obtain PGA values higher than the input one. The higher PGA values can be explained with the generation of the higher frequency peaks due to the non linear effects.

On Figure 4b spectra, some peaks are generated at high frequency, the fundamental frequency  $f_0$  being 3Hz. These peaks correspond to harmonics at  $3f_0$  and  $5f_0$ , but nothing at  $2f_0$ . We also see the effect of the filter on the high frequency components generated.

Figure 4c displays some energy curves during the propagation of the Ricker signal: the energy balance appears to be satisfied.

On the stress-strain curve in Figure 4d, we clearly observe the reduction of the shear modulus  $G_0$  induced by the hyperbolic constitutive law.



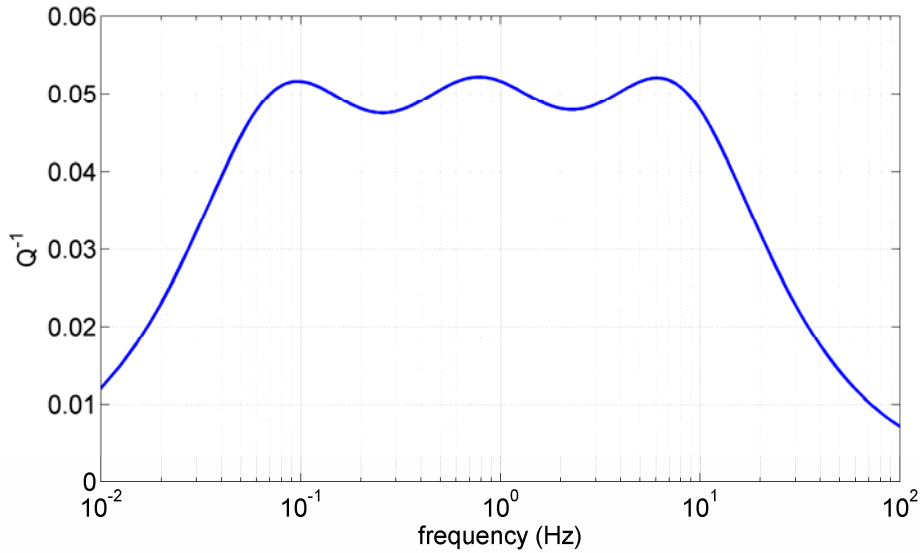
**Figure 4. (a): Time domain response (acceleration) to a Ricker signal ( $f_0=3\text{Hz}$ ) at the bottom, middle and top of a sedimentary layer (nonlinear elastic model  $\alpha = 1000$ ).**

**(b): Fourier transform of the signal. (c): Dissipated energy. (d): Stress-strain curves.**

The Fourier transform graph and stress strain curve of Figure 4, are in good agreement with the constitutive equation in the nonlinear elastic case (2<sup>nd</sup> order perturbation) studied by Van Den Abeele et al. (2000).

#### Layer response for non linear viscoelasticity

The analysis is then performed with a nonlinear viscoelastic model with a nonlinear viscosity. As depicted in Figure 5, we consider a nearly constant quality factor function for  $\gamma = 0$ . This quality factor is reduced when  $\gamma \neq 0$  (Equation 2).



