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PERFORMANCE-BASED PSEUDO-STATIC ANALYSIS OF SLOPES

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ABSTRACT

Current pseudo-static analysis of slopes requires the selection of an equivalent seismic coefficient k_{eq} that is demanded to globally represent the effects of the transient seismic action on the slope. In the paper a rational approach for the selection of k_{eq} is proposed introducing an equivalence between the pseudo-static analysis and the displacement-based approach. The procedure for the evaluation of k_{eq} was developed for the simple scheme of infinite slope assuming that the magnitude of earthquake-induced displacements represents a proper quantity for assessing the seismic stability of slopes.

Keywords: equivalent seismic coefficient, pseudo-static approach, displacement-based analysis

INTRODUCTION

Current methods for stability analysis of slopes subjected to seismic loading are based on the limit equilibrium or limit analysis methods applied using the pseudo-static approach. In these methods the effects of the transient seismic action, characterised by abrupt changes in modulus and sign, are represented by an equivalent static force that is demanded to represent the overall effect of the earthquake on the slope. This force is typically defined as a percentage of the weight of the potential sliding soil mass, through an equivalent seismic coefficient, k_{eq}, of horizontal and vertical components $k_{h,eq}$ and $k_{v,eq}$, respectively. Seismic stability of slopes is measured by a factor of safety F_{psd} that is defined as the ratio of the total resisting force to the total driving force acting on a potential failure surface. The result of a pseudo-static analysis is critically dependent on the value of k_{eq} whose selection is, to a certain extent, arbitrary and lacks a clear rationale. In current practice, values of the equivalent seismic coefficient given in seismic codes depend on the maximum acceleration of the design accelerogram, assumed to be applied to a rock outcropping, and on site conditions, through soil profile and topographical amplification factors.

In principle, the seismic coefficient should represent the amplitude of inertia forces induced in the potential sliding mass by the earthquake loading. Then, values of $k_{h,eq}$ and $k_{v,eq}$ should be selected using equivalent acceleration time histories obtained from seismic response analyses of the slope and, as a consequence, should depend on the main factors affecting its dynamic response: input accelerogram and bedrock depth, slope geometry and soil profile, soil stiffness and damping, pore water pressure conditions and failure mechanism. Moreover, different values of k_{eq} may be obtained depending on the analysis conditions (1D or 2D), assumptions for soil behaviour (linear-elastic, visco-elastic or elastic-plastic), boundary conditions and criteria adopted for evaluating the equivalent accelerogram and selecting an appropriate value for k_{eq}.

Alternatively, in the displacement-based approach, seismic stability of slopes is evaluated by comparing the earthquake-induced displacement with threshold values representing the maximum displacement that can be tolerated by the slope. The earthquake-induced displacement of the potential

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sliding mass is usually evaluated using the sliding block analysis, originally outlined by Newmark (1965). The permanent displacement developed during the earthquake can be assumed to represent a measure of the effect of the seismic event on the slope. The maximum tolerable displacement is related to the damage associated with slope instability, and depends on the behaviour of the slope and on that of structures and infrastructures possibly involved in the slope instability. In the literature, only a few studies attempt to relate earthquake-induced displacements with levels of damage (Idriss 1985), and definition of maximum tolerable displacements is rarely given by seismic codes.

The pseudo-static and the displacement-based analyses have been usually regarded as alternative methods for evaluating seismic stability of slopes. However, the seismic coefficient to be used in the pseudo-static approach may be related to earthquake-induced displacements. Such an attempt is presented in the paper with reference to the simple scheme of infinite slope, assuming that the earthquake-induced displacement is a proper quantity for assessing the seismic stability of slopes.

POSITION OF THE PROBLEM

A number of procedures are available in the literature for computing the equivalent seismic coefficients $k_{h,eq}$ and $k_{v,eq}$ for a given design earthquake and for a selected slope scheme (Marcuson, 1981; Hynes-Griffin and Franklin 1984). In most of them $k_{h,eq}$ is assumed to be a portion η_h of the horizontal peak ground acceleration coefficient $k_{h,max}$ while $k_{v,eq}$ is assumed to be a portion Ω of $k_{h,eq}$:

$$k_{h,eq} = \eta_h \cdot k_{h,max} \qquad\qquad k_{v,eq} = \Omega \cdot k_{h,eq} \tag{1}$$

A value of $\eta_h = 0.5$, suggested by Hynes-Griffin and Franklin (1984), is often considered as appropriate for most slope schemes and is assumed as a reference in many seismic codes, such as Eurocode EC8 (EN 1998-1, 2003); for $k_{v,eq}$ a few indications are available in the literature and values of Ω in the range ± 0.33 to ± 0.5 are usually adopted (Marcuson, 1981). Therefore, the only parameter describing the earthquake motion is the peak value of the horizontal acceleration coefficient $k_{h,max}$, the coefficient $k_{h,eq}$ being defined independently from the slope characteristics. Nevertheless, it can be anticipated that earthquake effects on a given slope are also related to its characteristics.

In the last decade, more advanced criteria have been proposed for selecting appropriate values of $k_{h,eq}$, based on a weighting procedure of 1D or 2D seismic response analysis results. Using these procedures, the horizontal seismic coefficient can be related to earthquake characteristics (magnitude, source distance, predominant period), slope seismic response (fundamental period) and earthquake effects on the slope (e.g.: Bray and Rathje, 1998; Stewart *et al.*, 2003; Melo and Sharma, 2004). Specifically, the earthquake effects are related to the permanent displacement of the potential sliding mass developed during the seismic shaking. Appropriate values of limit or threshold displacement can then be used for evaluating seismic slope stability.

The dependence of $k_{h,eq}$ on the earthquake characteristics is demonstrated in Figures 1(a) – (b), in which the acceleration time histories of 1994 Northridge (M_w=6.7) and 1979 Imperial Valley (M_w=6.5) earthquakes, both characterized by about the same value of $k_{h,max} = a_{max}/g \approx 0.36$, are shown. According to current procedures for evaluating the equivalent seismic coefficient, the same value of $k_{h,eq}$ is obtained for the two cases; a value of $k_{h,eq} = 0.5 \cdot k_{h,max}$ is plotted in the Figures as an example. However, different duration, frequency content and amplitude of the accelerograms result in different effects on a given slope. In fact, assuming a critical seismic coefficient $k_{h,c} = 0.085$, the displacement-based analysis shows significant differences in the slope response (Figure 2). If both the equivalent seismic coefficient $k_{h,eq}$ and permanent displacements are assumed to represent the earthquake effects on the slope, different values of $k_{h,eq}$ should be adopted in the two cases.

