

CYCLIC MECHANICAL BEHAVIOUR OF GRANULAR SOILS: EXPERIMENTAL RESULTS AND CONSTITUTIVE MODELLING

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ABSTRACT

The paper illustrates cyclic torsional shear experimental test results obtained on dry dense Toyoura sand specimens. The dependency of the soil specimen mechanical response on loading amplitude is discussed. Both equivalent/tangent elastic shear stiffness and damping ratio are evaluated by using different definitions. Their evolution with the number of cycles performed at different loading amplitudes is analysed: the irreversibility of the material mechanical response is discussed.

Experimental data are then reproduced by means of a non associated multiple mechanism elasto-viscoplastic constitutive model characterised by two generalized plastic mechanisms: one viscoplastic and one inner kinematic plastic which is capable of capturing the material mechanical response even when small cyclic tests are performed.

Keywords: cyclic torsional shear tests, shear stiffness, sand, constitutive modelling, viscoplasticity

INTRODUCTION

The comprehension of the mechanical behaviour of soils under cyclic and/or dynamic loading can be considered one of the most stimulating subjects of soil mechanics, in particular, when large strains as well as a wide range of frequencies are taken into consideration and the problem of the coupling between volumetric and shear irreversible strains is investigated. Usually, this subject is studied by starting from two opposite points of view. The cyclic stress-strain behaviour is highlighted by means of sophisticated constitutive models capable of reproducing the volumetric-deviatoric coupling and the irreversibility of the constitutive relationship, and by disregarding at all the time factor. On the contrary, the dynamic mechanical response is tackled either by means of visco-elastic or hysteretic-elastic constitutive models that are linear and allow us to solve boundary value problems in the frequency domain. In such perspective, only two constitutive parameters, the so-called equivalent parameters, are introduced to describe the material mechanical response: the shear modulus G and the damping ratio D .

Several data concerning the dependency of soil stiffness (both shear modulus G and Young's modulus E) and/or damping ratio at small strains on many different variables like strain amplitude, confining pressure and loading frequency (see for instance Iwasaki et al 1978, Isenhower 1979, Bolton & Wilson 1989, Tatsuoka & Shibuya 1991, Kim et al. 1994, Fioravante et al. 1994, Porovic & Jardine 1994, Tatsuoka et al. 1994, Tatsuoka et al. 1995, d'Onofrio et al. 1995, Shibuya et al. 1995, Lin et al. 1996,

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Di Benedetto & Tatsuoka 1997, d'Onofrio et al. 1999b, Tatsuoka et al. 2001, Vucetic & Tabata 2003) were published in the last three decades.

In a previous paper (Zambelli et al. 2006), the same authors discussed the influence of loading frequency on the cyclic mechanical response of the material; on the contrary, in this one we focus our attention on the shape of cycles at varying strain levels. This analysis is aimed at conceiving appropriate constitutive relationships capable of reproducing the material mechanical behaviour, both when small and large irreversible strains occur. The importance of this problem is due to the current availability of non-linear numerical codes which allow us to reproduce the seismic response of large domains, to simulate even the occurrence of unstable phenomena like liquefaction of loose deposits and to analyse dynamic soil-structure interaction.

The comparison between experimental data and numerical simulations is then used to discuss the constitutive modelling hypotheses.

EXPERIMENTAL DEVICE AND PROCEDURES

A resonant column-torsional shear apparatus developed at the University of Naples was employed to obtain the experimental data hereafter reported. Readers can refer to d'Onofrio et al. (1999a) for the description of this apparatus and its performances.

The specimens are prepared by employing the air pluviation method: dry sand is filled in a funnel with a metallic tube attached to the end. The terminal cross section of the tube is 2mm wide and 15mm long. A membrane is stretched taut to the inside face of a split mould which is attached to the base pedestal of the test apparatus. The pluviation falling height is equal to about 55cm, the mould is filled by the dry sand at constant speed. The falling height is chosen to obtain about 80% of relative density. The top of the sample is levelled with a knife to have an even surface. After the specimen is encased in the membrane with the top cap, a vacuum of 20 kPa is applied to complete the assembling operations of the apparatus.

Notice that in the experimental device employed specimen cannot be saturated in the cell. In fact, since a fixed-free scheme is imposed during resonant column tests, the drainage is permitted only downward.

The experimental results were obtained with the above mentioned torsional device; a series of six dry Toyoura dense sand specimens (35 x 72 mm) were tested, after an isotropic consolidation up to a cell pressure of 200 kPa. This phase was performed by imposing a constant stress rate of 10 kPa/hr. In order to verify the reproducibility of data, during isotropic compression, resonant column RC tests at a small level of excitation were performed.