**Figure 5. Quality factor (Q=20) as a nearly constant function of frequency**

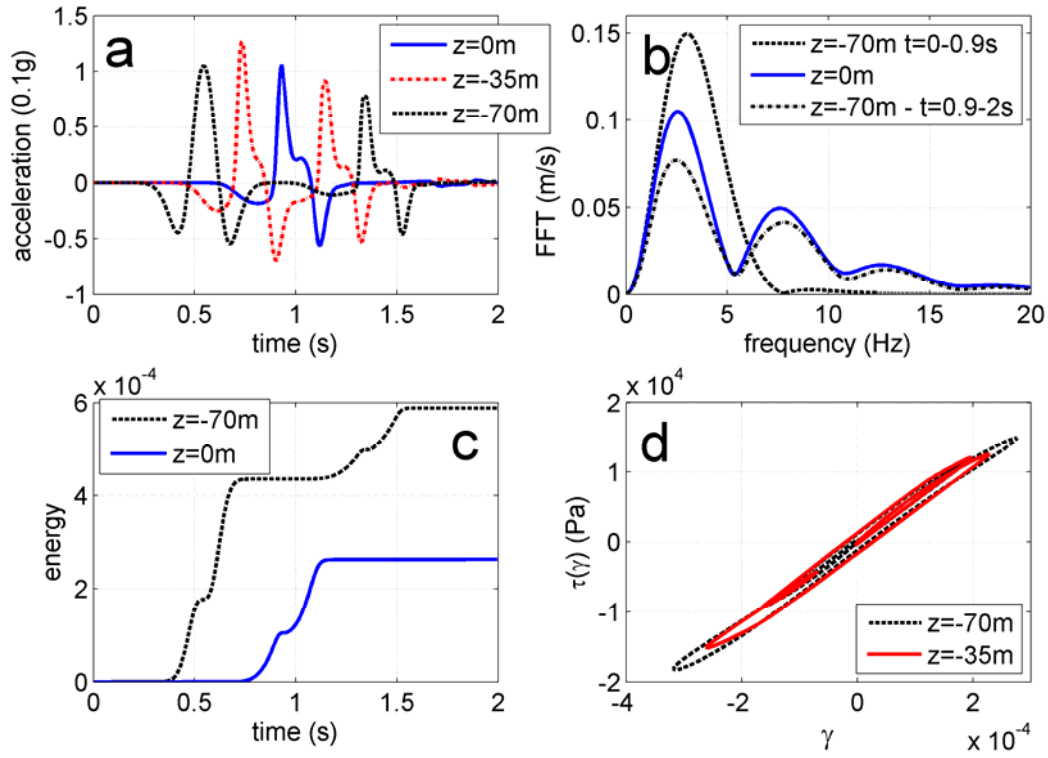
In Figure 6a, the spikes observed in the acceleration curves for the non linear elastic case are still present, but are decreased due to the viscosity. On the stress-strain graph (Figure 6d), the surface displayed corresponds to the dissipation in the medium.

As in the case of non linear elasticity, the Fourier spectra in Figure 6b show higher frequency components. Spectral harmonics due to the non linear behaviour are still found in the non linear viscoelastic response.

The influence of damping is also obvious on the energy curves in Figure 6c (dissipation during the propagation of the Ricker signal).

The stress-strain diagrams in Figure 6d show a reduction of the shear modulus of approximately 30%. Comparing to the non linear elastic case with the same input motion, larger strain values are observed with nonlinear viscosity.

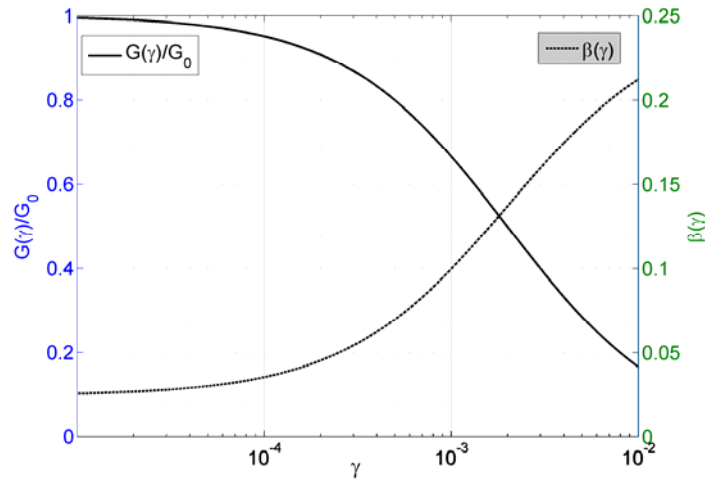
It should be mentioned that we obtain the same results with the implicit & the explicit formulations.



**Figure 6. a: Time domain response of the sedimentary layer (viscoelastic nonlinear model  $\alpha = 1000$  ,  $Q=20$  &  $\beta_{\max} = 0.25$  ) at the bottom-middle and top to a Ricker signal ( $f_0=3\text{Hz}$ ). b: Fourier transform of the signal. c: Dissipated energy. d: Stress-strain curves.**

### NON LINEAR RESPONSE FOR AN ACTUAL QUAKE

We complete the analysis with real acceleration data obtained during Northridge earthquake (1994), recorded at Topanga station. We consider the complete nonlinear viscoelastic model for a homogeneous soil layer, 30 m deep ( $V_s=200\text{m/s}$ ) overlaying an elastic bedrock ( $V_s=400\text{m/s}$ ). We use the  $G(\gamma)$  and  $D(\gamma)$  curves displayed in Figure 7 ( $\alpha=500$ ).