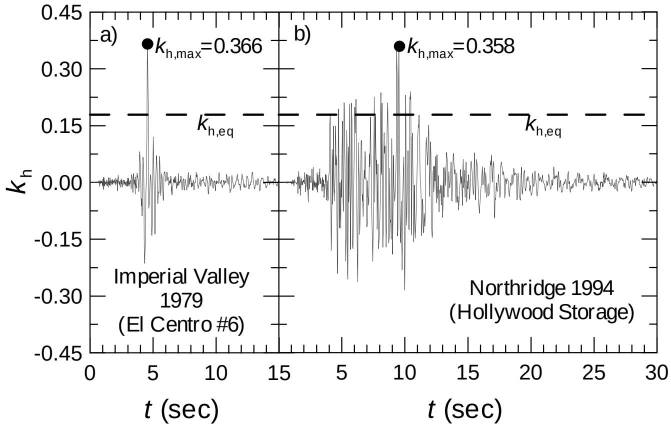

Figure 1. Dependence of $k_{h,eq}$ on earthquake effects

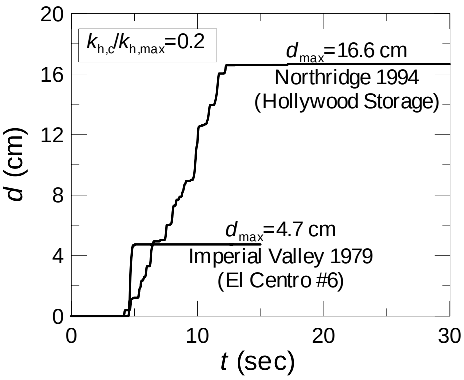

Figure 2. Influence of earthquake characteristics on slope displacements

In general, comparing the values of $k_{h,max}$, $k_{h,eq}$ and $k_{h,c}$, four different cases can occur. If the earthquake and slope characteristics results in the condition $k_{h,eq} = k_{h,c}$, the pseudo-static factor of safety is $F_{psd} = 1$ by definition of slope critical acceleration. In this case, since it is $k_{h,max} > k_{h,eq} = k_{h,c}$ permanent displacements will occur (Figure 3a) and evaluation of slope stability requires a comparison of computed displacements with threshold values. If $k_{h,eq} < k_{h,c}$, the safety factor F_{psd} is greater than unity, equilibrium being satisfied. Two different conditions may occur in this case: a) if $k_{h,c} < k_{h,max}$ (Figure 3b) permanent displacements will occur eventually leading the slope to an ultimate or a serviceability limit state despite $F_{psd} > 1$; b) if $k_{h,c} > k_{h,max}$ (Figure 3c) permanent displacements do not occur, the slope not experiencing any limit state consistently with the condition $F_{psd} > 1$. Finally, if $k_{h,c} < k_{h,eq} < k_{h,max}$ (Figures 3d), the safety factor is $F_{psd} < 1$ and permanent displacements will occur because $k_{h,c} < k_{h,max}$; however earthquake-induced displacement may not overcome threshold values, stability conditions for the slope being satisfied despite the pseudo-static equilibrium is not verified.

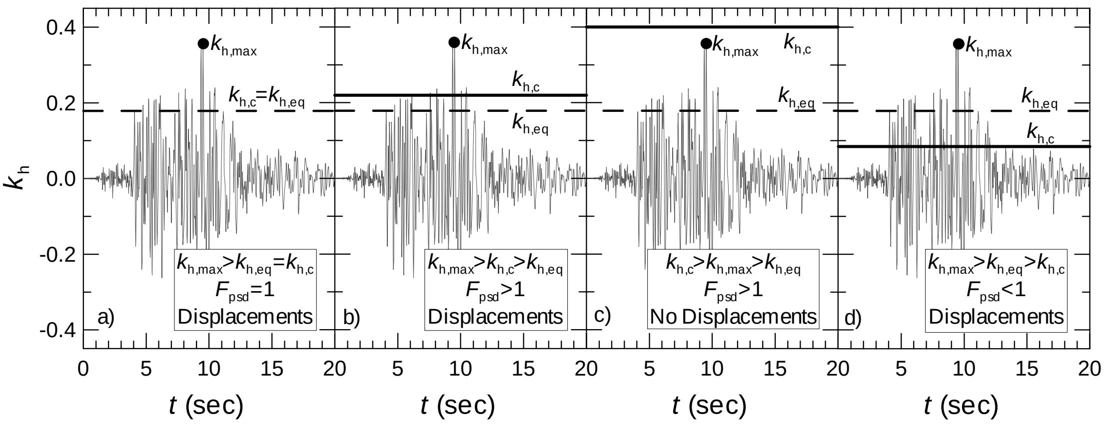

Figure 3. Pseudo-static versus displacement analysis

Therefore, a proper selection of the equivalent seismic coefficient $k_{h,eq}$ should take into account earthquake characteristics, threshold or limit values of earthquake-induced displacements d_{lim} and the ratio $k_{h,c}/k_{h,max}$ which represents a start out parameter for evaluating the occurrence of permanent displacement in the sliding block analysis.

A procedure for defining $k_{h,eq}$ as a function of $k_{h,c}/k_{h,max}$ and d_{lim} is proposed in the next paragraphs as follows: first the slope factor of safety is defined as the ratio of the limit to the earthquake-induced displacements; then an equivalence is proposed between the pseudo-static and the displacement-based analyses by introducing a safety factor defined as the ratio of appropriate seismic coefficients. The procedure is applied to the simple scheme of infinite slope using the notation summarised in Appendix A and in Figure A1.

ADOPTED RELATIONSHIPS FOR SIMPLIFIED DISPLACEMENT EVALUATION

In the displacement-based approach, the earthquake-induced displacement is calculated by integrating twice the acceleration time history with the critical acceleration used as the reference datum; more specifically, numerical integration involves those portion of the accelerogram in which the velocity of the sliding mass relative to the firm ground is positive. Therefore, the maximum computed displacement d_{max} depends on the slope characteristics, through the critical acceleration, and on the accelerogram characteristics (e.g.: strong motion duration, amplitude and frequency content). The permanent displacement of the sliding mass may be expressed as the displacement occurring on a horizontal surface multiplied by a slope factor which is function of the geometry of the failure surface and of the angle of shearing resistance ϕ'; values of slope factor have been shown to be often lower than 1.2 (Madiai and Vannucchi, 1997).

