

## GROUND RESPONSE ANALYSIS USING A MATRIX FORMULATION

J. YANG<sup>1</sup>, X.R. YAN<sup>2</sup>

### ABSTRACT

Evaluation of ground response is one of the most important problems in earthquake engineering. While a number of methods and computer programs have been developed for ground response analysis over the past three decades, SHAKE is probably the most popular one because of its simplicity in practical applications. The program is based on the one-dimensional wave propagation theory and computes the response of a horizontally layered soil-rock system to vertically traveling shear waves using a recursive algorithm. The nonlinear behaviour of soil during earthquake excitation is modeled through the iterative equivalent-linear approach. In this paper, an alternative formulation involving the use of dynamic stiffness matrix method is presented for earthquake response analysis of the same nonlinear ground model. A computer program named PASS is developed in the modern MATLAB environment to take full advantages of the matrix operations in MATLAB. Efficiency and accuracy of the new program are assessed by a detailed comparison of its results with those produced by SHAKE.

Keywords: ground response; site effect; nonlinear behavior

### INTRODUCTION

The characteristics of earthquake ground motions at a site are strongly influenced by site conditions such as subsurface soil properties and stratigraphy. This site effect is commonly known as soil amplification although the name may be misleading, since there is in fact amplification in certain range of frequencies and deamplification in others. Evaluation of ground response is one of the crucial problems in earthquake engineering. The response analysis can be used to predict ground surface motions and dynamic stresses and strains in the ground which are important information for seismic design.

SHAKE (Schnabel et al., 1972) was one of the first computer programs developed for ground response analysis. This program is based on the one-dimensional wave propagation theory and computes the response of a horizontally layered soil-rock system to vertically traveling shear waves using a recursive algorithm. The nonlinear behaviour of soils during earthquake excitation is modeled through the iterative equivalent-linear approach. While a number of methods and computer programs of various sophistications have been developed for ground response analysis over the past three decades, SHAKE, with its latest version known as SHAKE91 (Idriss and Sun, 1992), is still the most popular one in practical applications due to its simplicity in modeling. Recently, effort has been made to modify SHAKE using some modern development platforms such that it is more user-friendly (EduPro, 1998; Bardet et al., 2000). The new computer programs developed essentially follow the same concepts and formulations as SHAKE.

---

<sup>1</sup>Department of Civil Engineering, The University of Hong Kong, Email: junyang@hku.hk.

<sup>2</sup>Department of Civil Engineering, The University of Hong Kong.

In this paper, an alternative formulation is presented for earthquake response analysis of the same one-dimensional nonlinear ground model. The formulation involves the use of dynamic stiffness matrix method such that the recursive algorithm in SHAKE is avoided. A computer program PASS, standing for **P**RACTICAL **A**NALYSIS of layered **S**oil-rock **S**ystems, is developed in the modern MATLAB environment to take full advantages of the matrix operations in MATLAB. Feasibility of the new program is evaluated by a detailed comparison of its results with those produced by SHAKE.

## METHODOLOGY AND FORMULATION

For vertically propagating shear waves in a viscoelastic soil, the equation of motion is written by:

$$\rho \frac{\partial^2 u}{\partial t^2} = G^* \frac{\partial^2 u}{\partial z^2} \quad (1)$$

where  $u$  is the displacement at depth  $z$ ,  $\rho$  is the mass density of soil, and  $G^*$  represents the complex shear modulus, defined as  $G^* = G(1 + 2i\zeta)$ . Here  $\zeta$  is known as the critical damping ratio.

For steady-state harmonic waves with the frequency of  $\omega$ , the solution of Equation (1) is readily given as:

$$u(z, t) = (Ee^{ikz} + Fe^{-ikz})e^{i\omega t} = U(z, \omega)e^{i\omega t} \quad (2)$$

where the first term  $Ee^{ikz}$  represents the incident wave traveling in the negative  $z$ -direction (upward) and the second term  $Fe^{-ikz}$  represents the reflected wave traveling in the positive  $z$ -direction (downward). The parameter  $k$  is the complex wave number given by  $k = \sqrt{\rho\omega^2 / G^*}$ .