The second phase of the test is characterized by torsional shear cyclic tests. The loading frequency is fixed, while the torque amplitude is progressively increased. At each shear stress level, ten cycles are performed.

EXPERIMENTAL RESULTS

The non-linear and non-reversible pre-failure behaviour observed during cyclic shear tests is conventionally illustrated by putting in evidence the dependence of both equivalent shear stiffness G and damping ratio D on shear strain. Their definition is given in Figure 1: G represents the ratio between peak-to-peak shear stress and shear strain for a given cycle ($G = \tau_{pp} / \gamma_{pp}$); whereas G_{tan} is evaluated as the ratio between shear stress rate and strain rate in a determined point of the cycle ($G_{tan} = \dot{\tau} / \dot{\gamma} \cong \Delta\tau / \Delta\gamma$). This latter parameter has been numerically evaluated (Figure 1) by taking into consideration $\Delta\tau = 4$ kPa of shear stress. The comparison between the trends of G and G_{tan} versus γ_{pp} allowed us to put in evidence how the cycle shape evolves with both strain level and number of cycles performed.

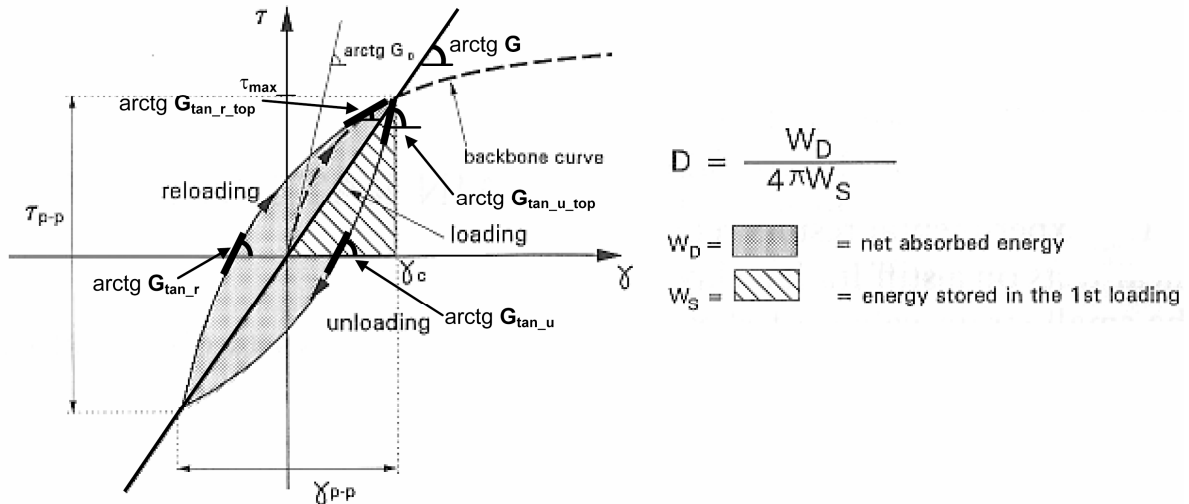


Figure 1. Definition of the equivalent parameters: equivalent shear stiffness G , secant shear stiffness G_{tan} and damping ratio D .

Figure 2 shows a summary of the experimental results obtained by testing the six dry Toyoura dense sand specimens at different loading levels. In this figure, as in all the following ones, γ_{SA} stands for the semi-amplitude/single-amplitude shear strain, i.e. $\gamma_{SA} = \gamma_{pp}/2$.

These results are very close to many others already published in literature regarding Toyoura sand in a dense state and illustrate the marked dependency both of shear modulus G and damping ratio D on γ_{SA} . In Figure 2 the two thresholds of the linear behaviour (γ_L) and of the stable cycles (γ_S), respectively, are put in evidence. As it will be commented in the following, when $\gamma_{SA} < \gamma_L$, G and D are approximately constant with both shear strain and number of cycles; they are respectively equal to the initial shear modulus G_0 and the initial damping ratio D_0 . When $\gamma_{SA} < \gamma_S$, both G and D do not depend on the number of cycles.

The evolution of the material mechanical response with the number of cycles, imposed for each shear stress amplitude, was considered to be important for capturing the occurrence of micro-structural rearrangements of the material internal fabric, i.e. the so-called ratcheting phenomenon.

