

## **RESPONSE ANALYSIS OF RIGID STRUCTURES ROCKING ON YIELDING FOUNDATION**

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### **ABSTRACT**

The rocking motion of slender/rigid structures stepping on nonlinear yielding foundation is examined. This work is the continuation of previous investigations on rocking structures where the foundation behaviour was restricted to linear viscoelastic. With yielding supporting springs, the geometric nonlinearities from the dynamics of a rocking block combine with the material nonlinearities of the foundation. This paper focuses in assessing the effects of the geometric and material nonlinearities and identifies various trends of the dynamic response. Selective results are presented.

**Keywords:** Non-linear dynamics, rocking motion, seismic stability, stepping bridges, yielding foundation.

### **INTRODUCTION**

The possibility of tall structures to separate from their foundation limits the earthquake generated forces and moments, and in most cases leads to more economical designs, assuming that stability prevails. For instance, the piers of the South Rangitikei Viaduct in New Zeland (Beck & Skinner, 1974; Fig. 1 left), the Rio Vista Bridge in Sacramento, California (Yashinsky & Karshenas, 2003; Fig. 1 centre), and the North Approach Viaduct of the Lions' Gate Bridge in Vancouver, British Columbia (Dowdell & Hamersley, 2000; Fig. 1 right) are allowed to rock on their foundation.

The analysis of the rocking response of a rigid block on a monolithic foundation is well known to the literature. Early studies on the rocking response of a rigid block supported on a base undergoing horizontal motion were presented by Housner (1963). Following this seminal work, a large number of studies have been presented to address the complex dynamics of the free-standing block (Zhang & Makris, 2001, and references provided therein), while a handful of studies examined the rocking response of structures stepping on a non-rigid foundation. Psycharis and Jennings (1983) presented an



**Figure 1. Examples of bridges allowed to rock on their foundation.**

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equivalent linear analysis of the rocking response of a rigid block stepping on a viscoelastic foundation, Yim and Chopra (1984) presented a linear analysis of a flexible structure supported on a base allowed to uplift, while Gazetas (1999) documented the permanent tilting of buildings due to foundation failure. More recently, Palmeri and Makris (2005) investigated in depth the rocking response of a rigid block which steps on linear viscoelastic supports via a fully non-linear analysis, which complements the linear one presented by Psycharis and Jennings (1983).

In this paper, we present a study on the rocking response of a rigid block which steps on yielding supports. It is assumed that the foundation system cannot take tension; therefore, separation occurs when the upward displacement of the heel of the structure is greater than the static deflection of the support. The developed methodology is applied to the response analysis of a bridge pier that is allowed to step, with the aim of investigating the effects that a non-rigid foundations has on the rocking response of tall/slender structures.

## STATEMENT OF THE PROBLEM

We consider in this study the rigid block of mass  $M$  shown in Fig. 2, in which  $R_0$  is the size and  $\alpha$  is the angle of slenderness. The nonlinear supports at the points L and R are yielding springs with vertical stiffness  $K = \omega_v^2 M / 2$  and yielding deflection  $\delta_y$ . The springs do not take tension, and the rigid block can pivot about points L and R when it is set to rocking. In Fig. 2, the horizontal dot-dashed line is used for the position where the supporting springs are unstressed.

When the motion is weak, then the body is in continuous contact with both supports at pivot points L and R. When the vertical uplift of the heel of the body exceeds the static deflection of the supporting springs,  $\delta_s$ , then separation occurs: e.g., in the stage B of Fig. 2 the separation is imminent, i.e. the supporting spring at the heel (point R) is unstressed, while in the stage C the body is supported only at its toe (point L); the contact at the heel is successively re-established in the stage E, where the support is waiting in the unstressed configuration (see stages C and D).