Alternatively, empirical relationships may be used for simplified evaluation of earthquake induced displacements. These are obtained from best-fit regression analyses of permanent displacements computed using a given set of accelerograms and assuming the relative motion to occur on a horizontal sliding plane. In this way, a unit slope factor is assumed in the computations without introducing significant errors. The reliability of the empirical relationships mainly depends on the accelerogram database adopted for computations and on the seismic parameters selected for representing the seismic shaking.

In the following, reference is made to the empirical relationships derived by Rampello *et al.* (2006) using a database of Italian earthquake records (Lanzo, 2006). The database consists of 214 EW and NS free-field accelerograms of 47 earthquakes occurred in Italy with magnitude of 4.0 to 6.9. The main characteristics of the database are summarised in Table 1, while a detailed description of it can be found in Scasserra *et al.* (2006). The accelerograms were grouped in three sets according to the lithology of the recording sites, classified as 'rock' for estimated values of shear wave velocity $V_s > 800$ m/s, 'stiff' for $360 < V_s \leq 800$ m/s and 'soft' for $V_s \leq 360$ m/s. The accelerograms were scaled to values of the maximum horizontal acceleration $a_g = 0.35g$, $0.25g$, $0.15g$ and $0.05g$ using scaling factors in the range 0.5÷2.

Table 1. Main characteristics of the database of Italian earthquake records

Soil type	Rock (V_s>800 m/s)	Stiff (360 ms<V_s≤800 m/s)	Soft (V_s≤360 m/s)	all
Number of records	74	98	42	214
Magnitude M_w	4.2 - 6.9	4.5 - 6.9	4.2 - 6.0	4.2 - 6.9
Magnitude M_s	3.5 - 6.9	3.4 - 6.9	3.5 - 6.1	3.4 - 6.9
Magnitude M_L	4.3 - 6.6	4.0 - 6.6	4.3 - 6.2	4.0 - 6.6
Focal depth (km)	3-16	2-24	2-15	2-24
Epicentral Distance (km)	5 - 80	2 - 71	7 - 48	1 – 87
PGA (g)	0.033 - 0.383	0.031 - 0.405	0.039 - 0.388	0.031 - 0.405
PGV (cm/s)	0.848 - 71.92	0.721 - 22.56	1.339 - 21.695	0.721 - 71.92
PGD (cm)	0.036 - 29.31	0.027 - 5.602	0.082 - 6.521	0.027 - 29.31
Arias Intensity (m/s)	0.005 - 1.434	0.006 - 1.326	0.005 - 0.347	0.005 - 1.434
Predominant period (sec)	0.04 - 1.10	0.04 - 1.26	0.06 - 0.66	0.04 - 1.26
Mean period (sec)	0.11 - 0.16	0.119 - 0.889	0.143 - 1.234	0.111 - 1.234
Bracketed duration (sec)	3.96 - 75.58	4.175 - 83.31	3.6 - 51.7	3.60 - 83.31

For each set of accelerograms and for each acceleration level, the permanent displacements of a rigid block sliding on a horizontal plane were computed for values of the ratio $k_{h,c}/k_{h,max}$ in the range 0.1 to 0.8. Linear best fit regression lines were then obtained in the semi-logarithmic plane $k_{h,c}/k_{h,max}$ – log d, in the form:

$$log\, d_{max} = C_1 \cdot (S \cdot S_T) - C_2 \cdot \frac{k_{h,c}}{k_{h,max}} \tag{2}$$

where C_1 and C_2 are the regression coefficients and S and S_T are the soil profile and the topographic amplification factors. Table 2 lists the mean values of C_1 and C_2 and their upper bound values, obtained for a non exceeding probability of 90% assuming a Beta distribution of the computed displacements around the mean value. Figure 4 shows the upper bound curves for values of $S = S_T = 1$.

Table 2. Regression coefficients of the displacement model

		Mean value		Upper Bound	
Soil type	**a_g (g)**	**C1 (cm)**	**C2**	**C1 (cm)**	**C2**
Rock (V_s>800 m/s)	0,35	1,322	3,257	2,107	3,257
	0,25	1,255	3,222	2,134	3,222
	0,15	1,000	3,249	1,839	3,249
	0,05	0,699	3,418	1,462	3,418
Stiff (360 m/s<V_s≤800 m/s)	0,35	1,505	3,431	2,021	3,431
	0,25	1,415	3,383	2,041	3,383
	0,15	1,255	3,414	1,857	3,414
	0,05	0,778	3,414	1,491	3,414
Soft (V_s≤360 m/s)	0,35	1,255	3,214	1,763	3,214
	0,25	1,230	3,275	1,778	3,275
	0,15	1,301	3,496	1,954	3,496
	0,05	0,845	3,505	1,663	3,505