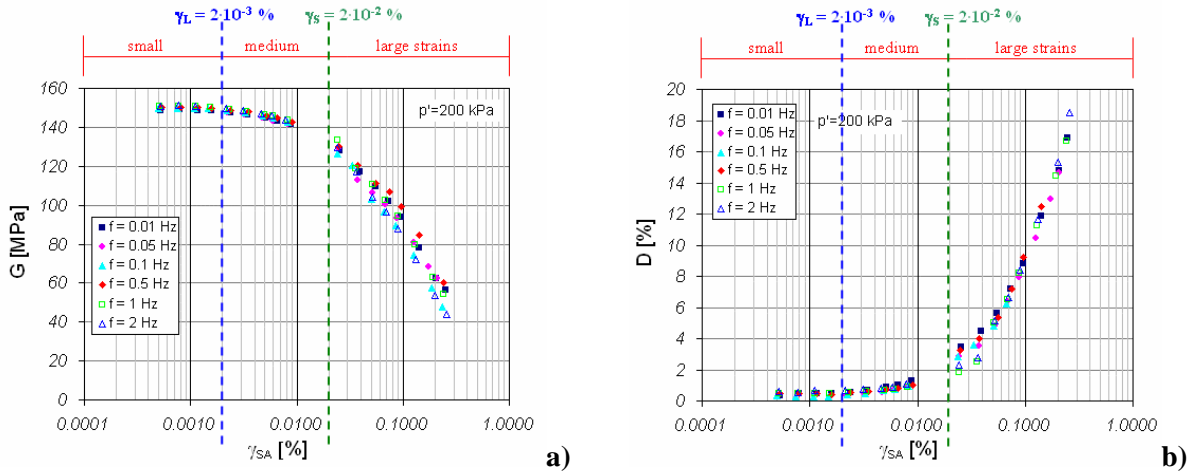


Figure 2. Shear modulus and damping ratio versus single amplitude shear strain, measured during cyclic torsional shear tests on dense dry Toyoura sand specimens.

At small and medium strain ranges, the shape of the cycles does not evolve whatever the number of loading cycles is applied (Figure 3a), and damping ratio is very small. On the contrary, when larger shear strain tests are performed (Figure 3b), the mechanical response (i.e. shape of cycles and

equivalent shear stiffness) of sand specimens changes progressively at increasing the number of cycles. In particular, the shear stiffness progressively increases, while the damping ratio decreases. At the same time the shape of the single cycle becomes more and more symmetric.

If we illustrated the overall mechanical response of the soil specimen, we obtained the curves of Figure 4. In Figure 4a the mechanical response of the soil specimen recording during the first cycles at increasing stress levels is plotted. On the contrary, in Figure 4b all the cycles are represented.

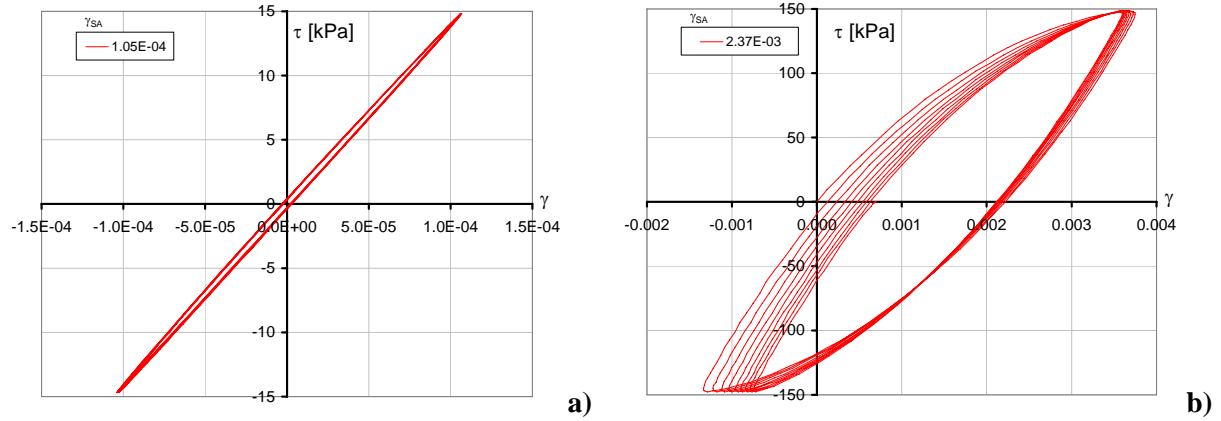


Figure 3. Cyclic torsional shear tests on a dense dry Toyoura sand specimen: stress-strain cycles at a) medium strains, b) large strains for a loading frequency 0.1 Hz.

The evolution of both shear modulus G and damping ratio D with shear strain correspondingly to the first and the ninth cycle is illustrated in Figure 5. We obtain four curves (two for G and two for D). When the previously defined threshold γ_s is overtaken, the experimental curves bifurcate. In particular, Figure 5a, b and c show test results performed at a frequency $f = 0.01$ Hz, $f = 0.1$ Hz and $f = 1$ Hz respectively. By comparing Figure 5a, b and c, we can derive that the position of the bifurcation point with respect to γ_{SA} is influenced severely by the frequency imposed. For the lower frequencies (i.e. at lower strain levels) this point shifts left, while for higher frequencies (i.e. at larger strain levels) shifts right.