For a multi-layered system as shown in Figure 1, the following recursive formulas can be established by using the above solution and considering the continuity conditions of displacements and tractions between layers  $m$  and  $m+1$ :

$$\begin{aligned} E_{m+1} &= \frac{1}{2}E_m(1 + \alpha_m)e^{ik_md_m} + \frac{1}{2}F_m(1 - \alpha_m)e^{-ik_md_m} \\ F_{m+1} &= \frac{1}{2}E_m(1 - \alpha_m)e^{ik_md_m} + \frac{1}{2}F_m(1 + \alpha_m)e^{-ik_md_m} \end{aligned} \quad (3)$$

where  $d_m$  is the thickness of layer  $m$  and  $\alpha_m$  the complex impedance ratio defined as  $\alpha_m = \sqrt{(\rho_m G_m^*) / (\rho_{m+1} G_{m+1}^*)}$ .

The recursive procedure is started at the top free surface and then applied successively to layer  $m$ . Further, the transfer function  $TR_{m,n}$  relating the displacements at tops of layers  $m$  and  $n$  is defined as:

$$TR_{m,n}(\omega) = \frac{U_m(\omega)}{U_n(\omega)} \quad (4)$$

[illegible]

Apparently there are  $(n-1)$  equations with  $n$  unknown variables. Given the bedrock motion, the motions in all layers can be computed conveniently from Equation (9). If rock outcropping motion is given, a radiation dashpot with the damping coefficient of  $\rho_{rock}v_{srock}$  is introduced to radiate the energy transmitted from the soil to the bedrock, where  $\rho_{rock}$  and  $v_{srock}$  are the mass density and shear wave velocity of the rock. The impedance of the rock to the soil is thus  $S_n = i\rho_{rock}v_{srock}\omega$ .

### Equivalent linear analysis

The nonlinear and inelastic behavior of soil is well established in geotechnical engineering (Seed and Idriss, 1970). To approximate the actual nonlinear response of a soil, a modulus reduction curve and a damping ratio curve need to be established and an equivalent linear approach can be utilized. In the analysis, the strain vector obtained in the frequency domain is transformed into the time domain by using the inverse Fourier transformation. The maximum strain level is then scaled by a factor to give an effective strain level, which is then used for the determination of the corresponding shear modulus and damping ratio for the next iteration. The process is repeated until the difference between the effective strains computed in two adjacent iterations is within an acceptable tolerance.

## VERIFICATION

A computer program named PASS is developed in the modern MATLAB environment to take full advantages of the matrix operations in MATLAB. The accuracy and efficiency of PASS as compared with SHAKE require assessment. Two hypothesized soil sites are considered (Figure 2): one is a homogeneous single layer overlying a rock half-space and the other is a two-layer system. Figure 3(a) compares the transfer function of the single layer site computed using PASS with that from SHAKE, and Figure 3(b) shows the comparison for the two-layer site. In the computation the soil is assumed either to be elastic with zero damping or viscoelastic with the damping ratio of 10%.

It is clear from Figure 3 that the results produced by PASS are in very good agreement with those by SHAKE. Both programs predict the predominant frequency of 1.67 Hz for the one-layer site and 2.62 Hz for the two-layer site. In the case of zero damping, resonance phenomenon is observed clearly.