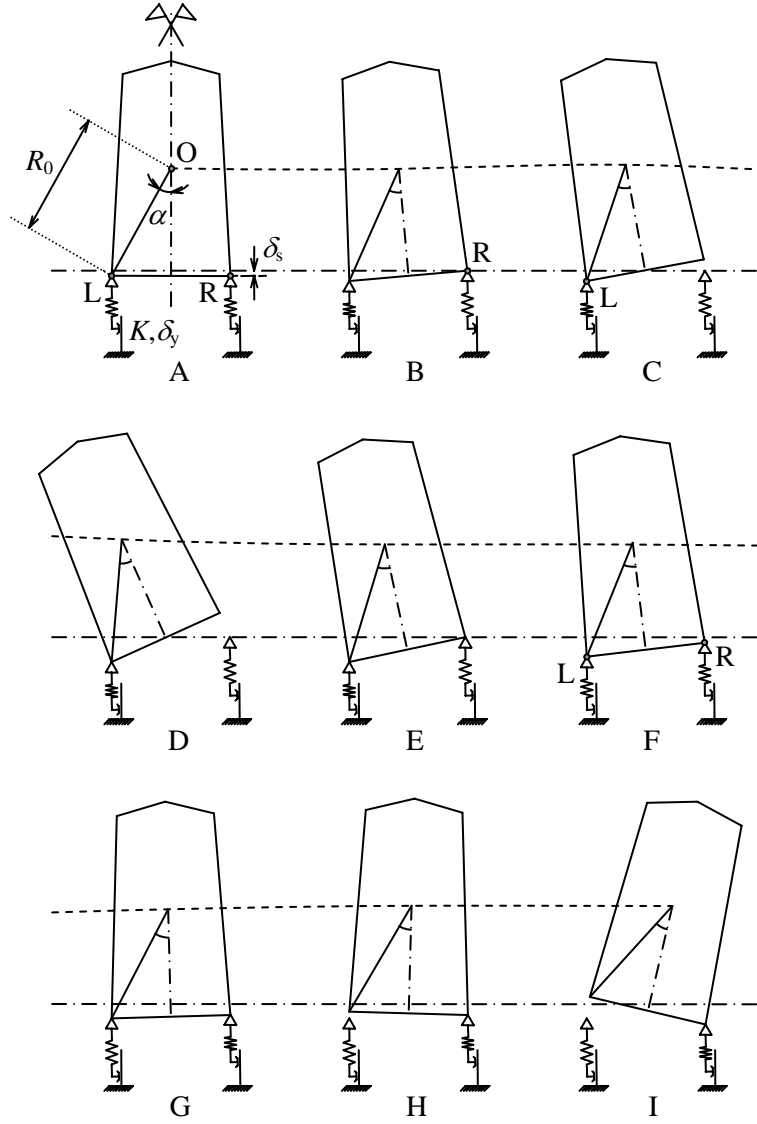
According to Fig. 2, the rigid block can experience three regimes of motion: (i) full-contact at both points L and R (stages A, B, and E to G in Fig. 2); (ii) contact only at point L (stages C and D, in which the block is tilting to the left); and (iii) contact only at point R (stage H and I, in which the block is tilting to the right). There is a fourth regime of motion, when the rigid body separates from both supports, which is not the subject of this study: when this happens, our analysis terminates.

It is worth noting that, given the elastoplastic behaviour of the supporting springs, Fig. 2 shows the plastic deformations accumulated at the pivot points (e.g., the transition between stage C and D), which may determine a permanent tilting of the block at the end of the motion. As an example, in the stage E of Fig. 2 the elastic deformations at the pivot points L and R are the same, as in initial stage A; however, in the stage E the rigid block shows a nonzero rotation due to the plastic deformation cumulated at the pivot point L, while prior to the stage A the supporting springs have not experienced plastic deformations.

The loss of energy caused by the impacts is governed in our analysis by the ratio of the velocity of the imminent pivot point before the impact to the velocity of the pivot point after the impact at the support. This ratio defines the coefficient of restitution,  $\varepsilon = \dot{v}_L^a / \dot{v}_L^b = \dot{v}_R^a / \dot{v}_R^b$ , which has been introduced by Psycharis and Jennings (1983). Given the abovementioned parameters, the rotation  $\theta$  of a rigid block rocking on a two-support yielding foundation can be expressed by:

$$\theta = f(\alpha, \mu, p, \varepsilon, \omega_v, \eta, \text{excitation}) \quad (1)$$

where  $\mu = M R_0^2 / I_0$  is the shape factor of the rigid block, being  $I_0$  the moment of inertia of the rigid block with respect to the centre of mass;  $\eta = \delta_y / \delta_s$  is the bearing capacity index of the foundation; and  $p$  is a frequency parameter, which results from its size,  $R_0$ , and the intensity of the surrounding gravitational field,  $g$ :



**Figure 2. Regimes of motion of a rigid block rocking on yielding foundation.**

$$p = \sqrt{\frac{\mu}{\mu + 1} \frac{g}{R_0}} \quad (2)$$

Eq. (1) indicates that there are six parameters of the rocking structure-foundation system that influence its response other than the characteristics of the ground excitation.

For a bridge tower, the frequency parameter can be as low as  $p = 0.5 \text{ rad/s}$  or even smaller; whereas for a household refrigerator  $p = 2.0 \text{ rad/s}$ . The slenderness of a 10 m tall classical column can be as low as  $\alpha = 10^\circ \cong 0.17 \text{ rad}$ ; whereas, the slenderness of a three-storey building which is 8 m wide is approximately  $\alpha = 36^\circ \cong 0.62 \text{ rad}$ . The vertical frequency,  $\omega_v$ , can originate from the vertical stiffness of an elastoplastic device or the stiffness of a conventional foundation, depending on the configuration of the rocking structure. Accordingly, practical values of interest for the ratio  $\beta = \omega_v / p$  are in the range  $10 < \beta < 100$ . Of course, there may be special cases where the ratio  $\beta$  lies outside this range. Finally, the values of the coefficient of restitution,  $\varepsilon$ , and the static safety factor  $\eta = \delta_y / \delta_s$  are in the range  $0 \leq \varepsilon \leq 1$  and  $\eta > 1$ .