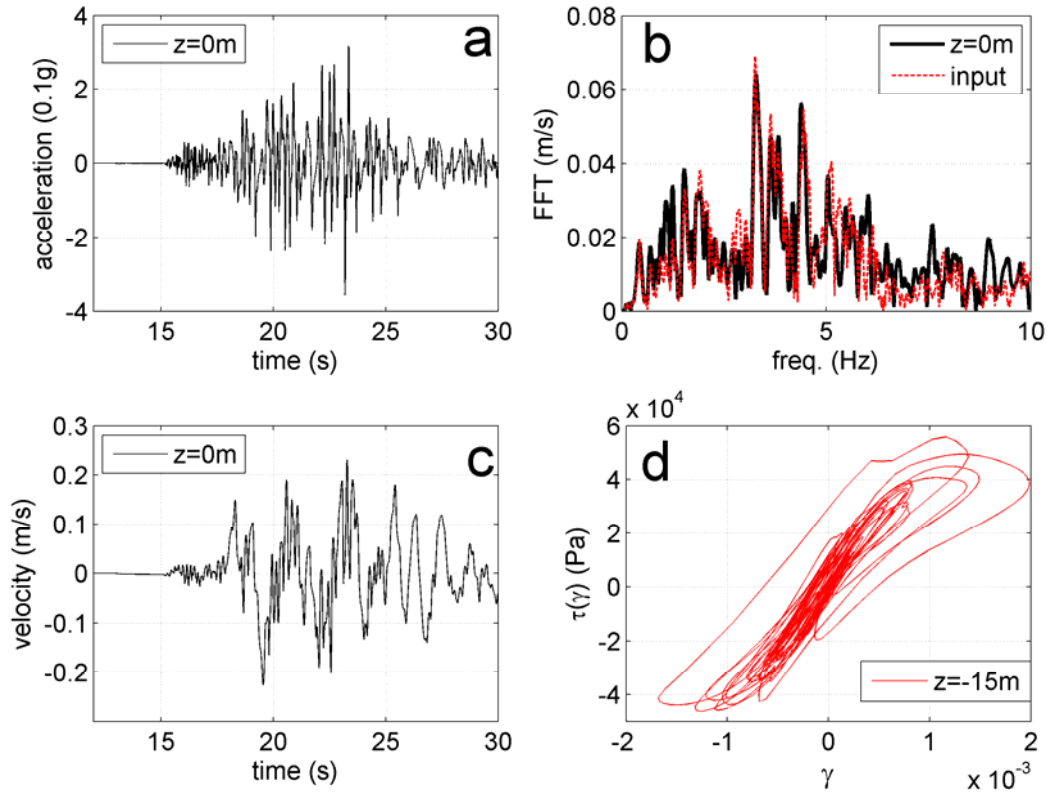
**Figure 7.  $G(\gamma)$  and  $D(\gamma)$  curves with:  $\alpha = 500$ ,  $\beta_0 = 0.0125$  &  $\beta_{\max} = 0.25$  )**

The results displayed in Figure 8 are filtered with a Butterworth band-pass filter (0.1-20Hz). In the time domain, we observe repetitive spikes, especially between 20-24s, the PGA is one of them. Such spike shapes have already been recorded on hysteretic behaviour of soil (Kushiro-Oki earthquake