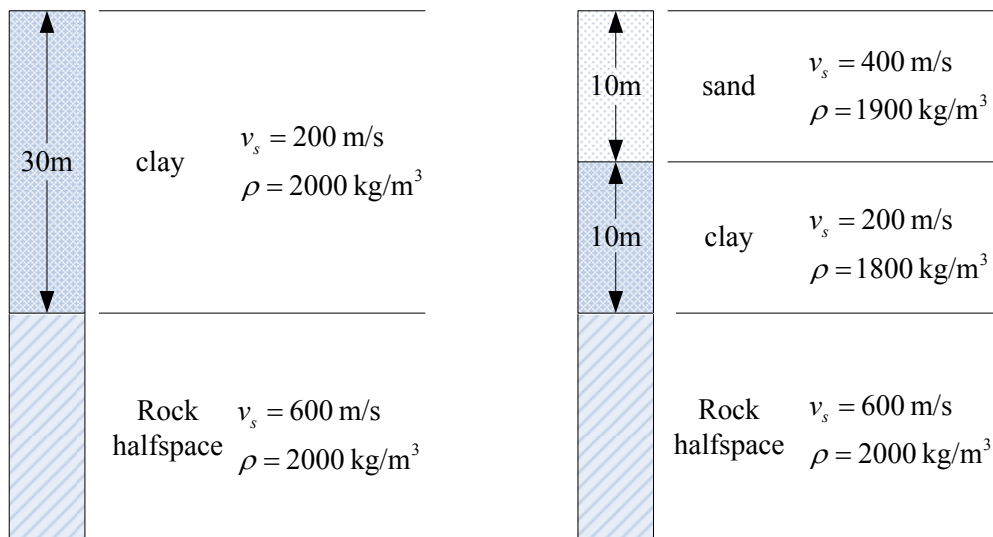
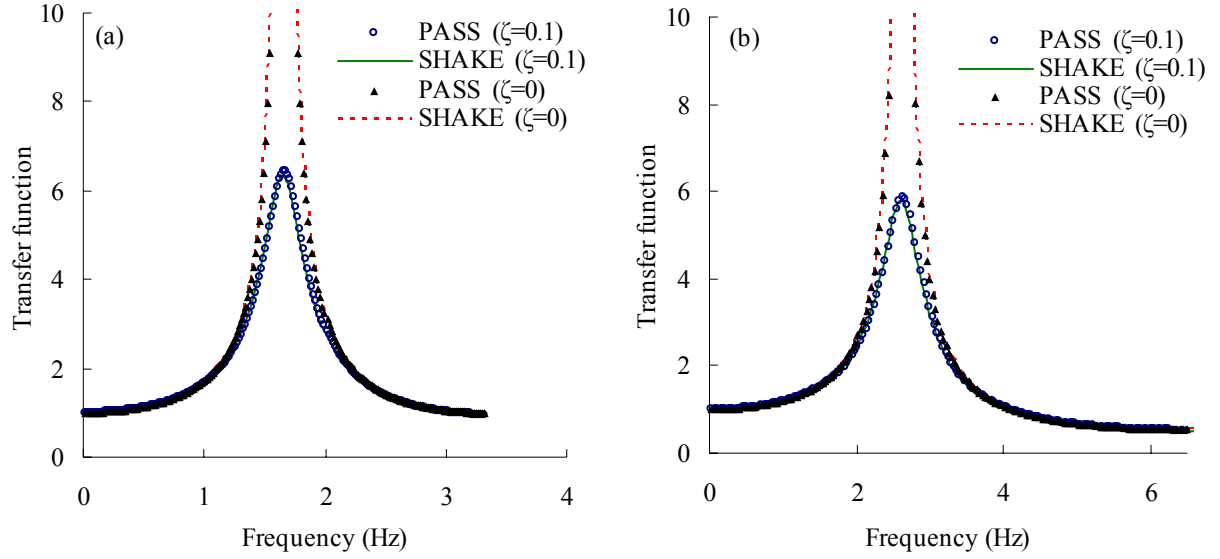
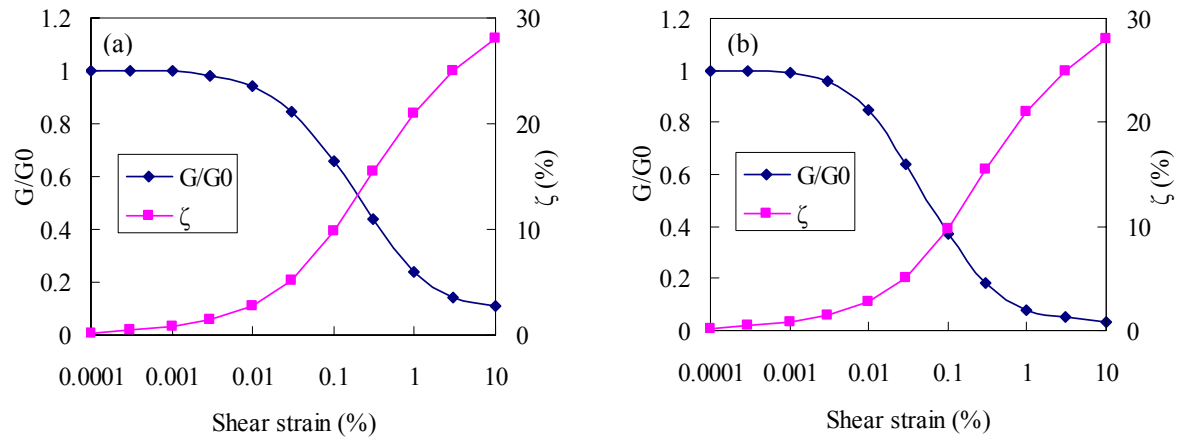