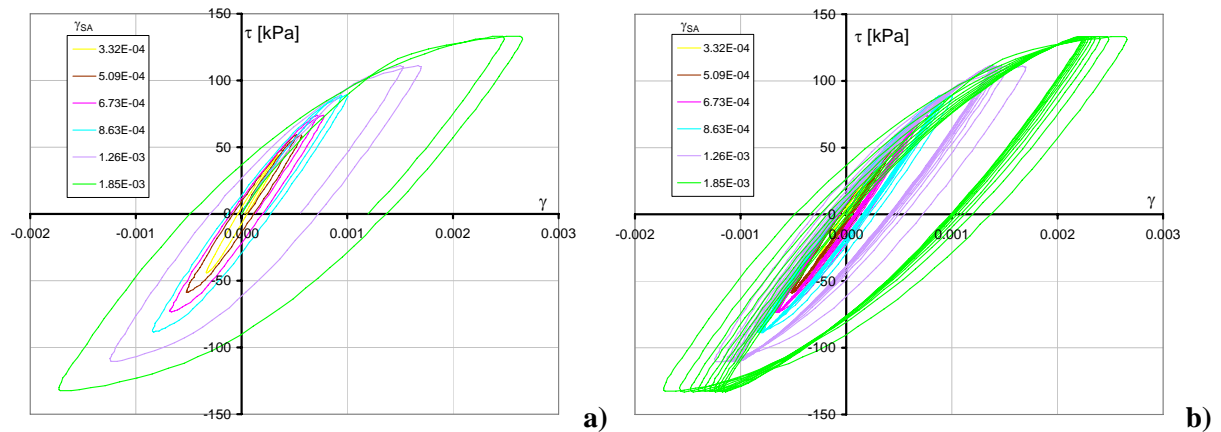


Figure 4. Cyclic torsional shear test at medium strains on a dense dry Toyoura sand specimen: stress-strain cycles a) 1° cycle, b) all nine cycles for a loading frequency 0.1Hz.

These experimental results are summarized in Figure 5d, where the dependency of damping ratio on the number of cycles is illustrated for the seven highest stress levels: by increasing γ_{SA} the number of cycles necessary for reaching a stable cyclic mechanical response increases.

According to the authors, when large strains occur, the shape of cycles should be discussed by taking into consideration the fact that the distribution of both shear stress and shear strain is not uniform within the cylindrical specimen. Within its nucleus the mechanical response of the material continues

to be elastic, whereas the external part begins progressively to behave plastically. On the contrary, it must be underlined that usually even when large strains are imposed, the soil specimen mechanical response is interpreted by assuming, within the specimen, a linear distribution of shear stresses. For this reason to interpret correctly these experimental data, the tests should be simulated by solving the corresponding boundary value problem.