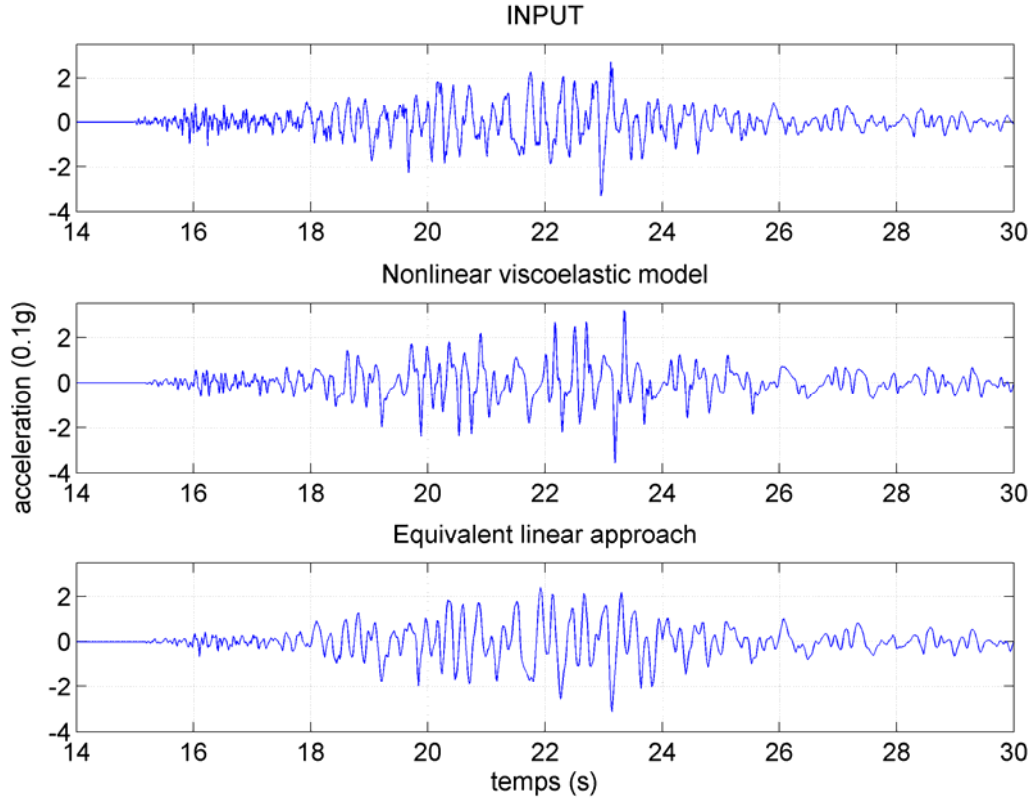
1993, Iai et al., 1995). On the stress-strain curve, we observe a large reduction of the shear modulus (2 times lower than in the linear viscoelastic case). On the Fourier spectra, high frequency peaks are generated at the top of the layer, and some frequency values are shifted to lower frequencies. Finally, the strain dependent damping is obvious on the stress-strain loops (Figure 8).

Considering the same parameters ( $G(\gamma)$  and  $D(\gamma)$  curves), but with an equivalent linear code (EduSHAKE), we did the same estimation. The acceleration response at the surface node is plotted in Figure 9. In this case, the linear equivalent model clearly overdamps the high frequency components when compared to the signal computed with our model.



**Figure 8. a: Time domain response (Accelerometer) of the sedimentary layer (nonlinear viscoelastic model  $\alpha = 500$ ,  $\beta_0 = 0.0125$  &  $\beta_{\max} = 0.25$ ) at the bottom to the Northridge earthquake (01/17/94-Topanga station).**

**b: Fourier transform of the signal. c: Velocity response. d: Stress-strain curves.**



**Figure 9. Top: input acceleration (Northridge earthquake). Middle: result at the free surface with the nonlinear viscoelastic model. Bottom: result at the same point with a equivalent linear approach (EduSHAKE)**

## CONCLUSIONS

In this paper, a nonlinear viscoelastic model is proposed to approximate the hysteretic behaviour of alluvial deposits. The hyperbolic constitutive law is considered to reproduce the nonlinear elastic behaviour. For the nonlinear viscosity, damping is chosen as a function of strain and meets the causality requirements.

With this model, we can take into consideration the generation of harmonics (3<sup>rd</sup> and 5<sup>th</sup>), as shown in the nonlinear elastic and nonlinear viscoelastic cases (Figure 4 & Figure 6).

In the general case of real seismograms, a lower deamplification of the high frequency components is observed with the nonlinear model when compared to the equivalent linear approach.

The interest of this simplified non linear model is to reduce the computational cost for the analysis of 2D site effects in alluvial basins. Furthermore, the characterization of the soil at a large scale is made easier since it is only necessary to determine the  $G(\gamma)$  and  $D(\gamma)$  curves. Considering a hyperbolic model, the  $G(\gamma)$  curve is controlled by only one parameter  $\alpha$  possibly quantified by cyclic laboratory tests. The dissipation properties are directly deduced from the  $G(\gamma)$  curve knowing 2 other parameters:  $\beta_0$  (linear viscosity) &  $\beta_{\max}$  (maximum viscosity term).

For site amplification at the geological scale, it will then be possible to combine basin effects and non linear soil response to investigate the interaction of the latter on the edge induced surface waves due to the former (Bard & Bouchon, 1985, Delépine & Semblat, 2006). In the future, it will be necessary to make the analysis at the seismological scale including source effects (EGF for instance) to get a complete strong ground motion analysis from the source to the soil. To do so, it will probably be necessary to couple various numerical approaches (e.g. FEM/BEM) since 3D analyses may be required (Dangla et al., 2005).

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