Figure 2. Two hypothesized sites used in verification



**Figure 3. Transfer functions from PASS and SHAKE: (a) one-layer site; (b) two-layer site**

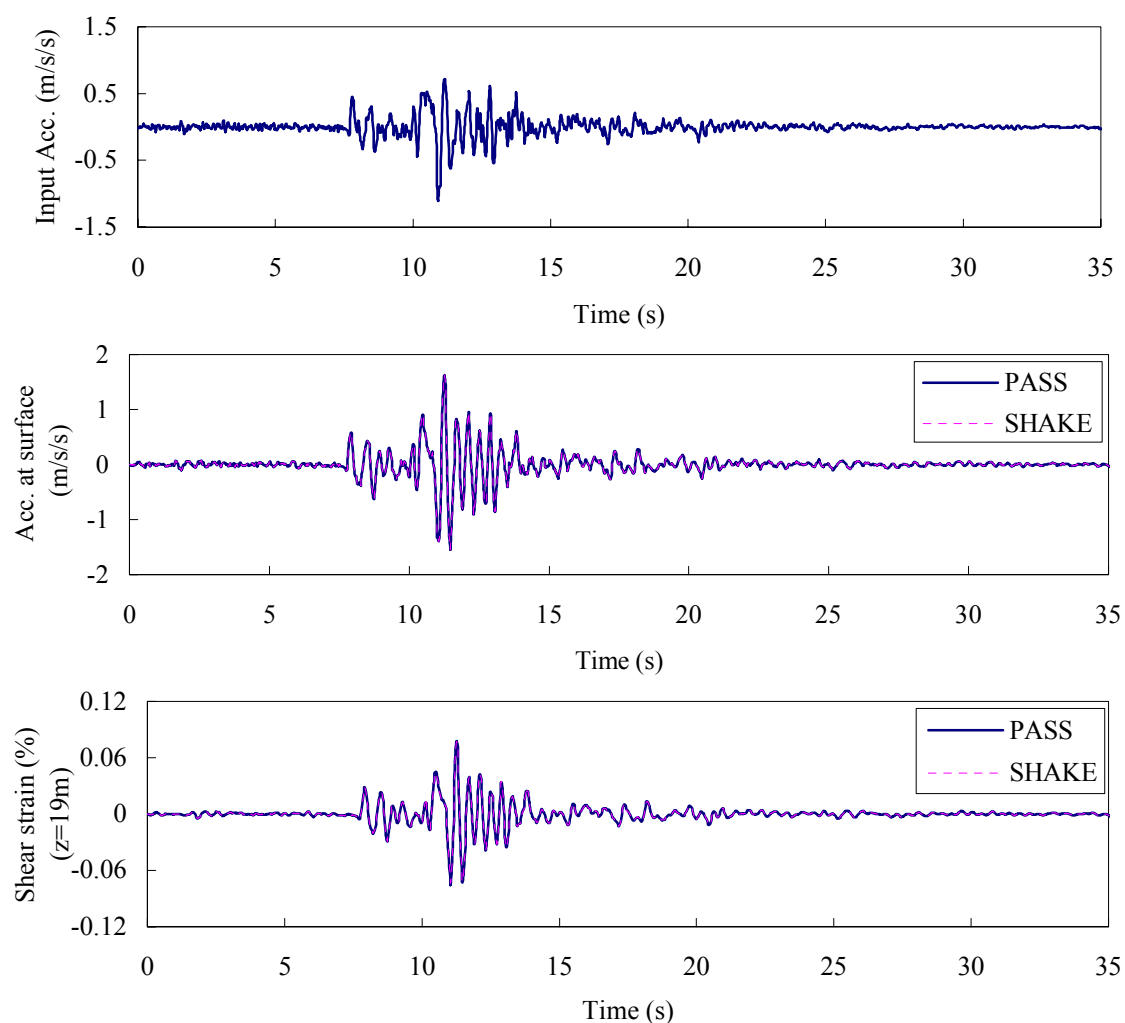


**Figure 4. Modulus reduction and damping ratio curves used in analysis: (a) sand; (b) clay**

Now consider the nonlinear response of the two-layer site subjected to earthquake loading. The modulus reduction curves and damping ratio curves for sand and clay, shown in Figure 4(a) and (b), are taken from Seed and Idriss (1970) and Sun et al. (1988), respectively. The bedrock motion used in the analysis is shown in Figure 5(a), which was recorded at Diamond Heights (EW component) during the 1989 Loma Prieta earthquake. The peak acceleration of this record is 0.113g. The effective shear strain factor is taken as 0.5 in the computation.