## EQUATIONS OF MOTION

Since the rigid block is not allowed to slide, the system has only two degrees of freedom. Among the severable possible choices, we select the vertical displacement of the centre of mass,  $v$ , and the rotation of the block,  $\theta$  (see Fig. 3). Using the two degrees of freedom, the equations of motion in the full-contact regime can be derived in the form:

$$\begin{cases} \ddot{v} + \frac{g\eta}{2}(z_L + z_R) = -g - \ddot{v}_g(t) \\ \left[ 1 + \mu \cos^2(\alpha) \right] \ddot{\theta} + \frac{1+\mu}{2} p^2 \eta \left\{ [\sin(\alpha) + \cos(\alpha)\theta] z_L - [\sin(\alpha) - \cos(\alpha)\theta] z_R \right\} = \\ -(1+\mu) p^2 \cos(\alpha) \frac{\ddot{u}_g(t)}{g} \end{cases} \quad (3)$$

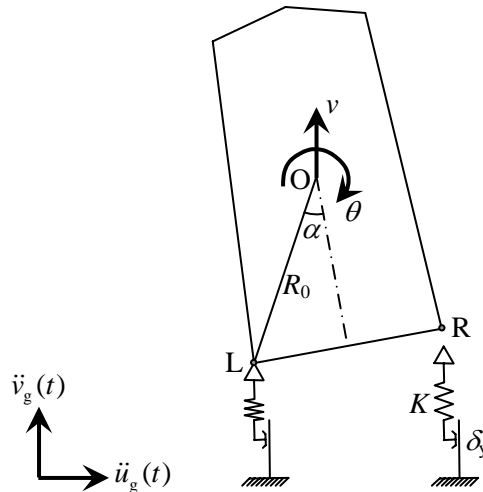
where  $\ddot{u}_g(t)$  and  $\ddot{v}_g(t)$  are the time histories of the horizontal and of the vertical components of the ground acceleration, respectively; while  $z_L$  and  $z_R$  are the dimensionless hysteretic variables associated with the yielding springs at the pivot points L and R, respectively, which are proportional with the corresponding reaction forces:

$$F_L = F_y z_L \quad ; \quad F_R = F_y z_R \quad (4)$$

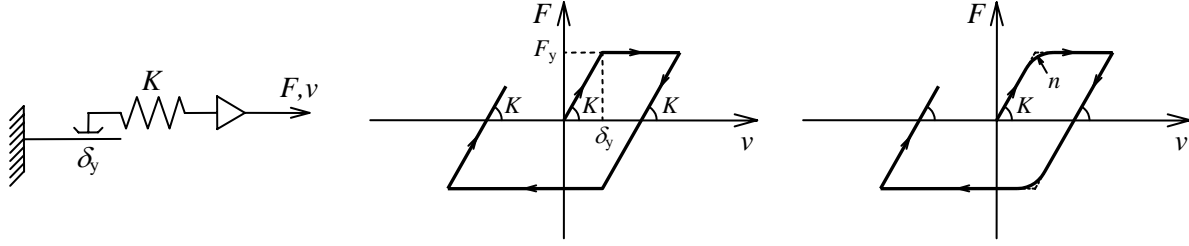
in which  $F_y = K \delta_y$  is the yielding force of the supporting springs. When the rigid block separates from the support on the right, the associated hysteretic variables goes to zero,  $z_R = 0$ , and the equations of motion are:

$$\begin{cases} \ddot{v} + \frac{g\eta}{2} z_L = -g - \ddot{v}_g(t) \\ \left[ 1 + \mu \cos^2(\alpha) \right] \ddot{\theta} + \frac{1+\mu}{2} p^2 \eta [\sin(\alpha) + \cos(\alpha)\theta] z_L = -(1+\mu) p^2 [\cos(\alpha) - \sin(\alpha)\theta] \frac{\ddot{u}_g(t)}{g} \end{cases} \quad (5)$$

In the same way, when the rigid block separates from the support on the left,  $z_L = 0$ , and the equations of motion are:



**Figure 3. Schematic of a rigid block rocking on yielding foundation.**



**Figure 4. Rheological model (right) and hysteresis loop (centre) for the ideal elastic-perfectly plastic behaviour. Hysteresis loop of the Bouc-Wen model.**

$$\begin{cases} \ddot{v} + \frac{g\eta}{2} z_R = -g - \ddot{v}_g(t) \\ \left[1 + \mu \cos^2(\alpha)\right] \ddot{\theta} + \frac{1+\mu}{2} p^2 \eta [\sin(\alpha) - \cos(\alpha)\theta] z_R = -(1+\mu) p^2 [\cos(\alpha) + \sin(\alpha)\theta] \frac{\ddot{u}_g(t)}{g} \end{cases} \quad (6)$$