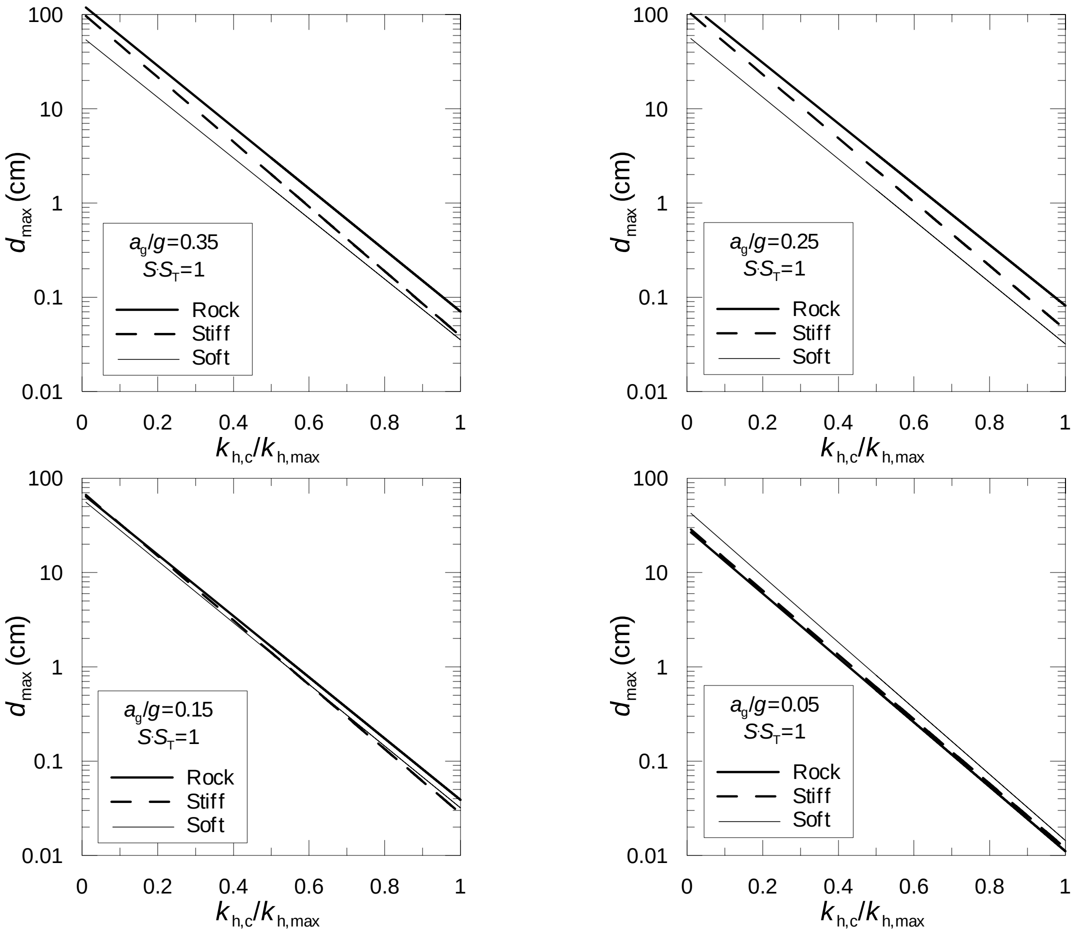

Figure 4. Empirical relationships for displacements (Rampello *et al.*, 2006)

Using equation (2) or curves in Figure 4, the portion η_h of the maximum design acceleration a_{max} can be evaluated assuming $k_{h,c} = k_{h,eq}$, once threshold values of displacement are selected for the slope (Rampello *et al.*, 2006). This implies that the pseudo-static analysis will be satisfied if the critical seismic coefficient for the slope at hand will be greater than the obtained value of $k_{h,c} = \eta_h \cdot k_{h,max}$. In the following, an alternative procedure is proposed, with reference to an infinite slope, in which the pseudo-static approach and the displacement-based method yield the same value of the safety factor.

PROCEDURE OUTLINE

Safety factor in terms of permanent displacements ratio

The sliding block analysis can be used for evaluating seismic slope stability by comparing the earthquake-induced displacement d_{max} with a threshold or limit value d_{lim}; a safety factor can then be defined in the form:

$$F_d = d_{lim} / d_{max} \tag{3}$$

Values of $F_d \geq 1$ means earthquake-induced displacements lower than the limit value corresponding to an ultimate or serviceability limit state for the slope, thus ensuring slope stability. Two limit cases can be distinguished in which the safety factor F_d is equal either to zero or to an infinite value:

- if no slope displacement is allowed ($d_{lim} = 0$), whatever the induced displacements d_{max}, the safety factor is $F_d = 0$ not allowing safety against a given limit state to be evaluated;
- if the design earthquake does not induce any permanent displacement in the slope ($d_{max} = 0$), $F_d = \infty$ does not provide any definite measure of slope stability.

Slope sustainable acceleration

Using an empirical relationship between d_{max} and the ratio $k_{h,c}/k_{h,max}$, a limit value of the seismic coefficient $k_{h,lim}$ can be associated to a given limit value of displacement d_{lim}. For example, with reference to equation (2), it is:

$$\frac{k_{h,lim}}{k_{h,c}} = \frac{C_2}{C_1 \cdot (S \cdot S_T) - \log d_{lim}} \tag{4}$$

Values of k_h lower than the limit seismic coefficient $k_{h,lim}$ yield earthquake-induced displacements smaller than d_{lim}, the adopted limit state not being achieved. If the slope cannot sustain any permanent displacement ($d_{lim} = 0$), the maximum horizontal acceleration tolerable by the slope coincides with the critical acceleration ($k_{h,lim} = k_{h,c}$). For values of $d_{lim} \neq 0$, the limit seismic coefficient $k_{h,lim}$ can be thought of as a generalised value of the critical acceleration related to a given limit displacement d_{lim}. It is worth noting that the condition $k_{h,lim} = k_{h,c}$ for $d_{lim} = 0$ cannot be obtained using most of the empirical relationships available in the literature due to their logarithmic formulation.

In Figure 5, the ratio of limit to critical seismic coefficients, $k_{h,lim}/k_{h,c}$, is plotted against d_{lim} for each soil type and acceleration level, using eqn. (4) and the upper bound values of coefficients C_1 and C_2. Figure 6(a) shows a comparison between different empirical relationships developed referring to Italian earthquakes only. Specifically, the curves were obtained using the relationships proposed by Rampello *et al.* (2006) for ‘rock’ ($S = 1$ and $S_T = 1.4$), stiff soil ($S = 1.25$ and $S_T = 1.2$) and soft soil ($S = 1.35$ and $S_T = 1$) for $a_g/g = 0.35$, by Simonelli and Viggiani (1995) and by Romeo (2000). A synthetic description of the regression models proposed by Simonelli and Viggiani (1995) and by Romeo (2000) is provided in Table 3 together with the expression fot $k_{h,lim}/k_{h,c}$. From the curves in Figure 6(a) it is apparent that the choice of the empirical relationships adopted for displacement computations represents a crucial step in the analysis.

Table 3. Displacement regression models developed referring to Italian earthquakes

Regression model $\log d_{max} = C_1 - C_2 \cdot \frac{k_{h,c}}{k_{h,max}} + C_3 \cdot \log I_a \pm \sigma_{\log d}$					Sustainable acceleration $\frac{k_{h,lim}}{k_{h,c}} = \frac{C_2}{C_1 - \log d_{lim} + C_3 \cdot \log I_a \pm \sigma_{\log d}}$
Reference	**C_1 (cm)**	**C_2**	**C_3**	**$\sigma_{\log d}$**	**Characteristics of the database of earthquake records**
Simonelli & Viggiani (1995)	2.652	3.333	-	-	10 records of the Irpinia 1980 (M=6.9) earthquake, $k_{h,max}$=0.107-0.326 epicentral distance = 27-137 km, Arias intensity I_a = 26.6 – 147.9 cm/s
Romeo (2000)	0.852	3.719	0.607	0.365	190 records of Italian earthquakes since 1972 (M=4.6-6.8) $k_{H,max}$=0.008-0.34, Arias intensity I_a = 0.15 – 105 cm/s