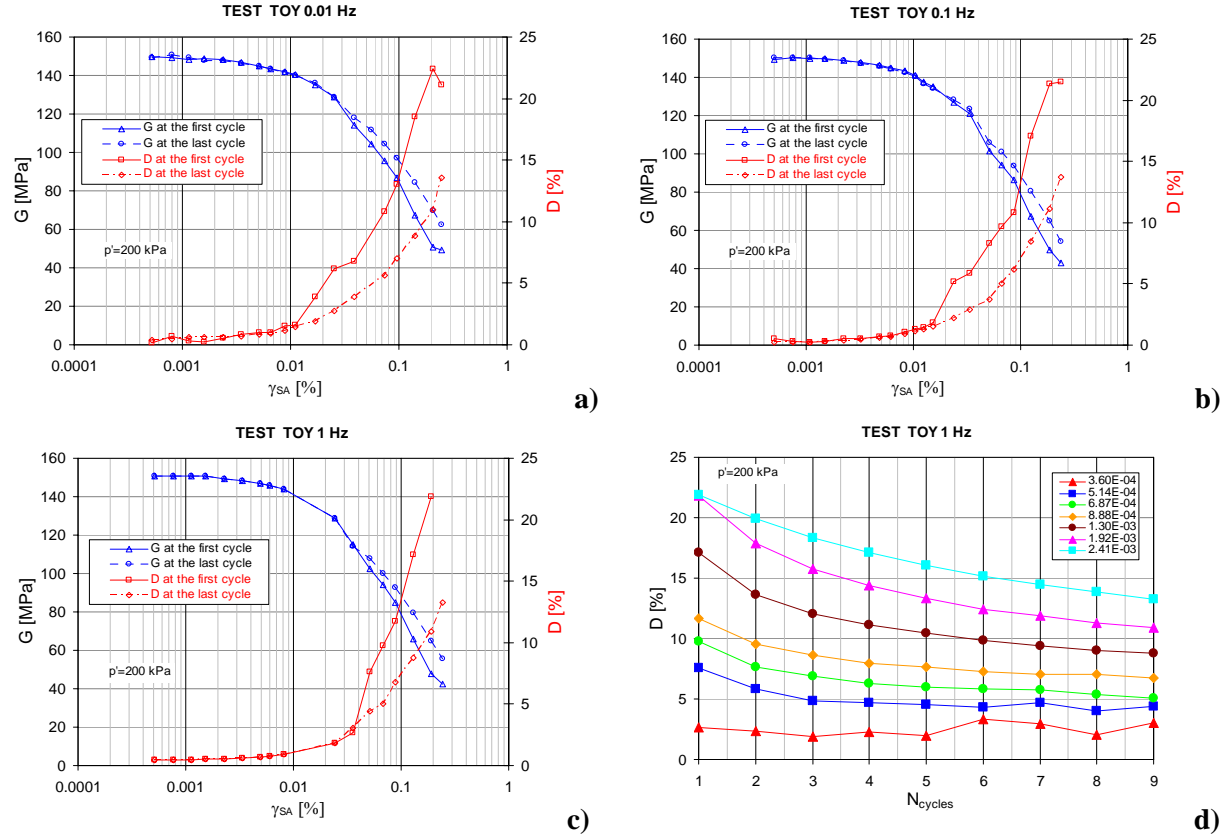


Figure 5. Shear moduli and damping ratios versus single amplitude shear strain, measured during cyclic torsional shear tests on dense dry Toyoura sand specimens, for the following loading frequencies: 0.01 Hz (a), 0.1 Hz (b), 1 Hz (c) in different loading cycles; (d) damping ratio for various number of cycles at different strain levels ($f = 1$ Hz).

At increasing strain levels the shape of cycles varies progressively; in particular the cycles rotate clockwise (Figure 4). To highlight this aspect, it is interesting to compare the degradation curves for G and for G_{tan} evaluated in different zones of the cycles (Figure 6). The tangent shear stiffness evaluated during the cycle for $\tau = 0$ kPa (Figure 1) is reported in Figure 6a, whereas for $\tau = \tau_{max}$ in Figure 6b. In Figure 6a G_{tan} has been calculated during both the first ($G(1)$) and the last cycle ($G(9)$), both in unloading ($_{u}$)/ reloading ($_{r}$). It is evident that reloading is more rigid independently of the number of cycles taken into consideration; this is likely to be due to the fact that the first loading (i.e. the increase in the stress level) is always imposed for positive values of τ . As is evident from Figure 3b, by increasing the number of cycles a loop rotation appears, but this evolution seems to be not sufficient to make a loop perfectly symmetric.

$G_{tan_r_top}$ (evaluated at the top of the cycle (Figure 6b) during reloading) presents a more evident degradation with γ_{SA} than G , whereas stiffness $G_{tan_u_top}$ (during unloading) increases progressively. When shear strain is sufficiently high, even if this was not reported in Figure 6b for the sake of clarity, $G_{tan_u_top}$ becomes negative. This is due to the viscous strains delayed with time induced by the previous loading.

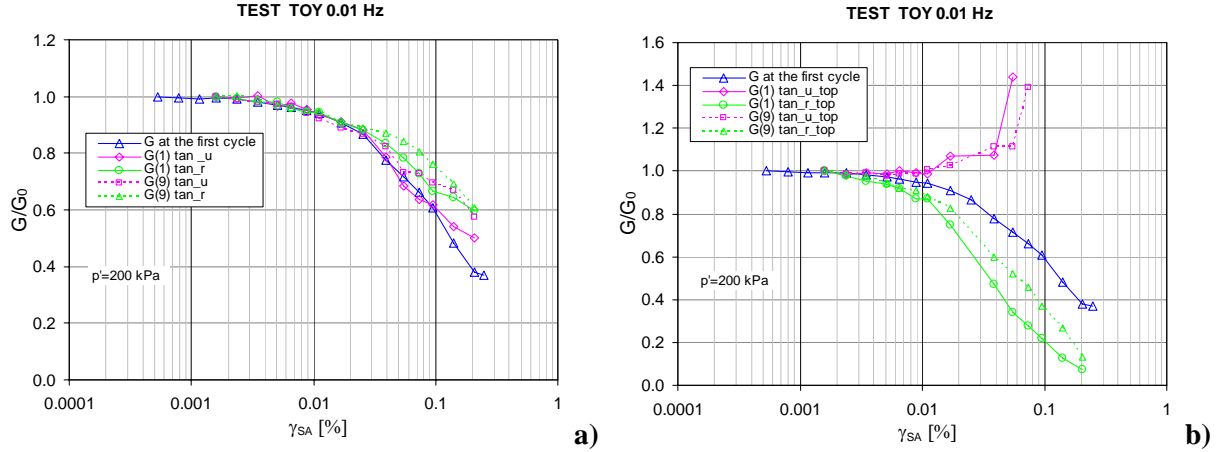


Figure 6. Normalised shear moduli versus single amplitude shear strain: comparison between G and the tangent shear stiffness G_{\tan} (a), G_{\tan_top} (b).