Each of Eqs. (3), (5) and (6) rules the dynamic equilibrium of the rigid block in one of the three regimes of motion depicted in Fig. 2. These equations, however, have to be coupled with those ruling the time variation of the hysteretic variables  $z_L$  and  $z_R$ , whose values depends in principle on the whole time histories of the vertical displacements of the pivot points L and R, respectively:

$$v_L = v + R_0 \sin(\alpha)\theta \quad ; \quad v_R = v - R_0 \sin(\alpha)\theta \quad (7)$$

Among the rheological models available in the literature for the hysteresis of yielding springs, the simplest one is the elastic-perfectly plastic model (Fig. 4 left and centre), made of an elastic spring with stiffness  $K$  in series with a Coulomb element with sliding force  $F_y = K \delta_y$ . The ideal elastic-perfectly plastic model can be effectively approximated by the Bouc-Wen model (Bouc, 1967; Wen, 1976), which is described by the equations:

$$\begin{aligned} F(t) &= K \delta_y z(t) \\ \dot{z}(t) &= \frac{\dot{v}}{\delta_y} \left[ 1 - 0.5 \operatorname{sign}(\dot{v}) |z|^{n-1} z - 0.5 |z|^n \right] \end{aligned} \quad (8)$$

where  $n$  is a dimensionless parameter that controls the rate of the transition between the elastic and the perfect plastic phases (Fig. 4 right). In the following, it is always assumed  $n = 5$ .

Taking into account Eqs. (7), the state equations ruling the hysteretic variables  $z_L$  and  $z_R$  when the rigid block is in contact with the supporting springs can be derived in the form:

$$\begin{cases} \dot{z}_L = \frac{\beta^2}{\eta} \left[ \frac{p^2}{g} \dot{v} + \frac{\mu \sin(\alpha)}{1+\mu} \dot{\theta} \right] \left\{ 1 - 0.5 \operatorname{sign} \left[ \dot{v} + \frac{\mu g \sin(\alpha)}{(1+\mu) p^2} \dot{\theta} \right] |z_L|^{n-1} z_L - 0.5 |z_L|^n \right\} \\ \dot{z}_R = \frac{\beta^2}{\eta} \left[ \frac{p^2}{g} \dot{v} - \frac{\mu \sin(\alpha)}{1+\mu} \dot{\theta} \right] \left\{ 1 - 0.5 \operatorname{sign} \left[ \dot{v} - \frac{\mu g \sin(\alpha)}{(1+\mu) p^2} \dot{\theta} \right] |z_R|^{n-1} z_R - 0.5 |z_R|^n \right\} \end{cases} \quad (9)$$

When the block separates from the right spring, the corresponding hysteretic variables takes the constant value  $z_R = 0$ , and then  $\dot{z}_R = 0$ . In the same way, when the block separates from the left spring  $z_L = 0$  and  $\dot{z}_L = 0$ .

## RESPONSE ANALYSIS TO TRIGONOMETRIC PULSES

The ability of distinct pulses to generate a structural response that resembles the earthquake induced response has been examined in past studies (Makris & Chang, 2000; Makris & Roussos, 2000; Mavroeidis & Papageorgiou, 2003, and references reported therein). Some of the proposed pulses are physically realizable motions with zero final ground velocity and finite acceleration whereas some other idealizations violate one or both of the above requirements. Physically realizable pulses can adequately describe the impulsive character of near-fault ground motions, both qualitatively and quantitatively. According to Makris and his coworkers, the minimum number of parameters that describe the pulses is two, which are the acceleration amplitude  $a_p$  (or velocity amplitude  $v_p$ ) and the duration  $T_p$ , while the more sophisticated model by Mavroeidis and Papageorgiou (2003) involves four parameters.