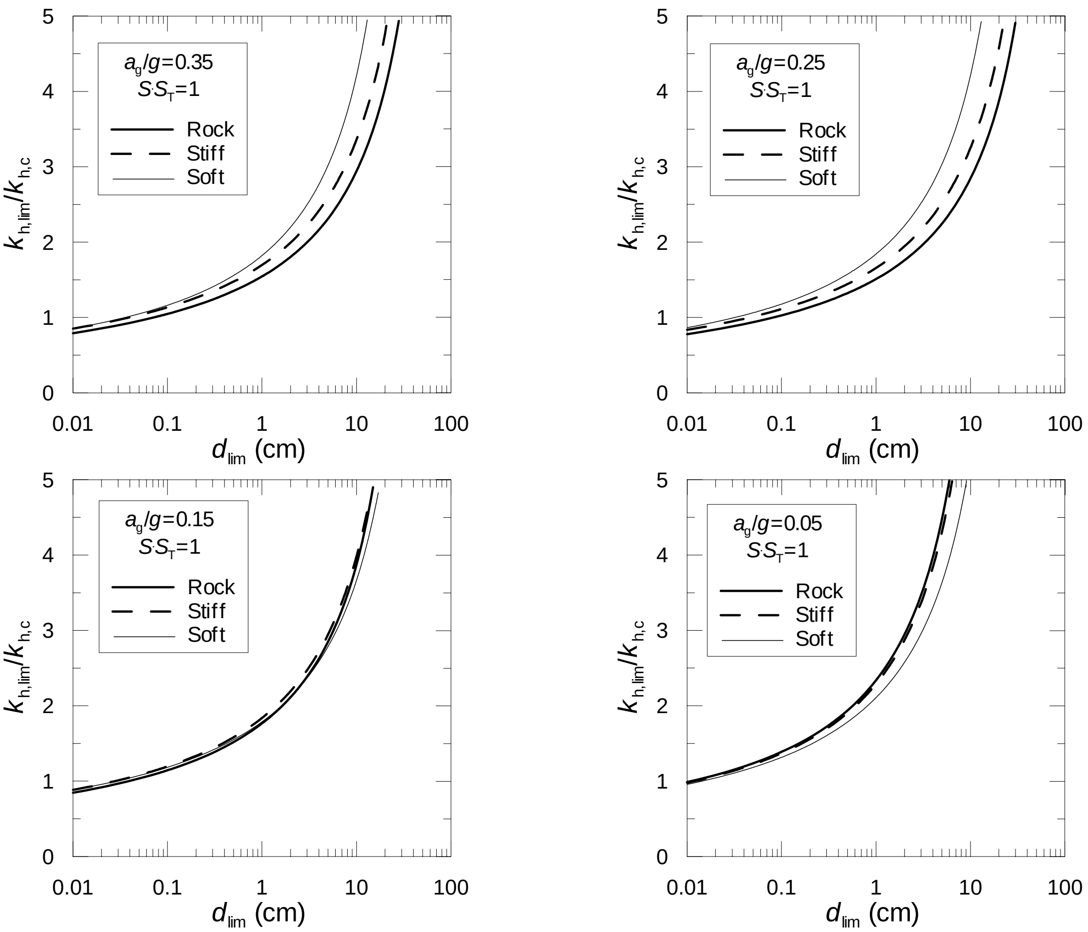

Figure 5. Sustainable horizontal seismic coefficient computed using eqn. (4).

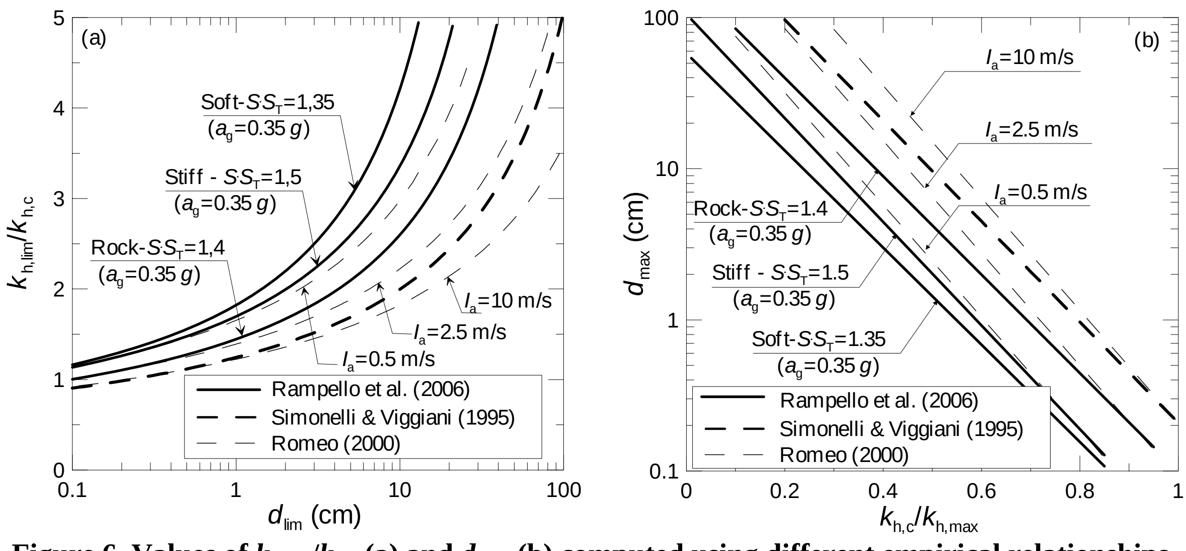

Figure 6. Values of $k_{h,lim}/k_{h,c}$ (a) and d_{max} (b) computed using different empirical relationships.

Safety factor in terms of accelerations ratio

The slope stability can be evaluated using a factor of safety F_k that is defined as the ratio between the limit seismic coefficient $k_{h,lim}$ that can be sustained by the slope without attaining any limit state and the maximum seismic coefficient $k_{h,max}$ to which it is subjected under earthquake loading conditions:

$$F_k = \frac{k_{h,lim}}{k_{h,max}} = \frac{k_{h,lim}}{(a_g/g) \cdot S \cdot S_T} \tag{5}$$

Since $k_{h,lim}$ and $k_{h,max}$ are related to the limit and to the earthquake-induced displacements, d_{lim} and d_{max}, respectively, the safety factor F_k evaluates safety against a limit state using an acceleration ratio rather than a displacements ratio. Values of $F_k > 1$ indicate slope stability; in fact, if $k_{h,him} > k_{h,max}$, the earthquake-induced displacement d_{max} will be lower than the tolerable displacement d_{lim}.