The acceleration time histories at the surface computed using PASS and SHAKE are compared in Figure 5(b). Figure 5(c) presents the shear strain time histories at the depth of 19 m from PASS and SHAKE. It is clear that the difference between the results produced by PASS and SHAKE is negligible. The excellent agreement is also observed in Figure 6 where the profiles of the shear strain, modulus reduction and damping ratio along the depth are illustrated. The results in Figure 6 show that the relatively weak clay layer exhibits greater modulus reduction and damping as compared with the strong sand layer. To make the comparison more apparent, the maximum shear strain and shear stress and the shear modulus at various depths after iteration are summarized in Table 1. It is of interest to

note that the iteration number required by PASS is significantly less than the iteration number required by SHAKE.

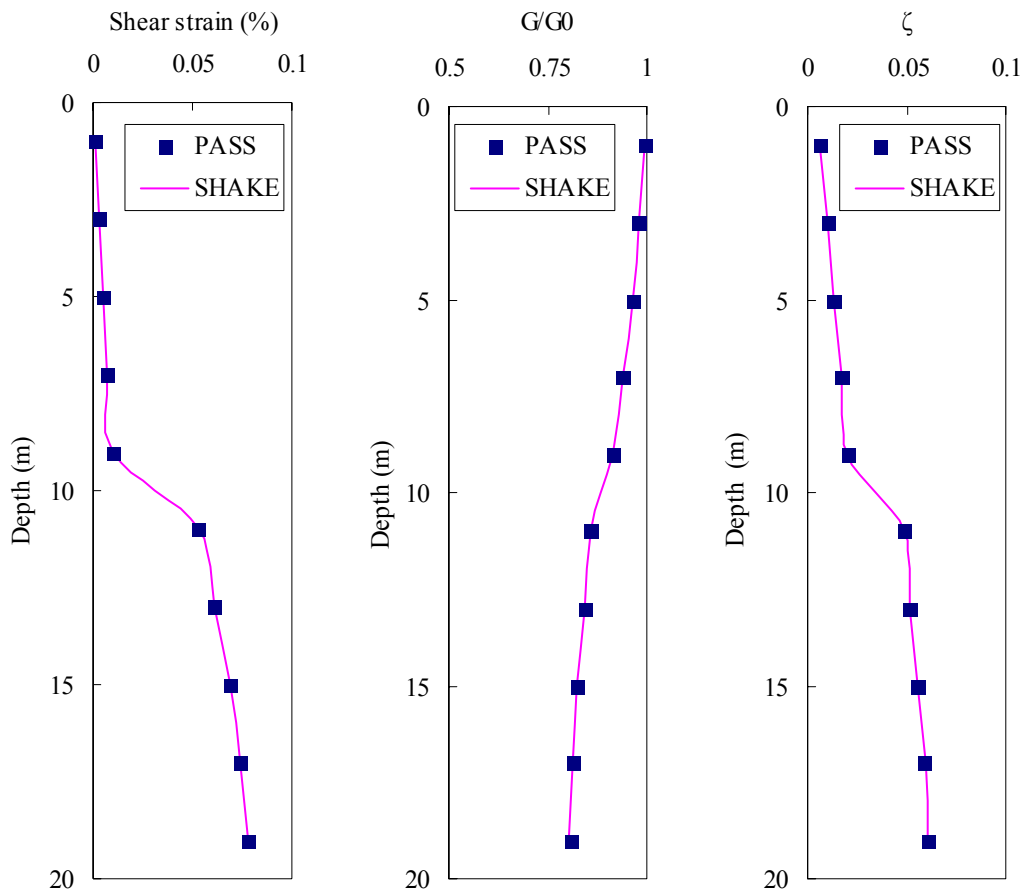


**Figure 5. Input motion, calculated acceleration at surface and shear strain at 19 m**

**Table 1. Comparison of results from SHAKE and PASS**

Depth (m)	Maximum shear strain (%)		Shear modulus (MPa)		Maximum shear stress (kPa)	
	SHAKE	PASS	SHAKE	PASS	SHAKE	PASS
1	0.0010	0.0010	302.66	302.65	3.09	3.10
3	0.0031	0.0031	297.30	297.27	9.25	9.27
5	0.0052	0.0053	292.97	292.94	15.35	15.39
7	0.0075	0.0075	285.74	285.66	21.34	21.41
9	0.0098	0.0098	278.25	278.17	27.24	27.30
11	0.0530	0.0528	61.75	61.77	32.70	32.64
13	0.0614	0.0612	60.72	60.75	37.28	37.19
15	0.0690	0.0687	59.40	59.44	40.95	40.81
17	0.0746	0.0742	58.49	58.55	43.65	43.46
19	0.0781	0.0776	57.97	58.04	45.27	45.06

Iteration number: SHAKE (8); PASS (4)



**Figure 6. Depth distribution of maximum shear strain, shear modulus reduction factor and damping ratio**

## SUMMARY AND CONCLUSIONS

This paper presents an alternative computational formulation for one-dimensional, equivalent linear ground response analysis. A new computer program PASS has been developed based on this formulation. A detailed comparison has been made between the results produced by PASS and the results produced by SHAKE, which has shown excellent agreement. The numerical implementation of the program PASS differs from that of SHAKE91 in the following major ways:

1. PASS assembles a dynamic stiffness matrix for the layered soil-rock system to compute the response and avoid the use of the recursive algorithm in SHAKE91. The responses of all layers are solved at one running. In SHAKE91, however, the user needs to specify beforehand the layers for which their responses needed to be computed.