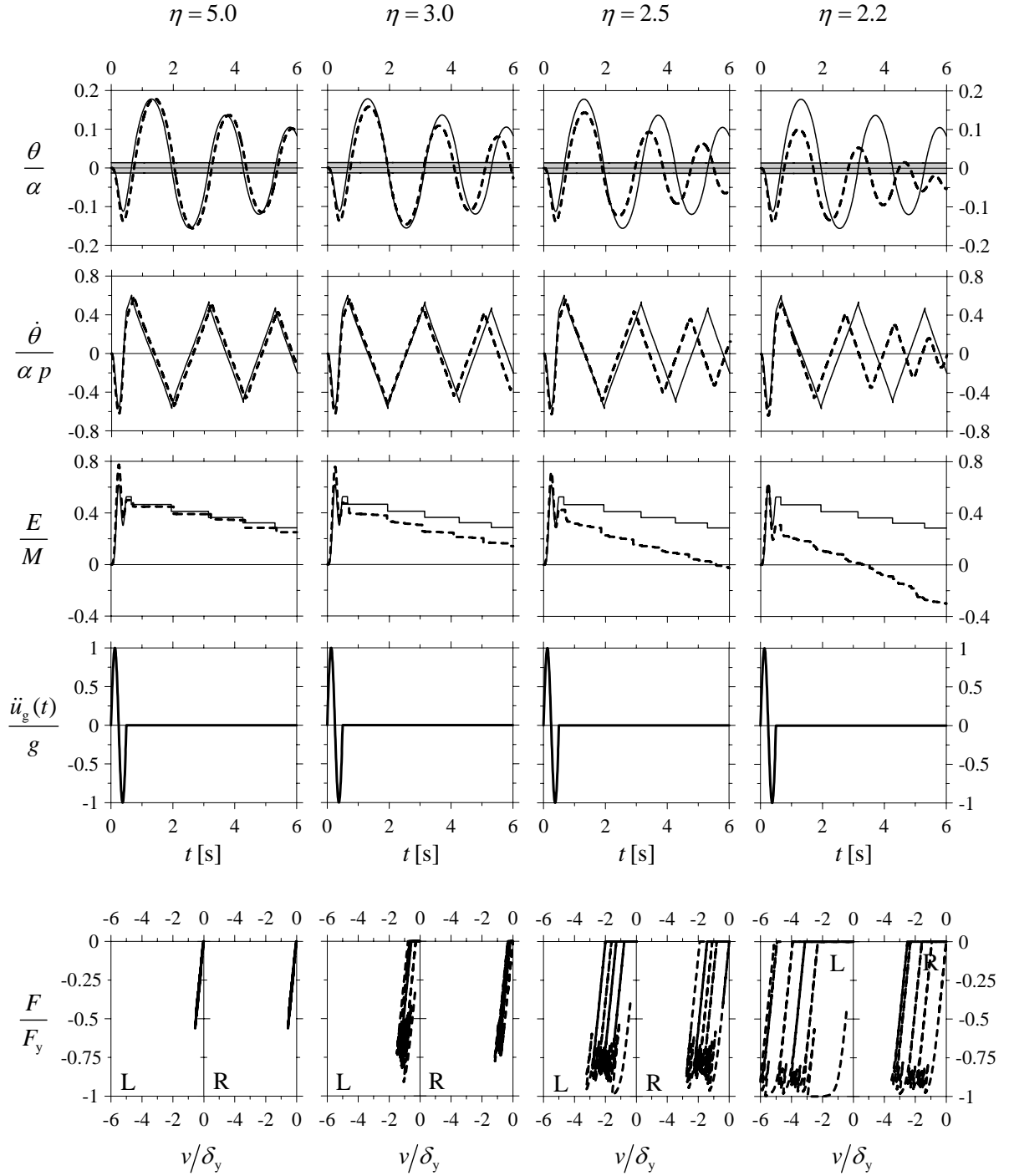
Pulse-type excitations are of key importance in this study because they allow investigating the effects of the governing parameters on the seismic response of the block-foundation system. Of particular interest are the forward motions, which can be approximated with a one-sine acceleration pulse, and the forward-and-back motions, which can be approximated with a one-cosine acceleration pulse (Makris & Chang, 2000; Makris & Roussos, 2000, among others).

As an example, Fig. 5 plots the responses to a one-sine pulse of a rigid block with slenderness angle  $\alpha = 0.20$  rad ( $\cong 11.5^\circ$ ), shape factor  $\mu = 3$  (rectangular shape) and frequency parameter  $p = 1$  rad/s ( $R_0 = 7.36$  m) rocking on a rigid foundation (solid lines) and a two-support yielding foundation (dashed lines), with different values of the static safety factor  $\eta = 5.0, 3.0, 2.5$  and  $2.2$ . The coefficient of restitution is assumed to be  $\varepsilon = 0.10$ , while the stiffness parameter takes the intermediate value  $\beta = \omega_v / p = 50$ . When the value of the static safety factor is higher ( $\eta = 5.0$ , first column from the left in Fig. 5) the rocking motion on rigid and non-rigid foundations are in good agreement, in terms of normalized rotation  $\theta/\alpha$  (first graph from the top), angular velocity  $\dot{\theta}/(\alpha p)$  (second graph) and total energy  $E/M$  (third graph, with values in J/kg). A more accurate inspection of these graphs, however, reveals that the flexibility of the foundation initially increases the amount of the energy that the ground motion transmits to the block (i.e., the peak of the total energy at the beginning of the motion is higher), while successively more energy is dissipated each time that the angle of rotation reverses (i.e., the total energy decays more rapidly). Interestingly, in this case ( $\eta = 5.0$ ) the total energy decreases only because the impacts, while no energy is dissipated in the yielding springs: their response, in fact, is purely elastic, and there are no hysteretic loops in the  $v - F$  plane (first graphs from the bottom in Fig. 5).