Differently from F_d, the safety factor F_k always provides a finite measure of slope stability:

- if no slope displacement is allowed ($d_{lim} = 0$), it is $k_{h,lim} = k_{h,c}$ and F_k, reducing to the ratio $k_{h,c}/k_{h,max}$, yields a finite measure of slope stability. In this case, the definition of F_k is consistent with the use of the critical acceleration as a safe design parameter for earth dams (Sarma and Bhave, 1974), and the use of the ratio $k_{h,c}/k_{h,max}$ as a safety factor, in place of the conventional pseudo-static factor F_{psd} (Baker *et al.*, 2006);
- if the selected earthquake does not induce any permanent displacement in the slope ($d_{max} = 0$), $k_{h,max}$ is smaller than $k_{h,c}$ and F_k is always greater than $k_{h,lim}/k_{h,c}$ thus providing a finite measure of safety against a limit state for the slope.

Convenience in using F_k rather than the pseudo-static safety factor F_{psd} is shown in Figures 7(a) – (b). For an infinite slope inclined of an angle $\beta = 25°$, with effective strength parameters $c' = 0$ and $\phi' = 30°$ and zero pore water pressure ($r_u = 0$, $\Delta u^* = 0$), the static safety factor and the critical acceleration coefficient are F_S=1.238 and $k_{h,c} = 0.0875$, respectively, the latter computed assuming a horizontal inertia force ($\Omega = 0$). Figure 7(a) shows the curves of F_k versus $k_{h,max}$ obtained for different values of the limit displacement d_{lim} using eqns. 4 and 5 and assuming that the soil of the slope is classified as 'stiff', with $S = S_T = 1$, and the acceleration level is $a_g/g = 0.25$. In the same figure the pseudo-static safety factor F_{psd}, computed assuming $\eta_h = 0.5$ and $\Omega = 0$, is also plotted with a thick dashed line for comparison.

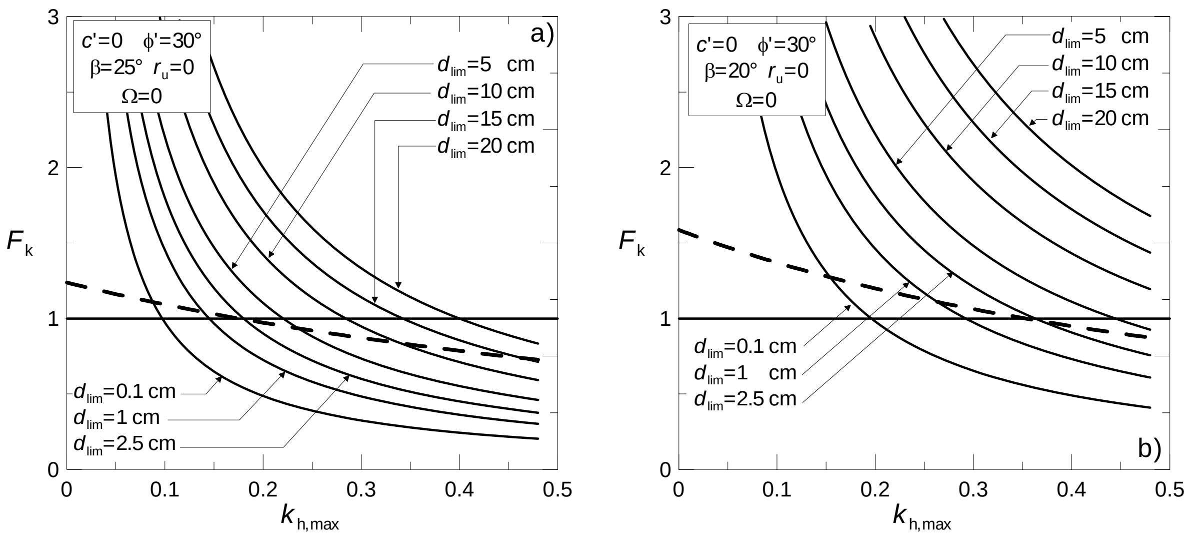

Figure 7. Comparison between F_{psd} and F_k

According to the pseudo-static approach, the slope is stable ($F_{psd} > 1$) if $k_{h,max}$ is smaller than 0.175. On the other hand, using F_k, the values of $k_{h,max}$ for which slope stability is ensured ($F_k > 1$) depend on the adopted values of the limit displacement d_{lim}, that is, on the permanent displacement which may be undergone by the slope without reaching a limit state. For example, for values of d_{lim} = 5, 10 or 15 cm the slope is stable for $k_{h,max}$ smaller than 0.22, 0.28 or 0.34, respectively, all of them being larger than the value obtained from the pseudo-static approach. Similarly, for a given value of $k_{h,max}$, different values of F_k are obtained depending on the adopted values of d_{lim}. For $k_{h,max} = 0.25$, eqn.2 provides d_{max} = 7.2 cm and F_k = 0.88, 1.14 and 1.37 for d_{lim} equal to 5, 10 and 15 cm, respectively, denoting that slope stability may be or may be not satisfied depending on the assumed limit displacement. Using the same value of $k_{h,max} = 0.25$, the pseudo-static approach yields $F_{psd} = 0.972$, denoting slope instability regardless the earthquake-induced displacement and the adopted limit displacement.

Similar results are shown in Figure 7(b) for a slope inclined of angle $\beta = 20°$, for which the static

safety factor and the critical acceleration coefficient are $F_S = 1.586$ and $k_{h,c} = 0.176$, respectively. Assuming $k_{h,max} = 0.25$ as above, eqn. (2) yields a maximum displacement $d_{max} = 0.45$ cm.
Again, different and not consistent evaluations of slope stability may be obtained using the safety factor F_k rather than F_{psd}. In fact, according to the pseudo-static approach slope stability is ensured with $F_{psd} = 1.127$. On the other hand, if the slope cannot sustain significant displacements, say for example $d_{lim} = 0.1$ cm, it is $F_k = 0.785$ and slope stability is not satisfied ($d_{max} > d_{Lim}$), while, if larger displacements are tolerated by the slope, say for example $d_{lim} = 1$ cm, a stable condition is predicted with F_k=1.17. Therefore, it is apparent that the agreement between the two approaches depends on the limit values of displacement assumed in the analysis.