In Figure 7, the energy density per unit volume W_D (Figure 1) dissipated during each cycle at different strain levels is finally reported. The trend of the experimental curves can be interpolated by means of the following polynomial function $W_D = c_1 \cdot \gamma_{SA}^2$ (in Figure 7 $c_1=15 \text{ kJ/m}^3$, $c_2=2.35$). This means that the shape of the cycle changes negligibly at increasing loading levels and grows omothetically. If c_2 was equal 2, the shape of the loop would remain perfectly unaltered (dashed line of Figure 7).

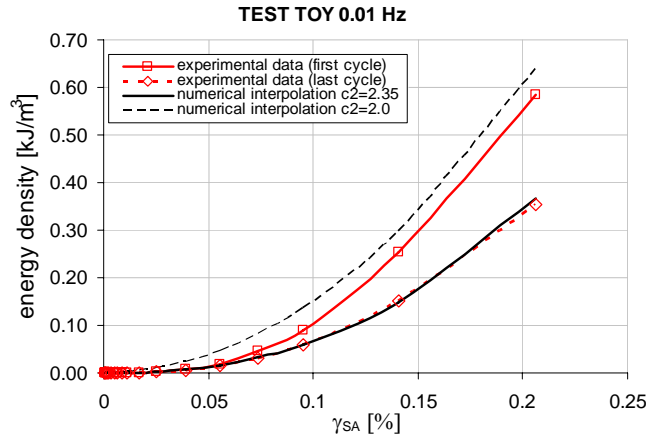


Figure 7. Energy density per unit volume dissipated in each cycle versus single amplitude shear strain.

NUMERICAL SIMULATIONS

In this section the previous experimental data will be commented by comparing them with the numerical simulations obtained by using the Milan Model 2002 proposed by Zambelli 2002, Zambelli et al. 2004, Zambelli 2006. It is about an elastoviscoplastic-cyclic model for sands characterized by two uncoupled plastic mechanisms: the former is associated to a global evolution of the material internal fabric, the latter to small strains and small loops taking into account the fabric rearrangements due to small size cyclic load disturbances. The approach of considering, perhaps artificially, as distinct the two contributes, allows the user to simply calibrate the constitutive parameters.

A detailed description of the viscoplastic part of the model can be found in di Prisco et al. 1993, 1995, di Prisco & Imposimato 1996, 1997 and di Prisco & Zambelli 2003, di Prisco et al. 2006.

The Milan Model is characterised by many parameters: three elastic (B_0 , λ , R^*), eleven plastic (γ , r_{c0} , $\hat{\theta}_c$, $\hat{\theta}_e$, ξ_c , ξ_e , t_p , c_p , B_p , β_{f0} , $\hat{\beta}_f$), two viscoplastic (α , $\bar{\gamma}$) and three cyclic (c_1 , c_2 , w_0). The plastic constitutive parameters have been calibrated on the basis of the results obtained by performing

one isotropic loading-unloading test, two drained triaxial compression/extension tests and two undrained triaxial compression/extension tests, all performed in strain-controlled conditions on saturated dense Toyoura sand specimens at University of Brescia. For satisfactorily capturing the experimental results with respect to the plastic constitutive model already cited, a modification of the plastic potential g was introduced and this justifies the large value of γ in Table 1. In particular, the parameters $\gamma, \hat{\theta}_c, \hat{\theta}_e, \xi_c, \xi_e$ are linked to the failure behaviour and have been calibrated on drained triaxial compression and extension tests. B_p , which is the plastic logarithmic volumetric compliance, can be derived from a loading-unloading isotropic test, while $\hat{\beta}_f$, linked to the yield function shape, can be determined by loading the specimen up to failure in triaxial compression and then unloading it. Parameters c_p and t_p control the rate of evolution of the yield locus and therefore the stiffness of the material mechanical response. The two viscous constitutive parameters α and $\bar{\gamma}$, which describe the system evolution rate, have been calibrated on the basis of creep test experimental data.