When the bearing capacity index reduces (i.e.  $\eta = 3.0, 2.5$  and  $2.2$ ), the rocking response on yielding springs becomes gradually different from that one on monolithic base. In particular, the peak rotation at the beginning of the motion decreases, while successively the block tilts to the left ( $\theta < 0$ ): in the first cycle of oscillation, in fact, the one-sine pulse induces a plastic (unrecoverable) deformation in the spring on the left which is larger than that one on the right. This is illustrated in the time history of the normalized rotation  $\theta/\alpha$  for the lower value of the bearing capacity index ( $\eta = 2.2$ , first column from the right in Fig. 5). In this case, in fact, at the ending the block oscillates completely under the shaded band which indicates the interval of rotation  $[-\bar{\theta}, \bar{\theta}]$  where rigid block and two-support foundation are in full-contact under a weak rocking motion, being:

$$\bar{\theta} = \arcsin \left[ \frac{1 + \mu}{\mu \beta^2 \sin(\alpha)} \right] \quad (10)$$

Moreover, Fig. 5 shows that the smaller is the bearing capacity index, the larger is the energy dissipated in the oscillations of the rigid block. When the ratio  $\eta = \delta_y / \delta_s$  reduces, in fact, the loss of energy is given not only at the impacts, but also during the plastic deformations in the supporting springs. This is confirmed by the hysteretic loops (first row from the bottom in Fig. 5), which encircle a larger area as the static safety factor  $\eta$  decreases. Finally, in the case where  $\eta = 2.2$  (first column from the right in Fig. 5) one can see that the total energy  $E$  takes negative values; indeed, the plastic



**Figure 5.** Normalized time histories of rotation  $\theta$ , angular velocity  $\dot{\theta}$  and total energy  $E$  (top graphs) of a rectangular block ( $\mu=3$ ) with parameters  $\alpha=0.20$  rad and  $p=1.0$  rad/s ( $R_0=7.36$  m) rocking on a yielding foundation with parameters  $\beta=50$ ,  $\varepsilon=0.10$  and various values of the safety factor  $\eta$ , when subjected to a one-sine type-A pulse ( $a_p=1.00$  g,  $T_p=0.5$  s). Hysteretic loops  $v-F$  (bottom graphs) of the yielding springs.

deformations cumulated in the yielding springs determine a permanent settlement of the rigid block, and then the potential energy becomes negative.

Fig. 6 plots the responses to a one-cosine pulse of a slender rigid block ( $\alpha = 0.15 \text{ rad} \cong 8.6^\circ$ ). The static safety factor is assumed to be  $\eta = 2.5$  and the stiffness parameter takes the values  $\beta = \omega_v / p = 20, 40, 60$  and  $100$ , while the other parameters are the same as in the previous example. In this case, interestingly, as stiff is the foundation (i.e., as large is  $\beta$ ), as large is the peak rotation of the rigid block rocking on yielding springs. All these values, however, are less than the peak rotation of the rigid block rocking on rigid base. In this circumstance, then, the use of rigid base allows to estimate in a conservative way the peak rotation of the rocking structure. More sophisticated models, however, are needed when the inelastic demand in the foundation is dealt with: the simple two-spring model considered in this study, for instance, shows that as stiff is the foundation (i.e., as large is  $\beta$ ), as large is the inelastic demand (see the hysteretic loops in the first row from the bottom in Fig. 6).

## RESPONSE ANALYSIS TO EARTHQUAKE LOADING

Following the response analysis to trigonometric pulses our study proceeds with the response analysis to earthquake loading after selecting six strong ground motions chronologically listed in Table 1. The proposed nonlinear formulation is used to estimate the seismic-induced response of a stepping bridge about 60 m tall. Frequency parameter, angle of slenderness and shape factor of the tower are assumed to be  $p = 0.38 \text{ rad/s}$ ,  $\alpha = 0.13 \text{ rad} (\cong 7.4^\circ)$  and  $\mu = 3$ , respectively. Moreover, in our analyses the coefficient of restitution takes the value  $\varepsilon = 0.05$ , while four combinations of parameters  $\beta$  and  $\eta$  are considered: 1)  $\beta = 40$  and  $\eta = 2.5$ ; 2)  $\beta = 40$  and  $\eta = 3.5$  (larger yielding deflection); 3)  $\beta = 80$  (stiffer foundation) and  $\eta = 2.5$ ; and 4)  $\beta = 80$  and  $\eta = 3.5$ . The peak rotations computed with these combinations of the foundation parameters are also compared with that one computed with a monolithic base.