Equivalence between pseudo-static and displacement-based analysis
Using F_k, an equivalence between the pseudo-static approach and the displacement-based analysis can be introduced by detecting the value of the equivalent seismic coefficient $k_{h,eq}$ for which the two approaches provide the same factor of safety ($F_k = F_{psd}$). Using eqn.(A3) given in the Appendix, the expression for $k_{h,eq}$ can be derived imposing the condition $F_{psd} = F_k$:

$$k_{h,eq} = \frac{c'/(\gamma \cdot D) + \cos\beta \cdot (1 - r_u) \cdot (1 - \Delta u^*) \cdot \tan\phi' - \dfrac{k_{h,lim}}{k_{h,max}} \cdot \sin\beta}{(\sin\beta \pm \Omega \cdot \cos\beta) \cdot \tan\phi' + \dfrac{k_{h,lim}}{k_{h,max}} \cdot (\cos\beta \mp \Omega \cdot \sin\beta)} \tag{6}$$

where the sign + is used if k_v points upwards.

Due to its definition, $k_{h,eq}$ satisfies the following conditions:
- $k_{h,eq} = k_{h,c}$ if $F_k = 1$ (due to the definition of $k_{h,c}$ and to the condition $F_k = F_{psd} =1$);
- $k_{h,eq} < k_{h,c}$ if $F_k > 1$ ($d_{lim} > d_{max}$);
- $k_{h,eq} > k_{h,c}$ if $F_k < 1$ ($d_{lim} < d_{max}$).

Equation (6) shows that the equivalent seismic coefficient depends on the parameter affecting static slope stability (c', ϕ', γ, D, β, r_u), on the effects induced by the earthquake ($k_{h,max}$,, Ω, Δu^*) and on the adopted limit displacement d_{lim} which is implicitly taken into account through $k_{h,lim}$ (eqn.4).
It is worth noting that:
- the expression for $k_{h,eq}$ (eqn.6) does not depend on the relationship adopted for simplified evaluation of earthquake-induced displacements;
- the relationship for displacement evaluation is only involved for computing $k_{h,lim}$ (eqn.4);
- if different empirical relationships for displacement evaluation are used, eventually incorporating the influence of strong motion duration and amplitude, frequency content and soil stiffness, a suitable expression for $k_{h,lim}$ should be derived, the effects of these factors being implicitly considered in the computation of $k_{h,eq}$ through eqn. (6).

Once the expression for $k_{h,eq}$ is known, the reducing factor η_h may be written in the form:

$$\eta_h = \frac{k_{h,eq}}{k_{h,max}} = \frac{c'/(\gamma \cdot D) + \cos\beta \cdot (1 - r_u) \cdot (1 - \Delta u^*) \cdot \tan\phi' - \dfrac{k_{h,lim}}{k_{h,max}} \cdot \sin\beta}{k_{h,max} \cdot (\sin\beta \pm \Omega \cdot \cos\beta) \cdot \tan\phi' + k_{h,lim} \cdot (\cos\beta \mp \Omega \cdot \sin\beta)} \tag{7}$$

The reducing factor η_h depends on the same quantities affecting $k_{h,eq}$.
In Figure 8, η_h is plotted against $k_{h,max}$ for the infinite slope with $\beta = 25°$ (Fig. 7a); the ratio $k_{h,c}/k_{h,max}$ and the value of $\eta_h = 0.5$ are also plotted in the figure with a thick line and a thin dashed line, respectively. For a given value of $k_{h,max}$ the reducing factor η_h clearly depends on the adopted limit displacement d_{lim}.
Specifically, assuming as above $k_{h,max} = 0.25$, the curves in Figure 8 yield values of $\eta_h = 0.58$ and 0.13

for d_{lim} = 5 and 10 cm respectively. Then, the greater is the value of d_{lim} that can be undergone by the slope, the lower is the factor η_h to be adopted in the pseudo-static analysis; as a consequence a single value of η_h = 0.5 can lead to erroneous evaluations of slope stability.

Recalling that for the slope at hand the maximum displacement computed by eqn. (2) was d_{max} = 7.2 cm, it is worth noting that:

- assuming d_{lim} = 5 cm, (η_h = 0.58) it is $F_d < 1$ ($d_{lim} < d_{max}$) and, consistently, $F_k < 1$, being $k_{h,eq} = \eta_h \cdot k_{h,max} = 0.145$ greater than $k_{h,c} = 0.0875$. In Figure 8, the point representing the condition $k_{h,max}$ = 0.25 and η_h = 0.58 is located above the thick line representing the ratio $k_{h,c}/k_{h,max}$;
- assuming d_{lim} = 10 cm (η_h = 0.13) it is $F_d > 1$ ($d_{lim} > d_{max}$) and, consistently, $F_k > 1$ ($k_{h,eq} < k_{h,c}$). In this case, the point representing the condition $k_{h,max}$ = 0.25 and η_h = 0.13 is located below the thick line representing the ratio $k_{h,c}/k_{h,max}$.

Then, the curve representing the ratio of $k_{h,c}/k_{h,max}$ divides Figure 8 in two zones: (i) in the zone above the curve, the equivalent seismic coefficient $k_{h,eq}$ is greater than the corresponding critical value $k_{h,c}$ and slope stability evaluated in terms of displacement ratio is not satisfied ($d_{lim} < d_{max}$, $F_d < 1$, $F_k < 1$); (ii) in the zone below the curve, $k_{h,eq}$ is smaller than $k_{h,c}$, and slope stability is ensured ($d_{lim} < d_{max}$, $F_d > 1$, $F_k > 1$).

The influence of the vertical component of seismic acceleration is shown in Figure 9 where, with reference to the infinite slope with angle β = 25°, the factor $\eta_h = k_{h,eq}/k_{h,max}$ and the ratio $k_{h,c}/k_{h,max}$ are plotted against $k_{h,max}$ for $\Omega = 0$ and $\Omega = \pm½$. The Figure shows that a vertical acceleration pointing upwards (positive values of Ω) leads to lower values of $k_{h,c}$ and to higher values of η_h representing the most conservative condition irrespective of the threshold or limit displacement adopted for the slope.