2. PASS is developed in the modern MATLAB environment. It uses 32-bit arithmetic rather than the 16-bit arithmetic used in SHAKE91.
3. PASS uses a different Fast Fourier Transform (FFT) routine. The routine computes the discrete Fourier transform (DFT) in one or more dimensions, of arbitrary input size, and of both real and complex data. This allows less restriction for the earthquake input motion than SHAKE91.
4. PASS uses a more efficient and robust procedure to iterate toward strain compatible soil properties, whereas in SHAKE91 the user has to specify the iteration number beforehand. The last iteration required to drop the modulus and damping error within the specified error tolerance

- in PASS will produce smaller final errors than the last iteration for the same error tolerance in SHAKE91.
5. In SHAKE91, the computational environment requires the user to set the maximum numbers of the input information (e.g. maximum number of soil layers, maximum number of input motion data points). There are generally no such pre-requirements in PASS as it uses robust matrix operations in MATLAB.

## **ACKNOWLEDGEMENTS**

The work described in this paper was supported by the Research Grants Council of Hong Kong (HKU7127/04E and HKU7191/05E). This support is gratefully acknowledged.

## **REFERENCES**

- Bardet, JP, Ichii, K and Lin, CH. EERA: A Computer Program for Equivalent-linear Earthquake site Response Analyses of Layered Soil Deposits, Department of Civil Engineering, University of Southern California, 2000.
- EduPro Civil Systems, Inc. ProShake: Ground Response Analysis Program----User's Manual, 1998.
- Idriss, IM and Sun, JI. User's Manual for SHAKE91, Department of Civil and Environment Engineering, University of California, Davis, 1992.
- Schnabel, PB, Lysmer, J and Seed, HB. SHAKE: A Computer Program for Earthquake Response Analysis of Horizontally Layered Sites, EERC 72-12, Earthquake Engineering Research Center, University of California, Berkeley, 1972.
- Seed, HB and Idriss, IM. Soil Moduli and Damping Factors for Dynamic Response Analyses, EERC 70-10, Earthquake Engineering Research Center, University of California, Berkeley, 1970.
- Sun, JI, Golesorkhi, R and Seed, HB. Dynamic Moduli and Damping Ratios for Cohesive Soils, UCB/EERC 88/15, Earthquake Engineering Research Center, University of California, Berkeley, 1988.
- Wolf, JP. Foundation Vibration Analysis using Simple Physical Models, Prentice Hall, 1994.
- Yang, J and Yan, XR. PASS: A Computer Program for Practical Analysis of Layered Soil-Rock Systems, Department of Civil Engineering, The University of Hong Kong, 2006.