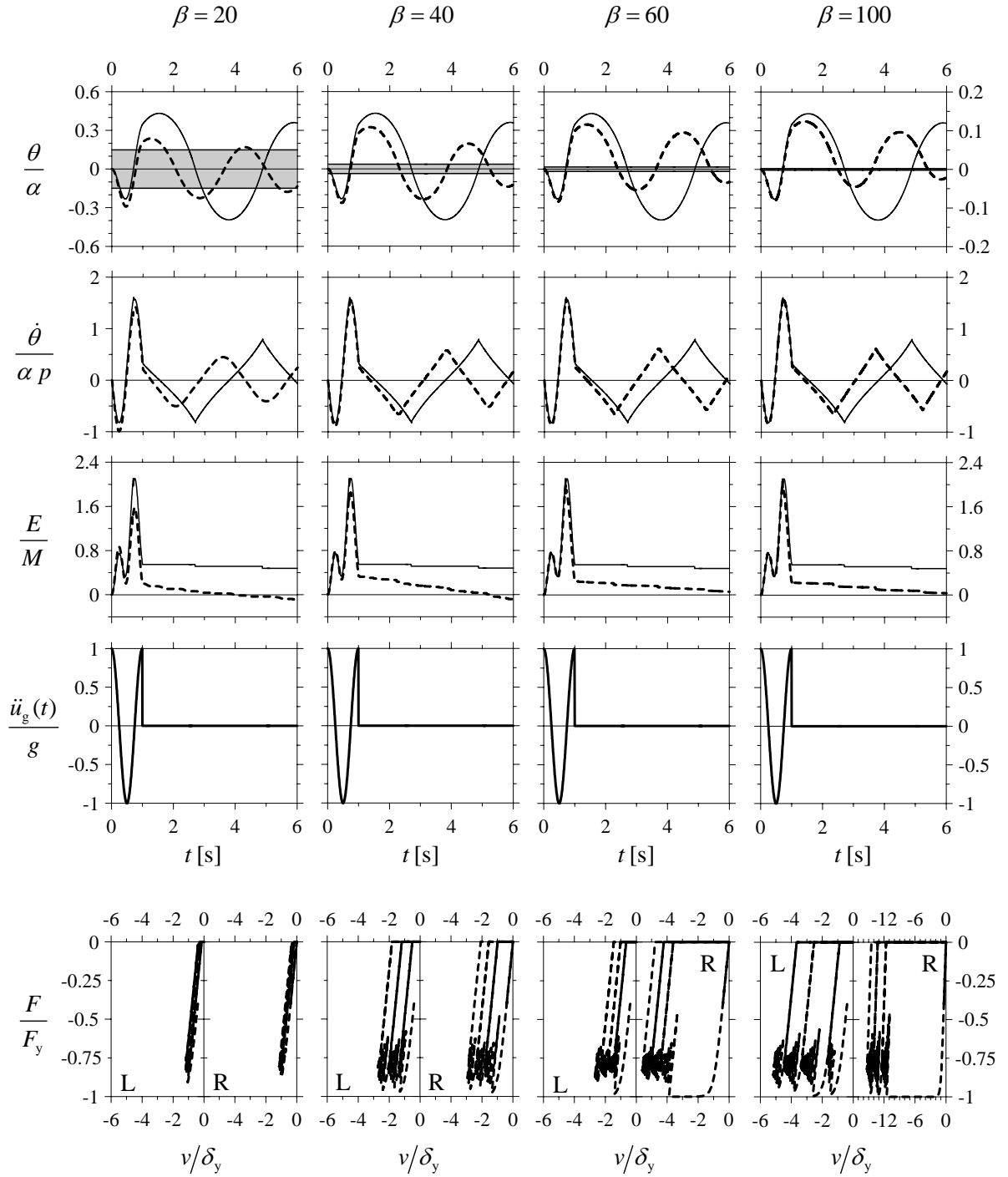
Fig. 7 plots the peak normalized rotations of the tower of interest when subjected to the six ground motions listed in Table 1, including also the vertical component. They are ordered with increasing peak ground acceleration, from  $\text{PGA} = 0.515 \text{ g}$  (1992 Erzikan) to  $1.497 \text{ g}$  (1992 Cape Mendocino). One can see that for the first three ground motions a foundation with larger values of stiffness and/or yielding deflection produces larger responses; in these cases, moreover, the model with rigid base