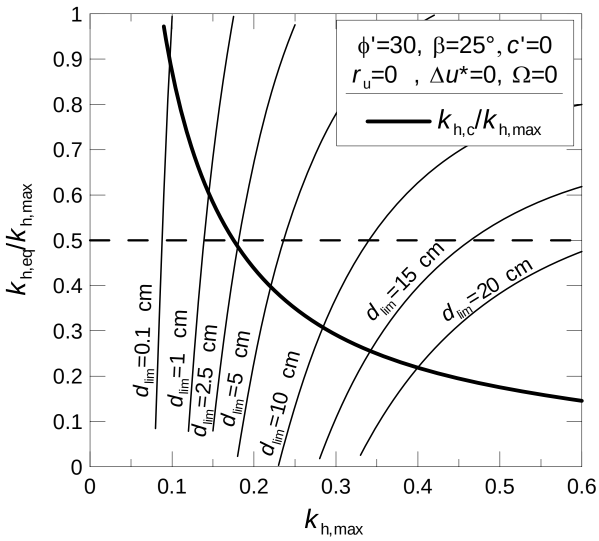

Figure 8. Dependence of η_H on d_{lim} and $k_{h,max}$

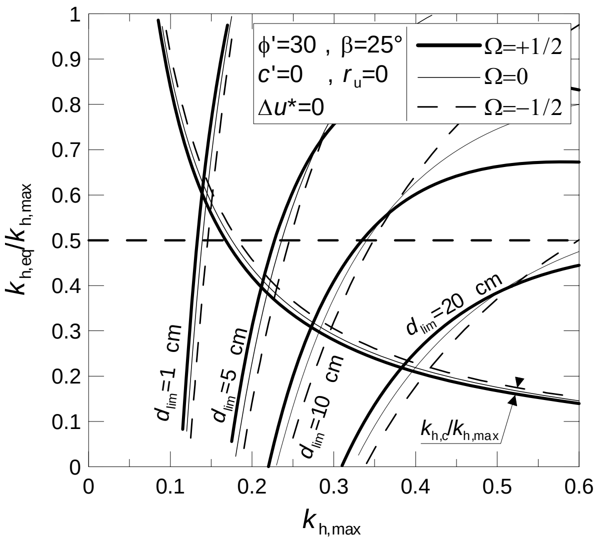

Figure 9. Influence of Ω on η_H

CONCLUDING REMARKS

A procedure for the evaluation of the equivalent seismic coefficient k_{eq} to be adopted in the pseudo-static analysis of slopes was presented with reference to the simple scheme of infinite slope. The proposed method relies on the assumption that the permanent displacement developed during the seismic event represents a proper quantity for assessing slope stability under earthquake loading conditions. It requires the use of a suitable relationship for simplified evaluation of earthquake-induced displacement d_{max}. In addition, limit values d_{lim} of tolerable displacements, related to ultimate and/or a serviceability limit states for the slope must to be defined.

For any given value of the limit displacement d_{lim}, a corresponding sustainable acceleration coefficient $k_{h,lim}$ can be computed by inverting the empirical relationship adopted for displacement evaluation; $k_{h,lim}$ represents the maximum acceleration coefficient which may be undergone by the slope without reaching a limit state. Slope stability condition can then be evaluated using a safety factor F_k, defined as the ratio between $k_{k,lim}$ and $k_{h,max}$ and incorporating the geometrical and mechanical characteristics of the slope and the earthquake characteristics. It has been demonstrated that the pseudo-static factors of safety F_{psd} and F_k may lead to different evaluation of slope stability, depending on the assumed values for the threshold or limit displacement.

However, the results of the pseudo-static analysis can be made to coincide with those of the displacement-based analysis by defining the equivalent seismic coefficient $k_{h,eq}$ as the value for which the condition $F_{psd} = F_k$ is attained. A solution was derived for the simple scheme of infinite slope, evaluating $k_{h,eq}$ as a function of the parameters affecting static slope stability, the maximum acceleration coefficient $k_{h,max}$ and the limit value of the permanent displacement d_{lim}. The expression of $k_{h,eq}$ can reflect the effect of as many seismic parameters as the displacement regression accounts for and can be used to plot stability charts that include the influence of static and earthquake-induced pore water pressures, as well as any inclination of the seismic acceleration.

APPENDIX A

Adopted notation for the infinite slope shown in Figure A1.

Notation

β: slope angle
H, D: failure surface depth, thickness of unstable soil
c': effective cohesion
ϕ': angle of shearing resistance
γ: soil unit weight
γ_w: water unit weight
r_u: pore pressure ratio = $(\gamma_w \cdot H_w)/(\gamma \cdot H)$
Δu^*: earthquake-induced pore pressure ratio $= \Delta u / [\gamma \cdot D \cdot cos\,\beta \cdot (1 - r_u)]$
k: seismic coefficient
k_h, k_v: horizontal and vertical component of k
$\Omega = k_v/k_h$; $D = H \cdot cos\beta$

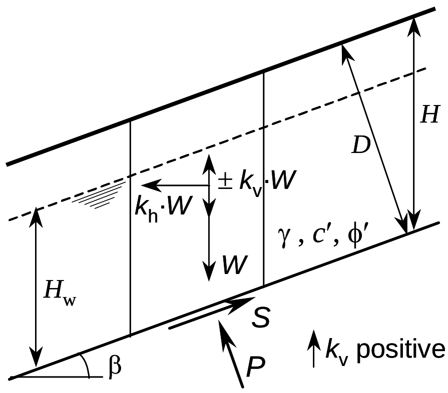

Figure A1. Scheme of infinite slope

Static safety factor:

$$F_s = \frac{c'/(\gamma \cdot D)}{sin\,\beta} + \frac{tan\,\phi'}{tan\,\beta} \cdot (1 - r_u) \tag{A1}$$

Pseudo-static safety factor:

$$F_{psd} = \frac{c'/(\gamma \cdot D) + \left[cos\,\beta \cdot (1 - r_u) \cdot (1 - \Delta u^*) - k_h \cdot (sin\,\beta \mp \Omega \cdot cos\,\beta)\right] \cdot tan\,\phi'}{sin\,\beta + k_h \cdot (cos\,\beta \pm \Omega \cdot sin\,\beta)} \tag{A2}$$

Horizontal critical acceleration coefficient:

$$k_{h,c} = \frac{c'/(\gamma \cdot D) + cos\,\beta \cdot (1 - r_u) \cdot (1 - \Delta u^*) \cdot tan\,\phi' - sin\,\beta}{(sin\,\beta \mp \Omega \cdot cos\,\beta) \cdot tan\,\phi' + (cos\,\beta \pm \Omega \cdot sin\,\beta)} \tag{A3}$$

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