The elastic parameters λ, R^* have been calibrated by means of one isotropic loading-unloading test, while the elastic stiffness B_0 has been set in order to obtain a value of the shear modulus comparable with the experimental result of Figure 2a in the range of small strains (numerical simulation of cyclic torsional shear test with $\tau = 4$ kPa).

The cyclic parameters w_0 and c_2 were calibrated on the experimental degradation curves of G and D (Figure 2) obtained from cyclic torsional shear tests in the range of medium strains.

Table 1. Constitutive model parameters for Toyoura dense sand

D_r [%]	e_{\max}	B_0 [kPa]		λ	R^*	c_1	c_2	w_0		
80	0.6403	2330		0.0263	0.25	150	6	1500		
γ	B_p	$\hat{\theta}_c$	$\hat{\theta}_e$	$\hat{\beta}_f$	β_{f0}	t_p	c_p	r_{c0}	$\bar{\gamma}$ [s ⁻¹]	α
8.5	0.0004	0.362	0.136	0.6	0.2	18	85	1	1·10 ⁻⁶	61

In the three different ranges of shear strain γ_{SA} (defined in Figure 2), small, medium, large, respectively, the constitutive model activates prevalently three different mechanisms: the elastic one, the cyclic elastoplastic and the anisotropic elasto-viscoplastic.

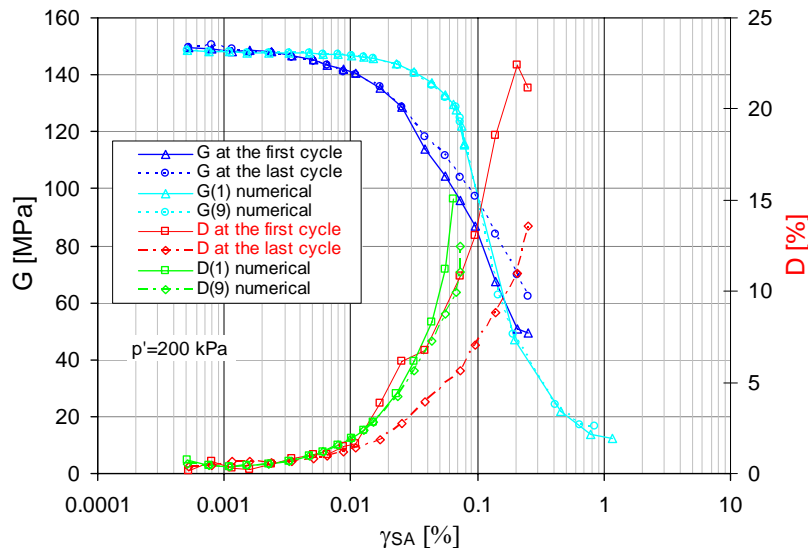


Figure 8. Experimental and numerical results: shear moduli and damping ratio versus single amplitude shear strain, measured during cyclic torsional shear tests on dense dry Toyoura sand specimens for loading frequency 0.01 Hz.

In Figure 8, the experimental and numerical results are compared. It is evident that the numerical simulations are not capable of reproducing the severe decrease in the shear modulus when $0.01 < \gamma_{SA}$

(%) < 0.1, whereas the damping ratio within the same range is slightly overestimated. As it can be derived from Figure 4 and Figure 9, the numerical model does not capture the progressive rotation of cycles when the shear stress amplitude is increased. In this range of γ_{SA} , the numerical curve is characterised by a sharp bend which is due to the activation of the anisotropic viscoplastic mechanism changing abruptly the shape of cycles.

On the contrary, it is worth noting that for values of γ_{SA} greater than 0.1 %, the experimental points are likely to overestimate G and severely underestimate D .

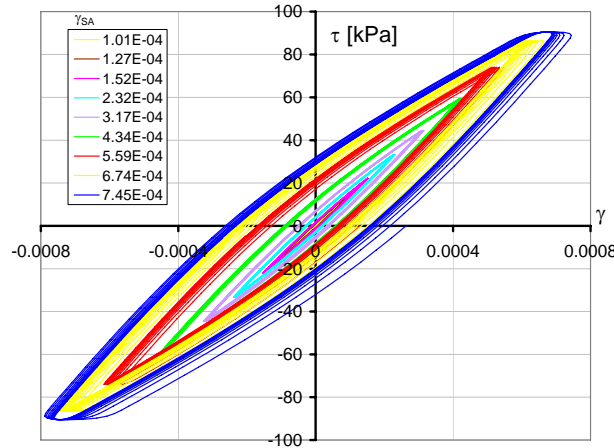


Figure 9. Numerical simulation of cyclic torsional shear tests on dense dry Toyoura sand specimens for loading frequency 0.01 Hz: stress-strain cycles at increasing strain levels.