**Table 1. Information pertinent to the strong motions selected in this study.**

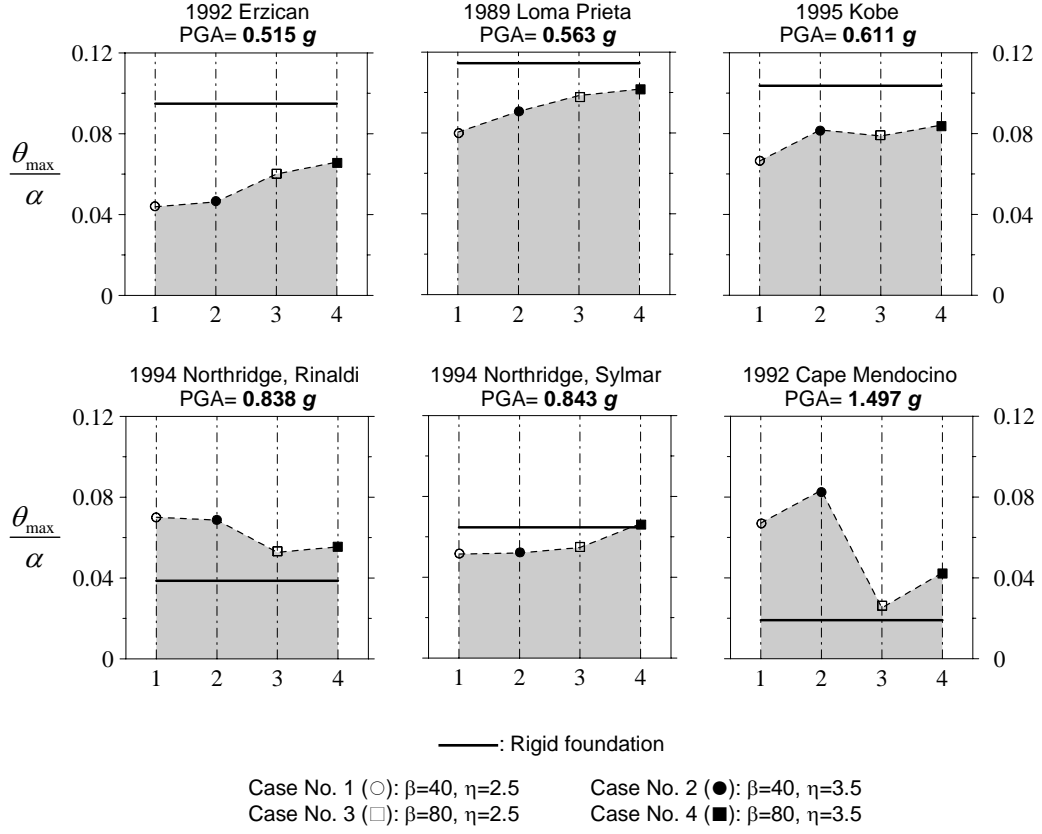
Earthquake (Date)	Station (Component*)	$M$	$D$ km	PGD m	PGV m/s	PGA g
Loma Prieta, California (18-Oct-1989)	16 LGCP (000)	6.9	6.1	0.412	0.948	0.563
Erzikan, Turkey (13-Mar-1992)	95 Erzikan (NS)	6.9	2.0	0.273	0.839	0.515
Cape Mendocino, California (25-Apr-1992)	89005 Cape Mendocino (000)	7.1	8.5	0.410	1.274	1.497
Northridge, California (17-Jan-1994)	24514 Sylmar Olive View Med FF (360)	6.7	6.4	0.327	0.130	0.843
Northridge, California (17-Jan-1994)	77 Rinaldi Receiving Station (228)	6.7	7.1	0.288	1.661	0.838
Kobe, Japan (16-Jan-1995)	0 Takatori (000)	6.9	0.3	0.358	0.127	0.611

\* NS: North-South.





**Figure 6. Normalized time histories of rotation  $\theta$ , angular velocity  $\dot{\theta}$  and total energy  $E$  (top graphs) of a rectangular block ( $\mu=3$ ) with parameters  $\alpha=0.15$  rad and  $p=1.0$  rad/s ( $R_0=7.36$  m) rocking on a yielding foundation with parameters  $\varepsilon=0.10$ ,  $\eta=2.50$  and various values of  $\beta$  when subjected to a one-cosine type-B pulse ( $a_p=1.00$  g,  $T_p=1.0$  s). Hysteretic loops  $v-F$  (bottom graphs) of the yielding springs.**



**Figure 7. Maximum rotation values of the bridge tower when subjected to the strong motions listed in Table 1.**

(solid lines) gives conservative estimations of the peak rotation. Remarkably, in two among the last three ground motions the opposite happens. Nevertheless, what is important to note is that large variations in the mechanical properties of the foundation have just a small effect on the value of the peak rotation. Furthermore, the tower, although very slender ( $\alpha = 0.13$  rad), experiences always small rotations because it has such large size ( $p = 0.36$  rad/s, and  $R_0 = 60.1$  m). More precisely: *i*) the maximum peak rotation of the stepping bridge under analysis ( $\theta_{\max} = 0.114\alpha$ ) is induced by the 1989 Loma Prieta earthquake (PGA = 0.563 g) when the base is assumed rigid; and *ii*), contrary to what one would intuitively expect, the minimum peak rotation ( $\theta_{\max} = 0.0191\alpha$ ) is induced by the 1992 Cape Mendocino earthquake, which exhibits the maximum peak ground acceleration (PGA = 1.497 g), and also in this case when the base is assumed rigid.

## CONCLUSIONS

In this paper, the rocking response of slender/rigid structures stepping on nonlinear yielding foundation is investigated. The response analysis of this study complements previous investigations where the foundation behaviour is restricted to linear viscoelastic. A novel set of equations of motion is presented, where two hysteretic variables are included, aimed to account for the plastic deformations cumulated in the supports. In the proposed model the geometric nonlinearities arising from the dynamics of a rocking block combine with the material nonlinearities of the foundation. As a result, the response of the rocking block to a given accelerogram depends on its size, shape and slenderness, together with the stiffness and the yielding deflection of the foundation, and with the amount of energy dissipated during the impacts.

The numerical examples presented herein demonstrate that a simple model with rigid base may give accurate estimations of the peak rotation, while more refined models are required when the inelastic demand in the foundation is of importance. Some counterintuitive results are also emphasized: for instance, an increase in the stiffness and/or in the strength of the foundation may indifferently produce larger or smaller rotations.

## ACKNOWLEDGEMENTS

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