From Figure 10c we can derive that even numerical simulations, although in a less marked manner than experiments, show a progressive evolution of the cycle shape. This is due to a partial activation of the viscoplastic mechanism when the shear stress imposed is sufficiently severe.

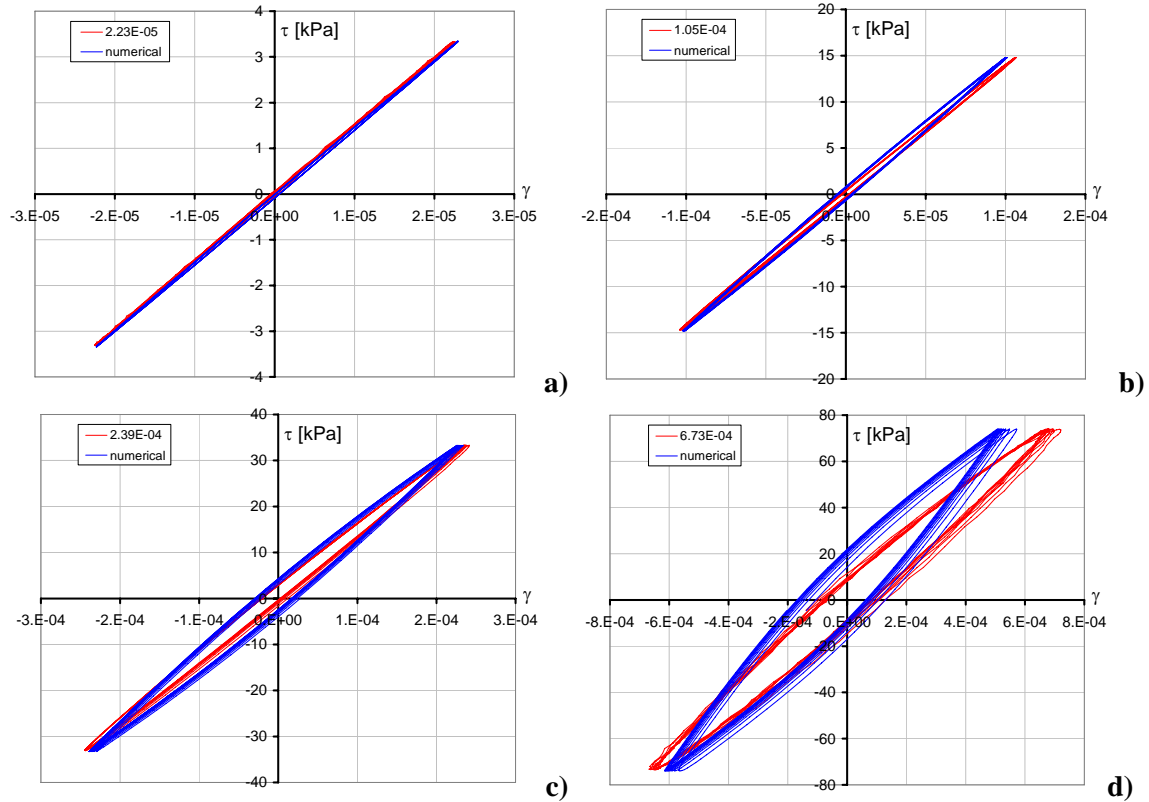


Figure 10. Numerical simulation of cyclic torsional shear tests on dense dry Toyoura sand specimens for loading frequency 0.01 Hz at different strain levels.

CONCLUDING REMARKS

The paper discusses the dependency of the mechanical response of dense Toyoura sand specimens on the loading amplitude during cyclic torsional shear tests. The material mechanical response is analysed putting in evidence the variation of both equivalent/tangent shear stiffness and damping ratio with shear strain. The experimental results are very close to many others already published in literature regarding Toyoura sand in a dense state. At small and medium strain ranges, the shape of the cycles does not evolve whatever the number of loading cycles is applied, and damping ratio is very small. On the contrary, when large shear strain tests are performed, the mechanical response of the sand specimens changes with the number of cycles: both shape and equivalent shear stiffness evolve progressively. Even the shape of cycles varies when strain level is increased, in particular a progressive rotation of cycles takes place. To highlight this aspect, it is interesting to compare the degradation curves for G and for G_{\tan} . Moreover the energy density per unit volume dissipated at each cycle at different strain levels has been considered. The change in shape of the cycle is then quantitatively described by means of a non dimensional coefficient c_2 . If this latter was equal 2, the shape of the cycles would be independent of γ_{SA} .

By employing a multiple mechanism elastoplastic constitutive model, the authors tried to simulate the dependency of G and D on the loading amplitude. At medium strain values, both G and D numerical values overestimate experimental data. In particular, the model is not capable of capturing the rotation of cycles when the shear stress is increased.

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